Module 4

Upper Primary Mathematics

Social Arithmetic

THE COMMONWEALTH of LEARNING

Science, Technology and Mathematics Modules

for Upper Primary and Junior Secondary School Teachers of Science, Technology and Mathematics by Distance in the Southern African Development Community (SADC)

Developed by
The Southern African Development Community (SADC)

Ministries of Education in:

- **Botswana**
- **Malawi**
- **Mozambique**
- **Namibia**
- **South Africa**
- **Tanzania**
- **Zambia**
- **Zimbabwe**

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SCIENCE, TECHNOLOGY, AND MATHEMATICS MODULES

This module is one of a series prepared under the auspices of the participating Southern African Development Community (SADC) and The Commonwealth of Learning as part of the Training of Upper Primary and Junior Secondary Science, Technology and Mathematics Teachers in Africa by Distance. These modules enable teachers to enhance their professional skills through distance and open learning. Many individuals and groups have been involved in writing and producing these modules. We trust that they will benefit not only the teachers who use them, but also, ultimately, their students and the communities and nations in which they live.

The twenty-eight Science, Technology, and Mathematics modules are as follows:

Upper Primary Science
Module 1: *My Built Environment*
Module 2: *Materials in my Environment*
Module 3: *My Health*
Module 4: *My Natural Environment*

Junior Secondary Science
Module 1: *Energy and Energy Transfer*
Module 2: *Energy Use in Electronic Communication*
Module 3: *Living Organisms' Environment and Resources*
Module 4: *Scientific Processes*

Upper Primary Technology
Module 1: *Teaching Technology in the Primary School*
Module 2: *Making Things Move*
Module 3: *Structures*
Module 4: *Materials*
Module 5: *Processing*

Junior Secondary Technology
Module 1: *Introduction to Teaching Technology*
Module 2: *Systems and Controls*
Module 3: *Tools and Materials*
Module 4: *Structures*

Upper Primary Mathematics
Module 1: *Number and Numeration*
Module 2: *Fractions*
Module 3: *Measures*
Module 4: *Social Arithmetic*
Module 5: *Geometry*

Junior Secondary Mathematics
Module 1: *Number Systems*
Module 2: *Number Operations*
Module 3: *Shapes and Sizes*
Module 4: *Algebraic Processes*
Module 5: *Solving Equations*
Module 6: *Data Handling*

A MESSAGE FROM THE COMMONWEALTH OF LEARNING

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Dato' Professor Gajaraj Dhanarajan
President and Chief Executive Officer

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UPPER PRIMARY MATHEMATICS PROGRAMME

Introduction

Welcome to the programme in Teaching Upper Primary Mathematics. This series of five modules is designed to help you strengthen your knowledge of mathematics topics and acquire more instructional strategies for teaching mathematics in the classroom.

Each of the five modules in the mathematics series provides an opportunity to apply theory to practice. Learning about mathematics entails the development of practical skills as well as theoretical knowledge. Each topic includes examples of how mathematics is used in practice and suggestions for classroom activities that allow students to explore the maths for themselves.

Each module also explores several instructional strategies that can be used in the mathematics classroom and provides you with an opportunity to apply these strategies in practical classroom activities. Each module examines the reasons for using a particular strategy in the classroom and provides a guide for the best use of each strategy, given the topic, context, and goals.

The guiding principles of these modules are to help make the connection between theory and practice, to apply instructional theory to practice in the classroom situation, and to support you, as you, in turn, help your students to apply mathematics to practical classroom work.

Programme Goals

This programme is designed to help you:

- strengthen your understanding of mathematics topics
- expand the range of instructional strategies that you can use in the mathematics classroom

Programme Objectives

By the time you have completed this programme, you should be able to:

- guide students as they work in teams on practical projects in mathematics, and help them to work effectively as a members of a group
- use questioning and explanation strategies to help students learn new concepts and to support students in their problem solving activities
- prepare your own portfolio of teaching activities

The relationship between this programme and the mathematics curriculum

The content presented in these modules includes some of the topics most commonly covered in the mathematics curricula in southern African countries. However, it is not intended to comprehensively cover all topics in any one country's mathematics curriculum. For this, you need to consult your national or regional curriculum guide. The curriculum content presented in these modules is intended to:

- provide an overview of the content in order to support the development of appropriate teaching strategies
- use selected parts of the curriculum as examples of the application of specific teaching strategies
- explain those elements of the curriculum that provide essential background knowledge, or that address particularly complex or specialised concepts
- provide directions to additional resources on the curriculum content

How to work on this programme

As is indicated in the goals and objectives, this programme requires you to participate actively in each module by applying instructional strategies when exploring mathematics with your students and by reflecting on that experience. There are several ways to do this.

Working on your own

You may be the only teacher of mathematics in your school, or you may choose to work on your own so you can accommodate this programme within your schedule. If this is the case, these are the recommended strategies for using this module:

1. Establish a schedule for working on the module. Choose a date by which you plan to complete the first module, taking into account that each unit will require between six and eight hours of study time and about two hours of classroom time to implement your lesson plan. For example, if you have two hours a week available for study, then each unit will take between three and four weeks to complete. If you have four hours a week for study, then each unit will take about two weeks to complete.
2. Choose a study space where you can work quietly without interruption, such as a space in your school where you can work after hours.
3. If possible, identify someone who is interested in mathematics or whose interests are relevant to it (for example, a science teacher in your school) with whom you can discuss the module and some of your ideas about teaching mathematics. Even the most independent learner benefits from good dialogue with others. It helps us to formulate our ideas—or as one learner commented, "How do I know what I'm thinking until I hear what I have to say?"

Working with colleagues

If there are other teachers of mathematics in your school or in your immediate area, then it may be possible for you to work together on this module. You may choose to do this informally, perhaps having a discussion group once a week or once every two weeks about a particular topic in one of the units. Or, you may choose to organise more formally, establishing a schedule so that everyone is working on the same units at the same time, and you can work in small groups or pairs on particular projects.

Your group may also have the opportunity to consult with a mentor, or with other groups, by teleconference, audioconference, letter mail, or e-mail. Check with the local coordinator of your programme about these possibilities so you can arrange a group schedule that is compatible with these provisions.

Colleagues as feedback/resource persons

Even if your colleagues are not participating directly in this programme, they may be interested in hearing about it and about some of your ideas as a result of taking part. Your head teacher or the local area specialist in mathematics may also be willing to take part in discussions with you about the programme.

Working with a mentor

As mentioned above, you may have the opportunity to work with a mentor, someone with expertise in maths education who can provide feedback about your work. If you are working on your own, communication with your mentor may be by letter mail, telephone, or e-mail. If you are working as a group, you may have occasional group meetings, teleconferences, or audioconferences with your mentor.

Resources available to you

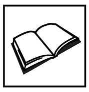
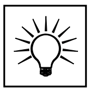
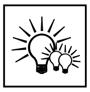
Although these modules can be completed without referring to additional resource materials, your experience and that of your students can be enriched if you use other resources as well. A list of resource materials is provided at the end of each module. You might also find locally available resource material that will enhance the teaching/learning experience. These include:

- manipulatives, such as algebra tiles, geometry tiles, and fraction tiles
- magazines with articles about maths
- books and other resources about maths that are in your school or community library

ICONS

Throughout each module, you will find some or all of the following icons or symbols that alert you to a change in activity within the module.

Read the following explanations to discover what each icon prompts you to do.

	**Introduction**	Rationale or overview for this part of the course.
	**Learning Objectives**	What you should be able to do after completing this module or unit.
	**Text or Reading Material**	Course content for you to study.
	**Important—Take Note!**	Something to study carefully.
	**Self-Marking Exercise**	An exercise to demonstrate your own grasp of the content.
	**Individual Activity**	An exercise or project for you to try by yourself and demonstrate your own grasp of the content.
	**Classroom Activity**	An exercise or project for you to do with or assign to your students.
	**Reflection**	A question or project for yourself—for deeper understanding of this concept, or of your use of it when teaching.
	**Summary**	
	**Unit or Module Assignment**	Exercise to assess your understanding of all the unit or module topics.
	**Suggested Answers to Activities**	

CONTENTS

Module 4: Social Arithmetic

Module 4
Social Arithmetic

Introduction to the Module

The term **Social Arithmetic** was chosen as the title for this module to emphasise the important role mathematics plays in the everyday lives of people. Although it can be difficult to distinguish between what is social mathematics and what is not, this module looks at topics in the primary mathematics curriculum that we meet in everyday life.

What is Social Arithmetic?

In Unit 6 of Module 3, we discussed currency as a measure of money. In this module, we will discuss the social aspects of mathematics involving money, including how it is earned and how it is spent. In addition to this, we will examine how governments earn money.

Statistics is an important topic in mathematics and is often applied to the study of society. Therefore, this module also looks at statistics as part of social mathematics.

Aim of the Module

Social mathematics involves the basic operations of addition, subtraction, multiplication, and division, yet many teachers find it an uncomfortable topic to teach. This module explores practical approaches to teaching social arithmetic, using activities that will make the topic more interesting and enjoyable for primary pupils.

Structure of the Module

Children learn about many aspects of their social lives in subjects such as Social Studies. Mathematics also provides opportunities for children to learn about life around them. In Unit 1 of this module, you will learn about ways to earn money, and how to work out calculations associated with earning money. Since people earn money in order to spend it for the betterment of their lives, Unit 2 discusses some of the ways money is spent. In Unit 3, you will learn how governments earn money.

Unit 4 looks at transportation in the context of social mathematics. Units 5 and 6 cover statistics, including methods to help children learn by collecting information from their environment. Transportation is part of the primary school curriculum because it is an important aspect of our everyday lives.

The unit activities in this module are aimed at helping you consolidate your content knowledge. There are practice activities for you to try with your pupils and self assessment exercises to help you test your mastery of the subject content. Answers to the self assessment exercises are given at the end of each unit.

Enjoy working through this module!

Objectives of the Module

After completing this module, you should be able to:

- state different ways of earning money
- carry out calculations involving money
- make simple household budgets
- interpret water and electricity bills
- explain the advantages and disadvantages of hire purchase
- outline various ways government earns money
- carry out calculations involving household bills and taxes
- use a problem-solving approach to teaching about transportation in mathematics
- represent data in pictographs, bar graphs, pie charts, and line graphs
- teach upper primary pupils the aspects of social arithmetic covered in this unit

Unit 1: Earning Money

Introduction

In Module 3, Unit 6 we discussed currency as a measure of money. In this unit we will discuss how to earn this money. Money is needed in many, if not ‘all’ situations of our everyday life. It is so important that even small children seem to ‘understand’ it at a very tender age.

Objectives

After working through this unit you should be able to:

- explain the following as ways of earning money:
  - selling
  - commission
  - salary
  - bank interest
  - royalties
  - services, as ways of earning money
- carry out calculations involving simple interest, commission, and royalties
- teach your upper primary pupils about the aspects of earning money that have been covered in this unit

Reflection

Have you ever been in a situation where you had money, but you could not use it? If not, do you know someone who has been in this situation? How did they feel about having money, but finding it was worthless? What is wrong about money in this instance?

Some Ways of Earning Money

Money can be earned in many different ways. Some of these are:

- business
- commission
- salary
- bank interest
- loan
- royalties

Business

There are many kinds of business, either selling goods or providing services. A shopkeeper buys goods at a lower price and sells them at a higher price. The difference between buying price and selling price is called **profit**. The profit is the money earned by the shopkeeper.

Commission

Commission is money paid to a sales person as a payment for selling goods on behalf of the business owner, and is calculated as a percentage of the value of goods sold. Commission can also be given to an employee for selling goods beyond the required number of sales.

Example 1

Mrs. Shamapango is paid a salary of K300 000 per month and an additional 3% commission on sales. In one month, she sold goods worth K960 000. What were her total earnings for that month?

$$\text{Total earnings} = \text{Salary} + \frac{3}{100} \times 960\,000$$
$$= \text{K}300\,000 + \text{K}28\,800$$
$$= \mathbf{K328\,800}$$

Quantity commission is a commission based on the amount of goods sold. For example, newspaper vendors are paid quantity commission.

Example 2

If a newspaper vendor receives a commission of K350 on every newspaper they sell, how much commission would they be paid after selling 150 newspapers?

Commission on 1 newspaper	= K350
Commission on 150 newspapers	= 150 × K350
	= **K52 500**

Salary

This is the most common way to earn money. This is calculated per year, and paid to the employee at the end of every month as $\frac{1}{12}$ of the annual salary.

Simple Interest

People keep their money in the bank for safe-keeping. Money kept in the bank is not only safe, but it also earns more money through interest. There are two types of bank interest—simple interest and compound interest. In this unit, we will discuss **simple interest**.

Simple interest is calculated by using the formula:

$$SI = \frac{P \times T \times R}{100}$$

where SI stands for Simple Interest (the bonus the bank gives you for using your money while they keep it), P is the Principal (the amount you deposit in your account), T represents Time (in years, during which the principal has been in the bank), and R is the Rate (the percentage rate at which simple interest is paid by the bank).

The total amount of money earned by depositing it in the bank is called **Amount.**

Amount = Principal + Simple Interest

Example 3

Calculate:

(a) Simple interest on a sum of K250 000 deposited for three years at the rate of $25\frac{1}{2}$% per annum.

(b) The amount after three years.

Solution:

(a) $$SI = \frac{P \times T \times R}{100} = \frac{\text{K}250\,000 \times 3 \times 25\frac{1}{2}}{1000}$$

$$= \frac{\text{K}19\,125\,000}{100}$$

$$= \text{K}191\,250$$

(b) K250 000 + K191 250 = K441 250

Example 4

A farmer put K20 000 000 in a bank at the rate of 40% per annum. If the farmer wants to earn interest of K4 000 000, how long will he have to keep his money in the bank?

$SI = P \times T \times R$ P = K20 000 000
T = ?
R = 40%
SI = K4 000 000

$$SI = 4\,000\,000 = \frac{\text{K}20\,000\,000 \times 40 \times T}{100}$$

$$100 \times 40\,000\,000 = 20\,000\,000 \times 40T$$

$$400\,000\,000 = 800\,000\,000T$$

$$800\,000\,000T = 400\,000\,000$$

$$T = \frac{\cancel{400\,000\,000}}{\cancel{800\,000\,000}} = \frac{4}{8} = \frac{1}{2}$$

$$T = \frac{1}{2} \text{ year} = 6 \text{ months}$$

Loans

Banks offer loans to customers who want to carry out a project that is expected to generate a profit. However, this loan has to be repaid with interest, calculated per annum or another agreed upon duration.

Example 5

Mrs. Mumba took out a medium-term loan of K9 000 000 to be repaid in two years time. If the rate is 30%, how much, including the K9 000 000, will she pay back to the bank at the end of two years?

$$SI = \frac{P \times T \times R}{100} = \frac{K9\ 000\ 000 \times 2 \times 30}{100}$$

$$SI = \frac{9\ 000\ 000 \times 60}{100} = 90\ 000 \times 60$$

$$SI = K5\ 400\ 000$$

Total amount to be paid = K9 000 000 + K5 400 000

= K14 400 000

Royalties

Publishers of books pay royalties to their authors. Normally, it is a percentage of not more than 15% on the sales of the book. This has to be agreed upon between the author(s) and the publisher, in advance. This royalties will continue to be paid to the author at the end of each financial year. As long as the book is selling, the royalties will continue to be paid, even if the author has passed away.

Unit Activity 1

Conduct some simple research to find out how people in your community earn their money.

Make a grid to show the following:

Methods of earning money	**Number of people**
Selling	
Banking	
Commission	
Royalties	

Which of the ways to earn money is safest and most stable? Justify your opinion.

Self Assessment 1

Explain the difference between commission and royalties.
Which of the ways to earn money are most safe and stable? Why do you think so?

Summary

In the modern world, money has a central role in the provision of goods and services. There are several ways to earn money. Some of these are:

- business
- employment
- bank interest
- borrowing from lending institutions such as banks
- royalties
- commission

Unit 1 Test

1. Name six ways to earn money.
2. When is a loss identified?
3. Calculate the **profit** or **loss** for each of the following:
   a) Cost price K75 000; Selling price K108 500
   b) Cost price K307 000; Selling price K350 750
   c) Cost price K47 370; Selling price K57 770
4. For each of questions 3(a), 3(b), and 3(c), express the profit or loss as a percentage of the cost price.
5. Calculate the Simple Interest on the following amounts:
   a) Principal = K260, Time = $2\frac{1}{2}$ years, Rate = 25%
   b) Principal = K466 000, Time = $1\frac{1}{2}$ years, Rate = 35%
   c) Principal = K760 100, Time = 3 years, Rate = 15%
6. Calculate the Time for the following:
   a) Simple Interest = K28 000, Rate = 35%, Principal = K40 000
   b) Simple Interest = K28 560, Rate = 28%, Principal = K68 000
   c) Simple Interest = K74 538.75, Rate = 22%, Principal = K104 250
7. Calculate the Rate in the following:
   a) Simple Interest = K18 000, Principal = K30 000, Time = 3 years
   b) Simple Interest = K15 937.50, Principal = K85 000, Time = $1\frac{1}{4}$ years
   c) Simple Interest = K59 812.50, Principal = K108 750, Time = $2\frac{1}{2}$ years
8. Ms. Mukandu is a salesperson. She is paid a salary of K420 000 per month and an additional 6% commission on sales. In the month of May 1999, she sold goods worth K846 000. What were her total earnings for May 1999?
9. A newspaper vendor is paid a commission of K425 for each newspaper sold after the first 50 copies. How much commission will this vendor receive after selling 240 papers?

Answers to Unit 1 Test

1. Business, commission, salary, bank interest, loan, royalties.
2. A loss occurs when goods are sold for less than the shopkeeper paid for them.
3. (a) profit – K33 500
   (b) profit – K43 750
   (c) profit – K10 400
4. (a) 31%
   (b) 12.5%
   (c) 18%
5. (a) SI = K162.5
   (b) SI = K244 650
   (c) SI = K342 045
6. (a) T = 2 years
   (b) T = 1.5 or $1\frac{1}{2}$
   (c) T = 3.25 or $3\frac{1}{4}$
7. (a) R= 20%
   (b) R = 15%
   (c) R = 22%
8. K470 760
9. K80 750

Unit 2: Spending Money

Introduction

In Unit 2 you learned about some of the ways people earn money. People earn money in order to spend it for an improved life. In this unit you will learn about some of the ways money is spent. It is important that children learn about these social aspects of mathematics in school as a way of preparing them for adult life. The social aspects of mathematics you will learn in this unit are:

- household budgets
- hire purchase
- bills, namely water, electricity, and telephone bills

Objectives

After working through this unit, you should be able to:

- prepare simple household budgets
- carry out calculations on hire purchase
- interpret water and electricity bills
- effectively teach the following aspects of social arithmetic to your pupils:
  - postal services
  - household budgets
  - hire purchase
  - water, electricity, and telephone bills

Household Budgets

Spending money wisely involves two things:

- planning how much money you will spend every month
- keeping a record of all money spent

Preparing a household budget can help families manage the money they earn every month, and help them save for future expenses. A monthly household budget is a plan for balancing the family's income, or money earned, with the expenses and other necessities for that month, including bill payments, food, school fees, clothing, etc. Before you can plan a monthly budget, you will need to have a record of your earnings and expenses.

Unit Activity 1

Before you begin to teach your pupils about the importance of budgeting, it will be useful for you to think about your own household budget. For one month, keep a record of all the money you spend. Compare this with your earnings. How much money do you have left after paying your expenses? Look at your list of expenses. Can you think of ways to spend less money?

Continues on next page

Now think ahead to the next month and plan a household budget based on your record of income and expenses.

Reflection

In Unit 1, we discussed some of the ways people earn money. However your money is earned, it should be properly spent. At the household level, parents should know how much money to spend on food, school fees, and other necessities for the family, and they should budget for their expenditures. They should also learn to save money for future use. To do this, they need to plan before they buy anything.

Practice Activity 1

1. Give your class a list of items that can be bought at a local shop. The list should include the unit cost of each item. Give them another list of the items they would actually be asked to buy.
   a) Discuss with the class how they can estimate their total expenditure to ensure that they carry the correct amount of money.
   b) Ask them to prepare a budget and determine the total amount of money required for their purchases. Discuss the importance of budgeting and over-estimating the expenditure so they carry a sufficient amount of money to the shop.

Budgeting for household expenditures is a necessary skill that requires a good knowledge of the average consumption of the family over a given period of time. This must be related to the family's actual earnings for the same period.

Practice Activity 2

Think of the average earning of the families in the community around your school. Using the following sample budget activity, prepare a budget activity for your class.

- Ms. Phiri earns US$600 per month.
- She pays US$20 on the electricity bill, US$30 on the water bill, and US$10 for the telephone.
- Ms. Phiri has three children who go to school. At the beginning of each year, in January, she pays school fees of US$30 per child.
- The monthly household consumption for Ms. Phiri is US$260 for food and US$70 for groceries.
- She rents a house for US$80 per month.

For the month of January, how much money does Ms. Phiri have left after she pays all her expenses?

Note: Modify this activity to suit the local environment of your pupils. In villages, parents earn little money, but they somehow manage to pay school fees for their children. How do they manage this? How do they plan the way they spend the little money they earn?

Hire Purchase

It is difficult for many people who earn little money to buy expensive items, such as a stove, a fridge, or bicycle, by paying cash to the shop. To enable the majority of people to buy these items, a system of paying a **deposit**, then paying the balance by monthly **instalments** was developed. The system is called **hire purchase**. A deposit is a fraction of the cost price of the item to be purchased and is paid at the beginning. The balance, plus interest, is paid in weekly or monthly instalments. If the amount owing is to be paid over a period of one year, there will be twelve equal monthly instalments to be paid. Because of the interest charged, goods bought on hire purchase are more expensive than those paid for by cash.

Example 1

The Supreme Furniture shop sells a set of dining table and chairs for ZK1 500 000 cash. The same set can be bought on hire purchase by paying a deposit of K500 000 and nine monthly instalments of K150 000 each.

a) Calculate the hire purchase price.
b) How much more does the set cost in hire purchase compared to cash price?

(a) Hire purchase price = Deposit + Total instalments
= K500 000 + (K150 000 × 9)
= K500 000 + (K1 350 000
= K1 850 000

(b) Hire purchase price – Cash price = K1 850 000 – K1 500 000
= K350 000

Therefore, the set costs K350 000 more on hire purchase than the cash price.

Practice Activity 3

Before you explain hire purchase to your pupils, introduce the topic to them by asking the following questions:

- What do you think happens in a situation where someone wants to buy an item but does not have enough money to pay for it?
- Has anyone witnessed such a situation? If so, what happened? How did the buyer arrange to pay for the item?

Your pupils will probably describe situations that are similar to hire purchase. You can then introduce the topic.

Note: The interest on hire purchase is usually calculated as a percentage of the cash price. In Example 1 above:

The cash price of the dining set is: K1 500 00.00
Cost by hire purchases is K1 850 000.00

Interest = K1 850 000 – K1 500 000
= K350 000.00

$$\therefore \text{ \% Interest} = \frac{\overset{70}{\cancel{350\,000}}}{\underset{3}{\cancel{1\,500\,000}}} \times \frac{\cancel{100}}{1}$$

$$= \frac{70}{3}$$

$$= 23\frac{1}{3}\,\%$$

Discount

Shop owners sometimes offer a reduced price on goods that are paid for with cash. This reduction in price is called a **discount**. A discount is usually expressed as a percentage of the cash price.

Example 2

The Supreme Furniture shop offers a 10% discount on goods bought by cash. How much does Ms. Phiri pay for her dining set if the cash price is K1 500 000 and she is able to pay cash.

Cost of dining set = K1 500 000

$$\text{10\% discount} = \frac{1\cancel{0}}{10\cancel{0}} \times \frac{1\,500\,00\cancel{0}}{1}$$

= K150 000

$\therefore$ Ms. Phiri's cash payment = K1 500 000 – K150 000
= **K1 350 000**

Self Assessment 1

a) Mr. Mwendapole wants to buy a plough that costs US$200. The Shop offers a 10% discount for paying cash. How much will the plough cost if Mr. Mwendapole pays cash for it?

b) Mr. Mwendapole does not have enough money to pay cash. The shopkeeper tells him that he can buy the plough on hire purchase. The conditions of payment for his purchase are as follows:

- the deposit is $\frac{1}{5}$ of the cash price
- the remaining amount, plus interest, is to be paid in five monthly instalments
- the interest rate is 5% of the cash price

i) How much interest will Mr. Mwendapole have to pay?

ii) Calculate his monthly instalments.

iii) How much more does he pay for the plough on hire purchase than by paying cash with a discount of 10%?

Practice Activity 4

Formulate two activities, similar to the one in Self Assessment 1, for your class.

Household Bills

Household bills spend money in many different ways. In addition to paying for things like rent, food, clothing, and school fees, parents need to pay for such services as water, electricity, and postage.

Water Bills

In many countries, local township councils are responsible for supplying water to their community. In some big cities, the supply of water is the responsibility of private companies. Whoever is responsible for supplying water, consumers pay towards maintenance of the services. Water is charged either at a fixed rate or according to the number of units consumed in a month. *Figures 2.1(a)* and *2.1(b)* show the water bills from Zambia and Botswana, respectively.

ZAMBIA WATER COMPANY

Customer's Name	**Account No:**
Mr. D. Phiri Eastern Road Lusaka	4029 7654

Account for Domestic Water Supply

Period 9 February – 8 August 1999	K2076.00
Fixed standing charge	5500.00
Account due	25 576.00

Figure 2.1(a): A bill for water supplied to a consumer in Zambia.

WATER UTILITIES COOPERATION
GABORONE

Date of bill 30 July '99
Plot number 5047/0/0
CONSUMER NUMBER 31010

READING RECEIPT DATE	REFERENCE NUMBER	DESCRIPTION	METER READING	WATER CONSUMED	WATER CHARGE	PAYMENTS	BALANCE
		B/forward					0.00
08/07/99	242581	Payment				0.00	
08/07/99	18804	Receipt				ˉ30.03	
19/07/99	2682628	Water charge	00684	14	25.58		

Victor Kokai
P/Bag 1
LOBATSE

THIS MONTH'S WATER CHARGE ˉ4.45

Figure 2.1(b): A water bill supplied to a consumer in Botswana. The currency for Botswana is Pula (1 Pula = 100 Thebe)

Unit Activity 1

For each of the bills in *Figures 2.1(a)* and *2.1(b)*, answer the following questions:

1. What is the name of the company supplying water?
2. What is the name of the consumer?
3. What is the account number for each consumer?
4. Is there a fixed charge in each case? If so how much?
5. Is there a unit price in each case? If yes, what is it?
6. What is the amount due on each bill? What differences have you observed in the amounts due? Explain the differences you have observed.

Notice that the bill in *Figure 2.1(a)* does not indicate the units of water consumed. There is a fixed charge on this bill and although the total charge is given, it is difficult to calculate the rate per unit.

The bill in *Figure 2.1(b)* gives the number of units consumed (14) and the total charge of P25.58. Therefore, the unit charge $= \frac{25.58}{14}$

$= 1.83$ Pula

In bill *2.1(a)* the amount due is K25 576.00. There is fixed charge of K5 500.00

In bill *2.1(b)* the amount due is $^{-}4.45$ Pula. There is no fixed charge. The amount due ($^{-}4.45$ Pula) is negative because the consumer paid more than the total charge for that month. This means the company owes him P4.45.

Practice Activity 5

1. Copy the two bills in *Figures 2.1(a)* and *2.1(b)* onto large charts.
2. Ask your pupils the same questions you answered in Unit Activity 2 above.

Electricity Bills

In each country, there are companies responsible for the supply of electricity. The charge for electricity is per unit consumed. Some companies also charge a fixed charge (or standing charge) in addition to the amount charged per unit.

ZAMBIA ELECTRICITY SUPPLY CORPORATION (ZESCO) LIMITED VAT: 10013869-13

CUSTOMER STATEMENT	MONTH OF BILLING	DATE OF BILLING	DATE DUE
MATAA MUKWE	JAN '99	26.04.'99	24.04.'99

KAFUE ESTATES
A76/10/C8
36 KAFUE MAIN

PLEASE NOTE THAT THIS BILL INCLUDES A FIXED CHARGE WHICH APPEARS JUST ABOVE THE TOTAL FIGURE.

ACCOUNT NO:
116457

SITUATION
KAFUE ESTATES, 36 KAFUE MAIN

DESCRIPTION	TARIFF CODE	DATE OF PREVIOUS READING	PREVIOUS READING	DATE OF PRESENT READING	PRESENT READING	NUMBER OF UNITS CHARGED	RATE K	CHARGE K
Balance b/f								216158.27
Energy	R	01.01	9265	31.01	9605	340		19720.00
charge						340	58.00	5 000.00
							5000.00	24720.00
TOTAL EXCISE DUTY AT 10%								2472.00
VAT AT 17.5%								4326.00

TAX INVOICE NO. 99170401259 METER NO. Z012303 AMOUNT DUE 247676.27

Figure 2.2: An electricity bill from the Zambia Electricity Supply Corporation Limited.

Unit Activity 2

Study the electricity bill shown above and answer the following questions:

1. What is the consumer's account number?
2. What is the standard charge?
3. How many units of electricity did the consumer use?
4. What is the total cost of electricity for the month of billing?
5. What are the different charges that add up to the total amount due?

Note: A **tariff** is a list of rates charged by a company. Electricity is charged at different rates for domestic consumers than for industries and businesses.

Practice Activity 6

1. Prepare a large chart of the electricity bill in *Figure 2.2*.
2. Ask your pupils the same questions you answered in Unit Activity 2.

Postal Services

Post office services are common in all countries. Post office services include sending and receiving letters, sending and receiving money, and in some cases even saving money. The tables in *Figures 2.3 (a)*, *2.3 (b)*, and *2.3 (c)* show various postage rates in Zambia.

Internal postal rates (in Zambia)		
Weight not exceeding	Charge	Each additional 250 g or part thereof
	K	K
30 g	400	
50 g	500	
100 g	700	
250 g	800	
500 g	900	
1000 g	2000	350

Figure 2.3.(a): Internal postage rates in Zambia.

Express Mail Service rates (in Zambia)			
Weight (max. 30 kg)	Local delivery K	Between Copperbelt towns Kabwe, Kapiri, and Lusaka K	Between participating provincial towns and the rest of delivery towns K
up to 50 g	700	1500	1500
50 – 200 g	1000	2600	2600
201 – 400 g	1400	5000	5000
401 – 500 g	1800	5600	5600
501 – 1000 g	3000	7000	7000

Figure 2.3.(b): Express mail services rates in Zambia.

International postal rates (outside Zambia)		
Category	Weight not over 100g K	Each additional 100g or part thereof K
letters	700	500
printed paper, newspapers, books, pamphlets, literature for the blind	600	400
small packets	600	400
aerogrammes	600	
postcards	600	

Figure 2.3(c): International postal rates in Zambia.

Unit Activity 3

1. Tabo posted two international letters by ordinary mail. One letter weighed 270 g and the other weighed 190 g. How much did Tabo have to pay?
2. Tebuho lives in Kabwe (Zambia) and she sent an important letter by express mail to her friend to Kitwe on the Copperbelt. The letter weighed 220 g. How much did she have to pay?
3. Naomi wanted to send a letter to her sister in the UK and an aerogramme to her brother in Australia. The letter weighed 120 g. How much did she have to pay in postage?

Practice Activity 7

1. Make large charts for postal rates, similar to those in *Figures 2.3(a)*, *2.3(b)*, and *2.3(c)*.

   Note: Modify the charts to reflect the postal rates in your country.
2. Formulate five questions from the charts for your class.

Unit 2 Test

Use the information in the tables in *Figures 2.3(a)*, *2.3(b)*, and *2.3(c)* to answer the following questions:

1. Yamba posted four letters by ordinary mail. Two of the letters weighed 25 g each and a third letter weighed 70 g. The fourth letter weighed 90 g. How much did Yamba pay to post the letters?
2. Monde posted seven letters, each weighing 20 g, from Mansa in Luapula Province to Kabwe on the Copperbelt by express mail. How much did she pay for postage?
3. How much does it cost Mwemba to send a parcel weighing 700 g from Lusaka to his sister in Germany?
4. A parcel weighs 17.5 kg. How much does it cost to send it by ordinary post?

Answers to Unit Activity 3

1. Tabo's postage:

   Cost of the 190 g letter = K800

   Cost of the 270 g letter = K900

   Total cost of postage = **K1700**

2. Tebuho's postage:

   The weight of the letter is between 201 – 400 g

   Therefore, it cost her **K5000**.

3. Naomi's postage:

   Cost of the letter weighing 120 g = K700 + K500 = K1200

   Cost of the aerogramme = K600

   Total cost of postage = K1200 + K600

   = **K1800**

Answers to Unit 2 Test

1. Yamba's postage:

   Cost of the two letters weighing 25 g each = K400 × 2 = K800

   Cost of the letter weighing 70 g = K700

   Cost of the letter weighting 90 g = K700

   Total cost of postage = **K2200**

2. Monde's postage:

   Cost of express post for seven letters weighing 20 g each = K1500 × 7

   Total cost of express postage = **K10 500**

3. Mwemba's postage:

   Cost of the package weighing 700 g= K600 for the first 100 g + K400 × 6

   = K600 + K2400

   = **K3000**

4. Cost to mail parcel weighing 17.5 kg:

   First 1000 g (1 kg) = K2000

   Additional 16 500 g (16.5 kg) at K350 per 250 g = $K350 \times \frac{16\ 500}{250}$

   Cost for ordinary post: K2000 + K23 100 = **K25 100**

Unit 3: Government Revenue

Introduction

Children can learn a lot about life around them through mathematics. This unit introduces another aspect of social arithmetic that should be taught to children in primary school. Governments need money to manage the country. One of the ways government earns money is by collecting taxes. Tax is a mandatory contribution made to a government by individuals and businesses. Governments use tax money in many different ways, including social goods and services that are provided to the public at no additional charge, such as schools, roads, and hospitals.

The tax money collected by governments is called **revenue**.

In this unit, you will learn about various forms of tax: personal levy, Pay-As-You-Earn (P.A.Y.E), value added tax (vat), and excise duty.

Objectives

After going through this unit you should be able to:

- explain to your pupils the meanings of the terms tax and revenue
- describe and carry out calculations on the following kinds of tax
    - personal levy
    - Pay-As-You-Earn
    - value added tax (vat)
    - excise duty
- teach the four forms of government revenue to upper primary pupils

Income Tax

A tax on the income of individuals or families is called **income tax**. Income tax can be paid in different ways. Look at Mr. Pilota's payslip for the month of March 1997 (*Figure 3.1*). Mr. Pilota's salary is in Zambian kwatcha.

GOVERNMENT OF THE REPUBLIC OF ZAMBIA

PAY DATE: MARCH 1997	PAYSLIP NO: 49707	MINISTRY
NAME: PILOTA	INTIALS: F.K.	DEPARTMENT: 900
PAY METHOD: NAT. COM BANK	N.R.C. NO: 164372/83/1	PAYPOINT: 0000
		MAN NO: 10297

APPOINTMENT DETAILS	**TAX DETAILS**	
CONDITIONS OF SERVICE: L	TAX CREDIT TO DATE: 60 000.00	KEY TO TAX BASIS
TERMS OF APPOINTMNET: 2	TAX BASIS: 2	0 NON TAXABLE
INCREMENTAL DATE: 01 OCT.	TAXABLE PAY TO DATE: 2558248.31	1 MONTH 1 BASIS
SALARY SCALE: EMS 05		2 CUMULATIVE BASIS

CODE	DESCRIPTION		SALARY AND ALLOWANCES	DEDUCTIONS	ACCUMULATED TOTALS
001	GROSS SALARY	999	185733.00		1919961.00
016	RETENTION ALLOWANCE	999	37146.60		459703.20
256	INSURANCE	999		19.91	238.92
258	LOAN			50.00	600.00
735	PERSONAL LEVY	000		7500.00	7500.00
967	P.A.Y.E	320		16.25	384195.00
970	RENT	008		17830.37	213964.44
971	PENSION	002		16158.77	193905.24
	TOTALS		222879.60	73575.30	NET AMOUNT PAYABLE 149304.30

Figure 3.1

Mr. Pilota's payslip shows, among other information, his gross monthly salary and allowances, deductions, and the net pay.

Among the deductions are personal levy and P.A.Y.E. (Pay-As-You-Earn).

Personal Levy is a tax on the income of individuals, paid to the local council in which the individual resides. In Zambia, it is deducted twice in a year. The council uses this money to provide services such as water and sanitation. Mr. Pilota's personal levy for the month of March 1997 is K7500.

P.A.Y.E. (Pay-As-You-Earn) is a tax on the income of individuals paid to the central government. This is deducted every month of the year.

Unit Activity 1

1. What percentage of Mr. Pilota's monthly gross salary goes to P.A.Y.E?
2. What percentage of Mr. Pilota's annual gross salary goes to P.A.Y.E.?
3. What fraction of Mr. Pilota's gross salary for March went to pay the personal levy?

   (Express your answer in decimals and correct to two decimal places.)

   Percentage of P.A.Y.E. is calculated as: $\dfrac{\text{P.A.Y.E.}}{\text{Gross salary}} \times \dfrac{100}{1}\%$

Personal levy expressed as a fraction of the gross salary is: $\dfrac{\text{Personal levy}}{\text{Gross salary}}$

Practice Activity 1

1. Draw Mr. Pilota's payslip on a large chart to show to your class. Use figures that are convenient for pupils to work with.
2. Formulate questions similar to those in the unit activity for your pupils to answer.

Other Taxes

ZAMBIA ELECTRICITY SUPPLY CORPORATION (ZESCO) LIMITED VAT: 10013869-13

CUSTOMER STATEMENT	MONTH OF BILLING	DATE OF BILLING	DATE DUE
MATAA MUKWE	JAN '99	26.04.'99	24.04.'99
KAFUE ESTATES			
A76/10/C8	PLEASE NOTE THAT THIS BILL INCLUDES A FIXED CHARGE		
36 KAFUE MAIN	WHICH APPEARS JUST ABOVE THE TOTAL FIGURE.		
ACCOUNT NO:	SITUATION		
116457	KAFUE ESTATES, 36 KAFUE MAIN		

DESCRIPTION	TARIFF CODE	DATE OF PREVIOUS READING	PREVIOUS READING	DATE OF PRESENT READING	PRESENT READING	NUMBER OF UNITS CHARGED	RATE K	CHARGE K
Balance b/f								216158.27
Energy	R	01.01	9265	31.01	9605	340		19720.00
charge						340	58.00	5 000.00
							5000.00	24720.00
TOTAL EXCISE DUTY AT 10%								2472.00
VAT AT 17.5%								4326.00

TAX INVOICE NO. 99170401259 METER NO. Z012303 AMOUNT DUE 247676.27

Figure 3.2: Mr. Mukwe's electricity bill for the month of January 1999 from Zambia Electricity Supply Corporation Limited (ZESCO).

The electricity bill in *Figure 3.2* illustrates how ZESCO charges for electricity in Zambia, and includes two items of tax—**excise duty** and **vat**.

Excise duty is a tax that government imposes on the sale of certain goods produced within the country. In Zambia, electricity is one of these select goods.

Excise duty is charged at 10% of the total electricity charge for the month. For example, Mr. Mataa's electricity consumption during the month of January 1999 was 340 units, and the unit charge is K58.00

Charge for January = K58.00 × 340 + (K5000 fixed charge)
= K19 720 + K5000
= K24 720.00
∴ Excise duty at 10% = **K2472.00**

Value Added Tax (VAT) is a tax imposed at each stage in the production and distribution of a particular commodity. It is based on the values added to the product at each stage. This tax is passed on to the consumer, as shown in

Figure 3.2. Vat is charged at a fixed percentage rate of the charge. For example, the Zambian government rate for vat is 17.5%.

Mr. Mataa is charged K24 720 for electricity consumed during the month of January 1999. He pays vat at 17.5%

$$= \frac{17.5}{100} \times \frac{24\ 720.00}{1}$$

$$= \mathbf{K4326}$$

Governments impose other forms of tax to raise revenue that have not been mentioned in this unit. For example, custom duty is a tax imposed on imported goods. It is also charged as a percentage of the total cost of the goods.

Practice Activity 2

1. Make a copy Mr. Mukwe's electricity bill (*Figure 3.2*) on a large chart. Use the chart to introduce excise duty and value added tax to your class. For teachers in other countries—find examples for your country to give to your pupils.
2. Formulate five questions for your pupil to calculate each of the following taxes:
   a) Excise duty
   b) Value added tax

Self Assessment

Mr. Mudala earns a gross monthly salary of US$2500 as the Managing Director of a supply company.

1. Twice a year, he pays 2% of his gross salary to the Council in personal levy. He also pays 10% of his gross salary to the central government in P.A.Y.E.
   a) How much does Mr. Mudala pay in personal levy annually?
   b) What is his annual P.A.Y.E.?
1. Mr. Mudala pays $30 for electricity and $15 for water every month.
   a) How much does he pay annually in:
      i) electricity bills
      ii) water bills
   b) Express:
      i) His monthly electricity bill as a percentage of his monthly gross salary.
      ii) His monthly water bills as a percentage of his monthly gross salary.

Unit 3 Test

1. Mr. Madubansi works for the American Embassy and receives his salary in US dollars. The local council in which he resides taxes its residents a personal levy of 5% of gross salary twice in a year. Mr. Madubansi also pays P.A.Y.E. to the central government at 10% of his gross salary. If his gross salary is $350 per month, find how much he pays annually in:
   a) Personal levy
   b) P.A.Y.E.

2. The table shows part of Mr. Madubansi's electricity consumption in a certain month:

Previous reading	Present reading
8147	8497

   a) Electricity is charged at 5 cents per unit ($1 = 100 cents), plus a fixed charge of $5. Find Mr. Madubansi's electricity charge for the month.
   b) The government imposed excise duty of 5% on household electricity consumption per month. How does Mr. Madubansi pay in excise duty, to nearest dollar?
   c) The government also imposes value-added tax of 7% on the monthly electricity household consumption. How much, to the nearest dollar, does Mr. Madubansi pay in vat?

Answers to Self Assessment

1. a) $100

   b) $3000

2. a) (i) $360 (ii) 180

   b) (i) 1.2% (ii) 0.6%

Answers to Unit 3 Test

1. a) $35

   b) $420

2. a) $22.50

   b) $1

   c) $2

Unit 4: Transport

Introduction

In this module, you have dealt with several aspects of earning and spending money. Travel is another aspect of life that requires careful planning and an understanding of distance charts, fare charts, and timetables. This unit deals with these aspects of transport.

Note: To benefit adequately from this unit, you need a variety of data on transport including timetables, distance charts, and fare charts.

Objectives

After working through this unit, you should be able to use a problem-solving approach to teaching mathematics on transport.

Distance Charts

To effectively plan a journey, you need to know how far you will be travelling. Other people who have already made a similar trip can be a useful source of information on the distance you have to travel. Distance charts, where available, are a more accurate source of information. A distance chart for Zambia is given in Appendix 1. All distances are in kilometres. The towns on the chart are shown on the map of Zambia in Appendix 2.

Unit Activity 1

1. From the distance chart in Appendix 1, find the distances between the following towns:

   a) Chipata and Lusaka

   b) Lusaka and Ndola

   c) Chipata and Ndola

   d) Do the distances from Chipata to Lusaka, and from Lusaka to Ndola satisfy the *transitive* property, that is:

   $$C + L = x$$
   $$L + N = y$$
   $$\therefore C + N = x + y?$$

   Do you think this result is always true for distance?

2. Use the distance chart in Appendix 1 to find the distances between the following towns:

Continues on next page

a) Mongu and Mwinilunga

b) Mwinilunga and Lusaka

c) Mongu and Lusaka

3. Study the map in Appendix 2. Do your answers to (a) and (c) agree with the geographical locations of the towns? Suggest explanations for the distances given on the distance chart.
4. Do the distances from Mongu to Lusaka to Mwinilunga satisfy the transitive property?

$$\begin{aligned} \text{Mongu} + \text{Lusaka} &= x \\ \text{Lusaka} + \text{Mwinilunga} &= y \\ \therefore\ \text{Mongu} + \text{Mwinilunga} &= x + y? \end{aligned}$$

Suggest explanations for the answer. What does this tell you about distances and the transitive property?

Unit Activity 2

1. Refer to the map of Zambia in Appendix 2. Using a ruler and referring to the scale in the lower right hand corner of the map, find the direct distances between the following towns:

   a) Kawambwa and Mansa

   b) Ndola and Choma

   c) Lusaka and Mansa

   d) Kabwe and Zambezi

2. You are travelling from Kabwe to Zambezi. What approximate distance will you plan for if you are travelling:

   a) On foot

   b) By car

   c) By air

   What are the reasons for your answers?

3. What are the advantages of travelling by road compared to travelling by air, and vice versa?

Timetables and Fare Charts

This section covers two important considerations when planning journeys by bus, train, or aeroplane. These are timetables and fare charts.

Reflection

What is the implication for travelling by road, rail, or air if timetables and fares are not taken into account when planning a journey?

Unit Activity 3

Do you know someone who has taken a journey without planning in advance? Interview this person and record, in writing, the inconvenience they experienced as a result of inadequate planning. Your account might include problems related to distance, timetables, or fare charts.

Train Timetable

Table 4.1 below shows the timetable for a Zambia Railway train. The times are stated in the 24-hour clock system. *Table 4.2* shows the fare chart for the same train.

Zambia Railways		
Kitwe	Depart	20:00
Ndola	Arrive Depart	21:50 22:11
Kapiri-Mposhi	Arrive Depart	02:11 02:16
Kabwe	Arrive Depart	04:20 04:40
Chisamba	Arrive Depart	07:00 07:06
Lusaka	Arrive Depart	08:59 09:19
Kafue	Arrive Depart	10:52 11:12
Mazabuka	Arrive Depart	12:42 12:47
Monze	Arrive Depart	14:38 14:43
Choma	Arrive Depart	17:38 17:58
Kalomo	Arrive Depart	19:14 19:21
Zimba	Arrive Depart	21:03 21:10
Livingstone	Arrive	23:52

Table 4.1: Timetable for a Zambia Railway train

Unit Activity 4

1. Use *Table 4.2* to answer the following questions:
   a) At what time does the train arrive at Monze?
   b) At what time does the train leave Monze?
   c) How long does the train stop at Monze?
   d) How long is the journey from Kabwe to Kalomo?
2. What is the shortest stop at a station on the journey from Kitwe to Livingstone?
3. A passenger travelling from Kitwe to Livingstone would like to keep in touch with a business partner by phone at every opportunity on the journey. There is a telephone at every station, but it takes the passenger at least twelve minutes to leave the train, make a call, and return to the train. At which stations will this passenger be able to make a telephone call?

Train fares on a railway line

To station	Distance Km	First Class K	Second Class K	Third Class K
Lusaka	00	000	0000	000
Kafue	48	1120	1030	680
Mazabuka	96	2020	1850	1150
Monze	156	2920	2670	1610
Pemba	192	3220	2940	1170
Choma	258	4420	4030	2390
Kalomo	328	5320	4850	2850
Zimba	383	6220	5670	3320
Livingstone	465	7420	6760	3940
Children below the age of 7 go free. Children between 7 and 15 years pay half the adult fare, plus K5.				

Table 4.2: Fare chart for a train. To determine the fare between two stations, subtract the smaller amount from the larger amount. For example the second class fare between Choma and Zimba is K5670 – K4030 = K1640.

Unit Activity 5

To answer the following questions, refer to *Table 4.2*.

1. Banda travels on a first class coach from Mazabuka to Kalomo. What is Banda's fare?
2. Tendai travels first class from Lusaka to Pemba, and returns to Lusaka second class. If there is a 10% discount on this particular trip, what is the return fare?
3. A group of five people are travelling second class from Lusaka to Choma.
   a) What is the fare for each of them?
   b) What is their total fare?
   c) How much would be saved if they travelled third class?
4. A couple wants to travel from Mazabuka to Livingstone, collect their five children aged 17, 12, 8, 7, and 4 years, and return to Mazabuka. How much is the total fare for the couple and their children?
5. Chanda is on holiday and would like to spend time in each town from Lusaka to Livingstone. If Chanda has to restart the journey at each town, how much will it cost him to travel the whole journey? How much would he save by travelling directly to Livingstone?

Practice Activity 1

This activity requires your pupils to plan an educational tour for the school holiday. Your role as a teacher is to help pupils secure the necessary information and give advice on the planning process. The pupils' role is to make calculations and make decisions on the alternatives that are available.

Discuss the destination with the children. It can be within the country or to another country, to multiple places or a single destination.

Have the class decide on a common destination, but have groups of pupils plan the tour independently. Supply them with information on possible discounts for school groups, discounts for return fares as opposed to one-way fares, and discounts for travelling during the off-peak season.

Let the groups present their plans to each other and discuss the "best" solution. Take note of what they consider to be the best alternative. Is it the least expensive fare, the safest route, the one that allows more time for the tour and less time travelling?

Encourage the idea that mathematics can be a tool for problem solving and decision making rather than strictly a process for finding scientifically "correct" answers.

Summary

- In planning a journey, it is important to consider distance, timetables, and fares.
- Unless on the same route, distance between towns is not transitive.

Unit 4 Test

1. Using Table 1 find:
   a) The distances between:
      i) Mpika and Mumbwa
      ii) Solwezi and Chingola
   b) The town that is 564 km from Luanshya.
   c) The two towns that are closest to each other.
2. Use Table 4.2 to solve this problem.

   A sports team with thirty members is travelling from Choma to Lusaka for a ten-day sports festival. Sports teams are entitled to a 20% discount. All passengers are entitled to a 25% discount (after any other discount) for purchasing return tickets for roundtrip journeys of more than seven days. How much will this sports team pay for its return trip to and from Lusaka?
3. The table below shows bus fares between Kitwe and Lusaka. Use the table to answer the questions that follow:

Intercity bus fares (in Kwacha), Lusaka - Kitwe

Lusaka					
2000	Kabwe				
2500	1000	Kapiri-Mposhi			
3000	1500	1000	Nyenyenzi		
3300	2300	1500	1000	Ndola	
3800	2800	2500	1500	700	Kitwe

   Sibeso wants to travel from Kabwe to see relatives in Ndola. Sibeso can save only K200 per week. How long will it take Sibeso to save enough money for a return journey to Ndola?

Answers to Unit 4 Test

1. a) (i) 687 km (ii) 173 km
   b) Mwinilunga
   c) Luanshya to Ndola
2. The fare from Choma to Lusaka is K2390 per person and the return fare is K4780.

   Team members receive a 20% discount (K956), reducing the fare to K3824. Since their trip will be longer than seven days, they receive an additional 25% discount (KK956) off the already discounted fare, further reducing the fare to K2868. The total fare for thirty team members will be K86 040.
3. The one-way fare from Kabwe to Ndola is K2800.

   $K2800 \times 2 = K5600$

   If Sibeso saves K200 per week, it will take:

   $\frac{K5600}{K200}$ = 28 weeks to save enough money.

Unit 5: Statistics 1
Data Collection and Presentation

Introduction

This unit is titled Statistics 1, yet you will probably notice that the word statistics does not appear anywhere in your primary mathematics syllabus. Some primary mathematics syllabuses do not approach the subject of statistics formally and therefore do not use the word statistics. The topic in this case is approached indirectly through other topics such as graphs and averages where calculations of mean are done. This unit approaches statistics in a formal way. Once pupils are comfortable with the operations of whole numbers and fractions and they are able to carry out all types of measurements, they should be ready for statistics—data collection and representation. The four types of data presentations covered in this unit are pictographs, bar graphs, pie charts, and line graphs.

Objectives

After working through this unit, you should be able to:

- represent data in pictographs, bar graphs, pie charts, and line graphs
- teach upper primary school pupils how to present data in pictographs, bar graphs, pie charts, and line graphs

Statistics in Primary School

Statistics in primary school begins with making pictures and diagrams to show information. But it is important to make the information they are dealing with as relevant and meaningful as possible, as this increases the motivation of the pupils and makes the activity more worthwhile.

Stage One

Pictographs

Pictographs use pictures to represent statistical information. When introducing pictographs to your pupils, always begin with simple situations.

Ask pupils to go out and collect different types of leaves. Use the common names for the trees in your area and specify the number of leaves they should bring from each tree. Small numbers, between one and five, are convenient to start with. Ask them also to bring leaves from a type of that is not found in your area—they will obviously bring nothing.

Ask each pupil to draw the leaves in their exercise books, providing guidelines for how they should arrange their drawings on the page.

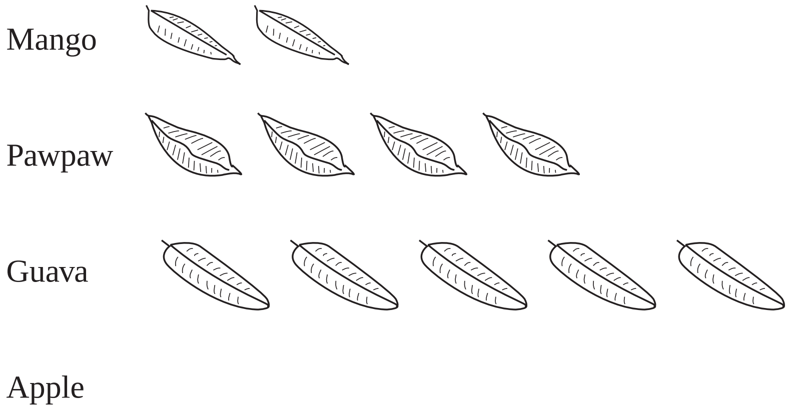
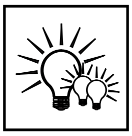


Figure 5.1: Pictographs

Each child will come up with drawings of the different types of leaves they have collected, but no drawing for the type of leaves they could not find (in this case, apple). Explain to your pupils that what they have drawn is a pictograph. Ask them to label it, as shown in *Figure 5.1*.

Now in a whole-class discussion, ask them the following questions:

- Which type of leaves are the highest number?
- Which leaves are the smallest number?
- Which type of leaves are not represented?
- How many paw-paw leaves are there?



Practice Activity 1

Design five pictograph activities similar to the one above, using different objects (seeds, flowers, fruits, etc.) for each.

Bar Graphs

After a series of activities representing objects in pictographs, you can introduce bar graphs using the same examples. Begin with simple bar graphs in block form. Show the class how you would represent the leaves in a bar graph, as shown in *Figures 5.2(a)* and *5.2(b)*. Notice that the bars can either touch each other as in *Figure 5.2(a)* or can be separate as in Figure *5.2(b)*.

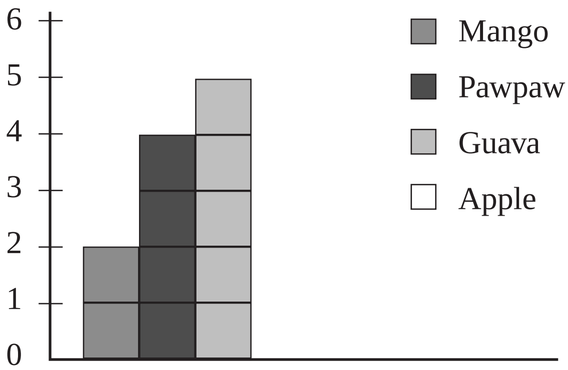


Figure 5.2(a)

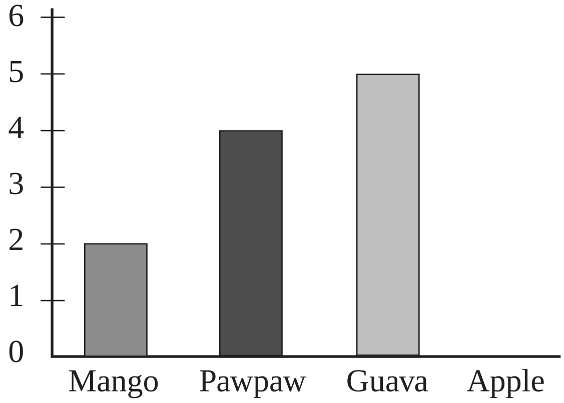


Figure 5.2(b)

Explain to your pupils that in the block diagram (*Figure 5.2*), each bar represents a leaf type, and the number of blocks in the bar represents the number of leaves of that type. For example, there are two mango leaves but no apple leaves.

Now ask your pupils to draw a bar graph for each of the pictographs they drew in the previous activity.

Stage Two

At this stage, pupils can be introduced to using representative numbers in the pictures and bars, thus introducing the idea of scale. Start with a simple example of how many pupils have brothers and how many have sisters. Pupils who have brothers and sisters are counted in both groups. Record the information as shown in this simple table:

	No. of pupils
Brothers	20
Sisters	15

For this example, your pupils would have to draw twenty pictures of pupils with brothers and fifteen pictures of pupils with sisters. To solve the problem of having to do so much drawing, tell them they can draw one picture to represent five pupils. This will result in a pictograph similar to the one shown in *Figure 5.3*. It is important for pupils to understand that if one picture represents a number of people or objects, their pictograph must include a scale.

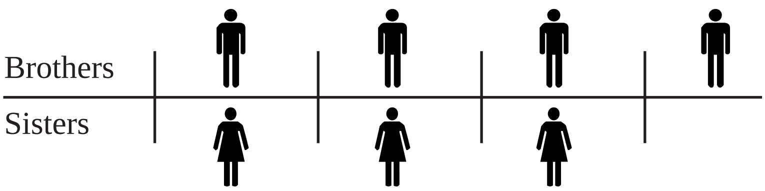


Figure 5.3: Scale: each figure represents 5 pupils

In the pictograph in *Figure 5.3* one picture represents five pupils. Discuss the pictograph with pupils by asking the following questions:

- How many pupils have brothers?

answer: 4 pictures × 5 pupils = 20 pupils

- How many pupils have sisters?

  answer: 3 pictures × 5 pupils = 15 pupils

Now change the numbers in the example, so that twenty pupils have brothers and eighteen pupils have sisters. In case of eighteen pupils, you can draw three pictures representing fifteen pupils, with a remainder of three pupils. How do we represent the remainder? Three is about half of five, so you can use half a picture, as shown in *Figure 5.4*.

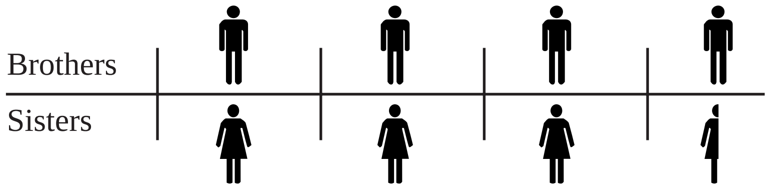


Figure 5.4: Scale: each figure represents 5 pupils

Practice Activity 2

Design five activities for pupils to collect data that can be represented in pictographs.

Bar Graph

It might be necessary to use many pictures, or even parts of pictures, to represent data in a pictograph. For this reason, they are not an ideal way to present information. To demonstrate this to your pupils, draw two bar graphs on the chalkboard, using the information from *Figures 5.3* and *5.4*. Your bar graphs should resemble those in *Figures 5.5* and *5.6*.

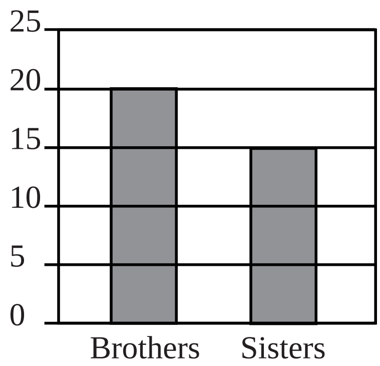


Figure 5.5

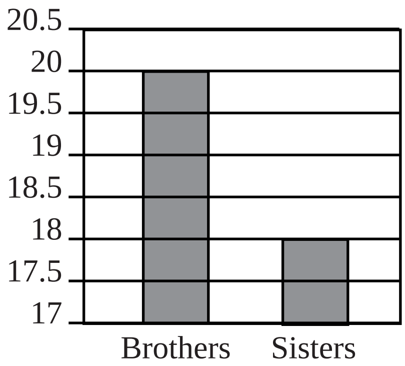


Figure 5.6

In *Figure 5.5*, the number of pupils with brothers is 20 and the number of pupils with sisters is 15. In *Figure 5.6* the number of pupils with brothers is 20 and the number of pupils with sisters is 18.

The problem of having to draw part pictures has now been resolved, so it seems that bar graphs are more accurate than pictographs in presenting information. You can also show your pupils that the blocks in the bars are no longer necessary. Show pupils how this happens by giving examples as illustrated in *Figure 5.7* and *5.8*.

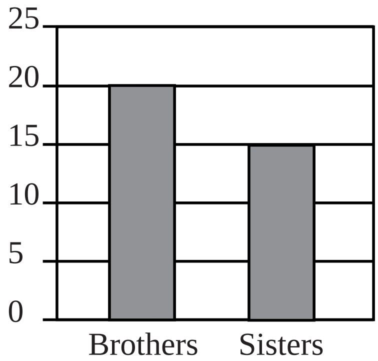


Figure 5.7

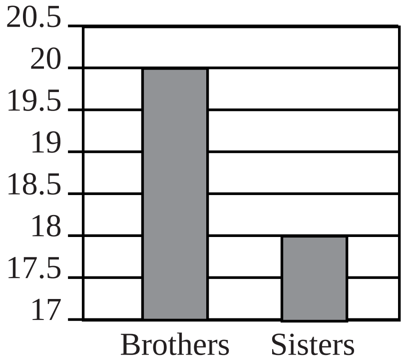


Figure 5.8

Explain the importance of using a uniform scale. Always mark the axes at equal intervals starting at zero. It is important to give pupils the horizontal form of bar graphs as well the vertical form.

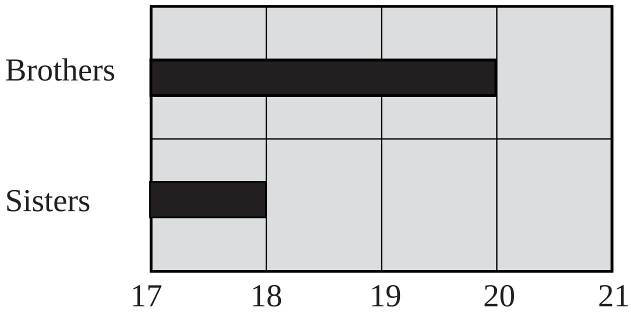
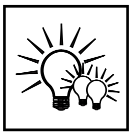


Figure 5.9

Figure 5.9 shows the number of pupils who have brothers and those who have sisters. In this example, the bars are drawn horizontally.



Practice Activity 3

Prepare five activities for your pupils to practice drawing bar graphs without blocks. They should draw both the vertical and horizontal bar graphs.

Note that the terms **bar chart** and **bar graph** mean the same thing. Be careful **not** to introduce the term histogram to pupils at this stage.

Suggested Class Activities—extracts from *Health into Mathematics* (Gibbs & Mutunga, 1991).

1. Put children in groups according to their position in the family—the first born in one group, the second born in another, and so on. Tabulate the information for pupils on the chalkboard and ask your pupils to draw pictograms.
2. Give your pupils a piece of paper (all pieces should be the same size) and ask them to draw pictures of themselves. Have them name an illness they once suffered from. Construct a pictograph, similar to the example shown

Continues on next page

below, by listing the illnesses on the chalkboard and pasting the pupils' pictures next to disease they named.

Illness	Number of Pupils				
	1	2	3	4	5
Chicken Pox					
Malaria					
Measles					

3. Ask pupils to tell you their favourite foods and make a list of them.

   Growth Foods: beans, chickens, eggs, millet, meat, fish, groundnuts, flying ants, caterpillars.

   Energy (strength) Foods: cooking oil, bread, maize porridge, sugar, rice, bananas, potatoes, margarine, millet, sugarcane, nuts.

   Food for Protection against Diseases: pumpkins, pineapples, pawpaw, guavas, green leaves, lemon, mango, pepper, tomatoes, onions, peas, cabbage.

   Tabulate this information on the chalkboard and ask pupils to draw pictographs and bar graphs.

Unit Activity 1

Gibbs and Mutunga (1991) outlined a wide range of activities on collection and presentation of data. These activities provide opportunities to teach mathematics in real life situations. Try these activities yourself, before you give them to your pupils.

1. Take a survey in your class to find out which pupils have been immunized. The pupils will have to ask their parents. Let each pupil prepare a record to be filled in at home.

	Tick (✓) or Cross (✗)
BCG	
Polio	
DPT	
Measles	

   My Immunisation Record

   Pupils will tick (✓) if they have been immunised against the disease or cross (✗) if they have not been immunised.

   Prepare a bar graph to display in class, showing the number of pupils who have been immunised and those who have not been immunised against the given diseases. Discuss with pupils the importance of immunisation.

Continues on next page

2. Take a survey of the common illnesses among your pupils and keep a record of the illness within the class during each month of the three school terms.

Term 1, 2000	Stomach ache	Malaria	Diarrhoea	Coughing	Sore eyes
January	✓✓✓✓ ✓✓✓✓	✓✓✓✓ ✓✓✓✓	✓✓✓✓✓✓ ✓✓✓✓✓✓✓✓	✓✓✓✓	✓
February					
March					
April					

Each time a pupil falls ill with a given illness in a given month, place a tick in the appropriate box. At the end of each term, prepare a bar graph to display in your classroom. Your bar graph might look like the following example in *Figure 5.10*.

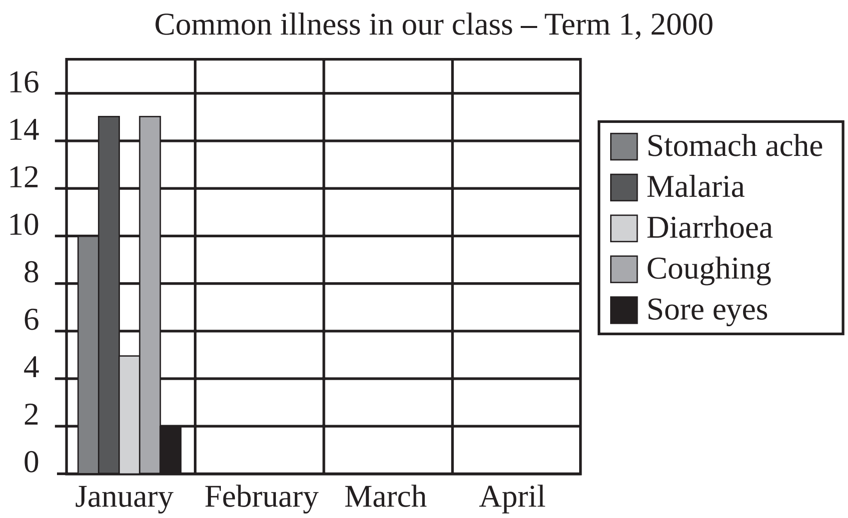


Figure 5.10

Note: This activity produces an example of another form of bar graph, where more than one piece of information is represented. A key is necessary to explain what each bar represents.

Note: The information represented in this graph provides opportunities to ask questions that will encourage your pupils to further develop their skills in mathematics:

- Which is the most prevalent (common) illness during the first term?
- What fraction of the pupils suffered from coughing:
  - in the month of January?
  - in the whole term?
- What percentage of the pupils suffered from malaria during the term?

Practice Activity 4

1. Ask your pupils to carry out a survey of children in their household between the ages of 1 and 4 years who have not been immunised. They can obtain this information from the under five clinic records card that each child will normally have. The pupils can ask their parents to tell them which immunisation each child has received, or they can read the information from the record card. Have each pupil tabulate the results of their survey in a table, as shown below:

   Immunisation record for my young brothers and sisters

Name of child	Age	BCG	Polio	DPT	Measles
Naomi	2				✗
Tele	4			✗	

   In class, help your pupils to tabulate all the information in one table as shown below:

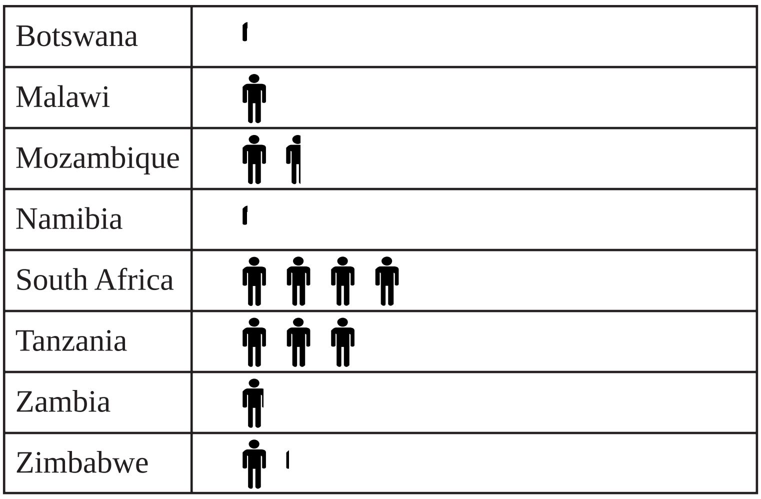
   Immunisation record for children of ages of 1 to 4

Age	BCG	Polio	DPT	Measles
1				
2				
3				
4				

   Put the ticks and crosses in the boxes, then ask pupils to draw a bar graph in their exercise books. Ask appropriate questions to help your pupils interpret the information in their bar graph, as in the unit activity above.

Self Assessment 1

1. The following pictograph represents the populations of eight Commonwealth member countries in southern Africa.



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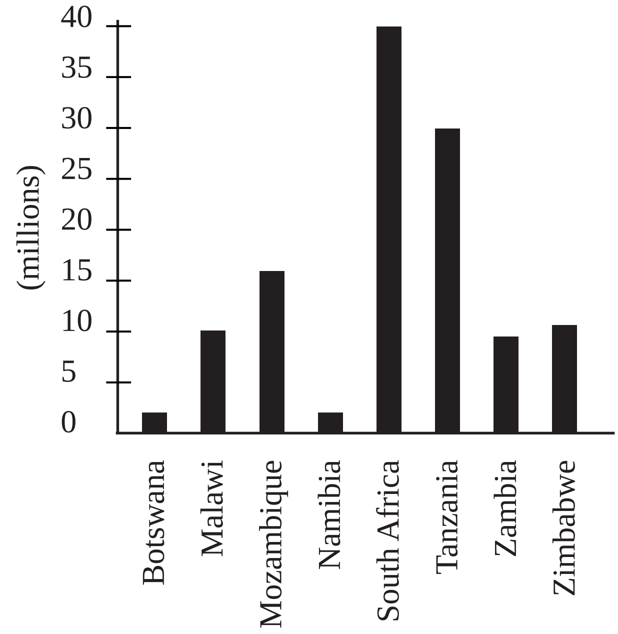
2. The bar graph represents the populations of eight commonwealth member countries in southern Africa:



From each graph:

a) Write down the population of each country.

b) Comment on the difficulties you experienced in determining populations from the pictograph and bar graph.

3. Life expectancies for eight southern Africa Commonwealth member countries are given in the table below. Life expectancy is the average number of years that people live.

Botswana	Malawi	Mozambique	Namibia	South Africa	Tanzania	Zambia	Zimbabwe
68	43	47	59	64	51	46	57

Represent this data in a:

a) pictograph

b) bar graph

Pie Chart

To introduce pie charts, begin with a simple pie chart showing the proportion of boys and girls in a class of 50 boys and 50 girls as shown in *Figure 5.11*.

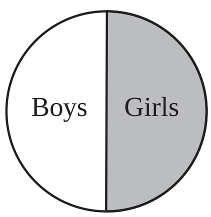


Figure 5.11

Tell pupils that the circle represents the total number of pupils in the class. One half of the circle represents the number of girls, the other half represents the number of boys in the class. Ask your pupils the following questions about the pie chart:

- What fraction of the class are boys?
- What fraction are girls?

Now present another pie chart to your pupils, representing a class in which $\frac{3}{4}$ are boys and $\frac{1}{4}$ are girls, as shown in *Figure 5.12*.

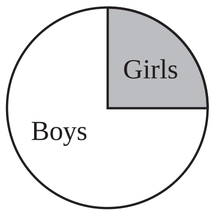


Figure 5.12

Discuss the pie chart with them by asking the following questions:

- What fraction of the class are girls?
- What fraction are boys?

Now talk about angles. Ask pupils how many degrees there are in a complete turn. How many degrees are there in a half-turn? in a quarter turn?

There are 360˚ in a complete turn, 180˚ in a half turn and 90˚ in a quarter turn. Therefore, in the first pie chart in *Figure 5.11* the angle representing the boys is 180˚ and the angle representing the girls is also 180˚.

In the pie chart in *Figure 5.12*, the angle representing the girls is 90˚ and the angle representing the boys is 270˚. Draw pictures of both pie charts making the angles as shown in *Figures 5.13* and *5.14*.

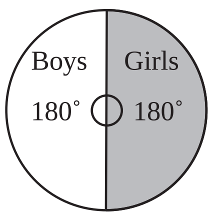


Figure 5.13

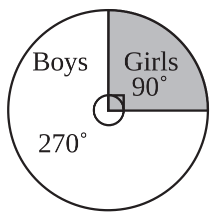


Figure 5.14

Discuss the following questions with your pupils:

- What fraction of a complete turn is 180˚?
- What fraction of a complete turn is 90˚, 270˚?
- What fraction of a complete turn is 60˚, 120˚?

$180^\circ = \frac{1}{2}$ turn ($\frac{1}{2}$ of 360°) because $\frac{180}{360} = \frac{1}{2}$

$90^\circ = \frac{1}{4}$ turn ($\frac{1}{4}$ of 360°) because $\frac{90}{360} = \frac{1}{4}$

$270^\circ = \frac{3}{4}$ turn ($\frac{3}{4}$ of 360°) because $\frac{270}{360} = \frac{3}{4}$

$60^\circ = \frac{3}{4}$ turn ($\frac{3}{4}$ of 360°) because $\frac{60}{360} = \frac{3}{4}$

$120^\circ = \frac{1}{3}$ turn ($\frac{1}{3}$ of 360°) because $\frac{120}{360} = \frac{1}{3}$

Draw more examples on the board and ask pupils to work out the fractions for boys and girls as shown in *Figure 5.15*.

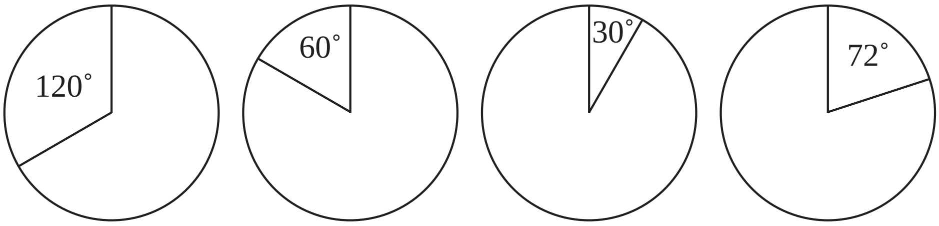


Figure 5.15

Discuss other examples such as the one given below.

Example 1

A gardener plants three types of vegetables in her garden: rape, tomato, and cabbage. The pie chart in *Figure 5.16* shows the proportions of her garden taken up by the different vegetables.

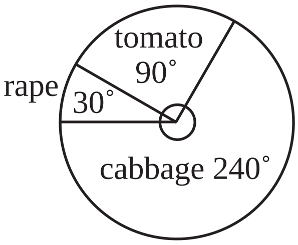


Figure 5.16

What fraction of her garden is used for growing rape? for growing tomatoes? for growing cabbage?

Discuss this example with your pupils, then give them a similar exercise to work out on their own. Use numbers that will cancel easily so the calculations are not too difficult.

Now change the question and ask them to find the angles given by each fraction. For example, a farmer grows beans and maize on his farm. If one quarter of the farm is used for growing beans, what angles will represent beans and maize on the pie chart?

Do the following calculations on the board with your pupils:

The angle for beans must be $\frac{1}{4} \times 360^\circ = 90^\circ$. The angle for maize must be $360^\circ - 90^\circ = 270^\circ$. Ask a pupil to sketch the pie chart, which is shown in *Figure 5.17*.

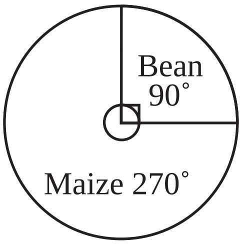
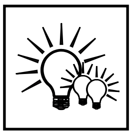


Figure 5.17

If the farmer decides to change crops next year and uses $\frac{1}{5}$ of the farm to grow groundnuts and the rest to grow cotton, what angles will represent the two crops on a pie chart?

$$\text{Groundnuts} = \frac{1}{5} \text{ of } 360^\circ = \frac{1}{\underset{1}{\cancel{5}}} \times \frac{\overset{72}{\cancel{360}}}{1} = 72^\circ$$

Cotton = $360^\circ - 72^\circ = 288^\circ$



Practice Activity 5

Prepare questions for your pupils to do as classwork on:

1. Finding the fractions of a pie chart, given the angles.
2. Finding the angles of a pie chart, given the fractions, and sketching the pie chart.

Constructing a pie chart

Consider a school with 300 pupils of whom 180 are boys and 120 are girls. To show this on a pie chart, first calculate the angles to represent the boys and girls. There are 300 pupils altogether.

The fraction of the total number of pupils made up of boys = $\frac{180}{300} = \frac{6}{10}$

Therefore, the angle representing boys = $\frac{6}{10} \times 360^\circ = 216^\circ$

The angle representing girls = $360^\circ - 216^\circ = 144^\circ$

Now draw a circle and mark the angles using a protractor, as shown in *Figure 5.18*.

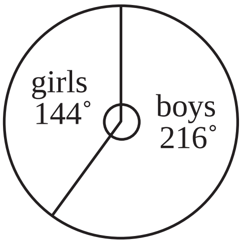


Figure 5.18

You can check your calculations by adding the angles. If the sum is 360˚, then they are correct.

Example 2

Pupils in a class of 40 were asked to give the number of meals they have each day. Ten pupils said they only have one meal a day. Sixteen said they have two meals and fourteen said they have three meals in a day. Illustrate this information in a pie chart.

First find the angle for each group of pupils:

Angles for those who have only one meal in a day $= \frac{10}{40} \times \frac{360}{1} = 90^\circ$

Angle for those who have two meals in a day $= \frac{16}{40} \times \frac{360}{1} = 144^\circ$

Angle for those who have three meals in a day $= \frac{14}{40} \times \frac{360}{1} = 126^\circ$

Draw a circle and mark in the angles 90˚, 144˚, and 126˚ as shown in *Figure 5.19*.

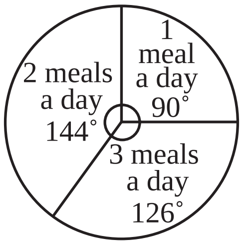


Figure 5.19

Pupils in primary school are unlikely to have a protractor to help them measure and mark angles on a pie chart. Exact measurements may not be necessary for this purpose. What is important to help pupils to get a sense of the size of angles. You can do this by using a clock face. Look at the angles between the numbers as shown in *Figure 5.20*

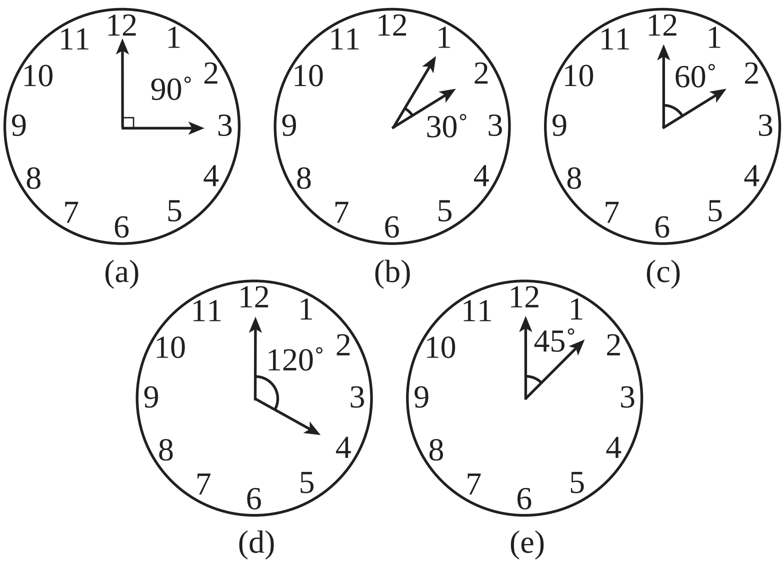


Figure 5.20

Pupils will be able to see that at the numbers 12 and 3 on the clock face make 90˚ as in *Figure 5.20(a)*.

The space between 12 and 3 is divided into equal angles. Dividing 90˚ by 3, we get 30˚.

- Therefore, the angle at the centre of the clock face between any two numbers which are next to each other is 30˚, as shown in *Figure 5.20(b)*.
- The numbers 12 and 2 make an angle of 60˚, as shown in *Figure 5.20(c)*.
- The numbers 12 and 4 make an angle of 180˚, as shown in *Figure 5.20(d)*.
- 45˚ is half of 90˚, as shown in *Figure 5.20(e)*.

These ideas will help pupils draw angles for pie charts without using a protractor.

Practice Activity 6

1. Describe how you would make the following angles on the clock face:
   (a) 75˚ (b) 10˚ (c) 100˚
2. Formulate five questions for your pupils about pie charts.

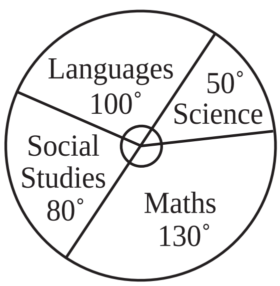
Self Assessment 2

1. The populations of eight southern Africa Commonwealth member countries are given below. The figures have been rounded to the nearest million:

Botswana	2 000 000
Malawi	10 000 000
Mozambique	16 000 000
Namibia	2 000 000
South Africa	40 000 000
Tanzania	30 000 000
Zambia	9 000 000
Zimbabwe	11 000 000

   a) Calculate the angles representing each population on a pie chart.
   b) Draw the pie chart for the populations of the eight countries.

2. A group of 72 pupils were asked to name their favourite subjects in school, and the results are represented in the following pie chart.



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a) What fraction of the group of pupils likes maths?
b) Which subject is the most popular?
c) Which is the least popular?
d) How many pupils like social studies?
e) Find the number of pupils whose favourite subject is languages. Express this number as a percentage of the total number of pupils in the groups.

Line Graph

When a baby is born, the clinic gives the mother the *under five clinic card*. A record of the baby's growth is kept on this card in the form of a graph, similar to the one in *Figure 5.21* below.

Weight (kg)	Age (months)
3.4	0 (birth)
6.0	3
7.8	6
9.2	9
10.4	12
10.9	15
11.4	18

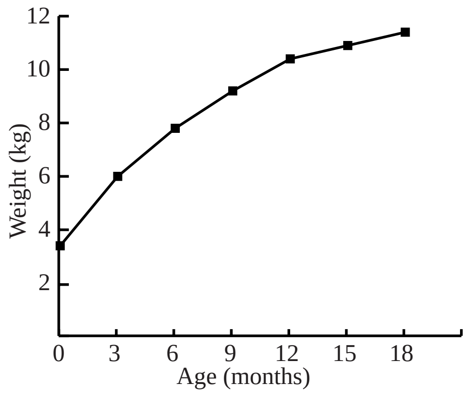


Figure 5.21: A line graph showing the growth of an infant from birth to eighteen months.

The baby's growth is indicated by the amount of weight it gained over a period of eighteen months. The baby is weighed each time it is taken to the clinic, and the weight is plotted on the graph by marking a point for the baby's weight and age. These points are joined with straight lines. This kind of data representation is called a **line graph.**

A line graph is a useful way to present data that has been collected over a period of time.

Example 3

Mutina, a grade five pupil, was taken to hospital. The doctor suspected she had malaria and her body temperature was monitored every two hours. The following table shows the change in her temperature over a period of ten hours.

Time (hrs)	08:00	10:00	12:00	14:00	16:00	18:00
Temperature (˚C)	40	42	40	39	38	37

Draw a line graph to represent this information.

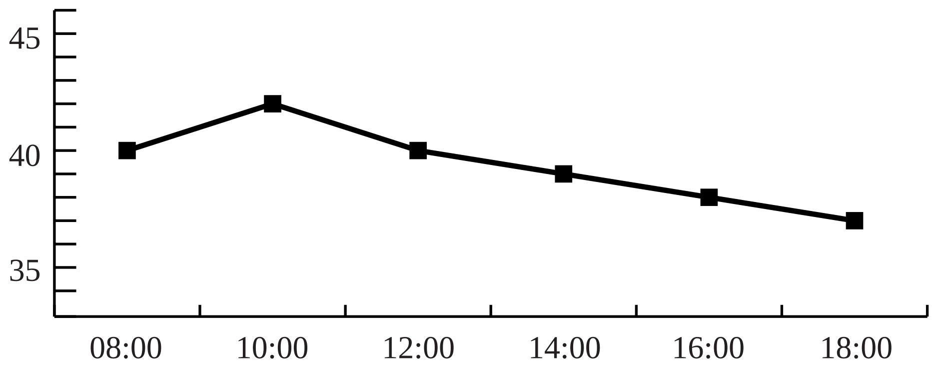


Figure 5.22

The change in Mutina's temperature is easily seen on the graph in *Figure 5.22*. Her temperature rose to 42˚C, then fell steadily until it reached normal body temperature.

From the line graph, we can predict Mutina's approximate temperature between the two hours intervals. For example, her temperature at 11:00 hours is about 41˚C, as shown by the line on the graph.

Example 4

The table below shows the marks obtained by a grade seven pupil in six weekly mathematics tests.

Week	1	2	3	4	5	6
% Mark	35	46	55	40	62	60

Show this information on a line graph.

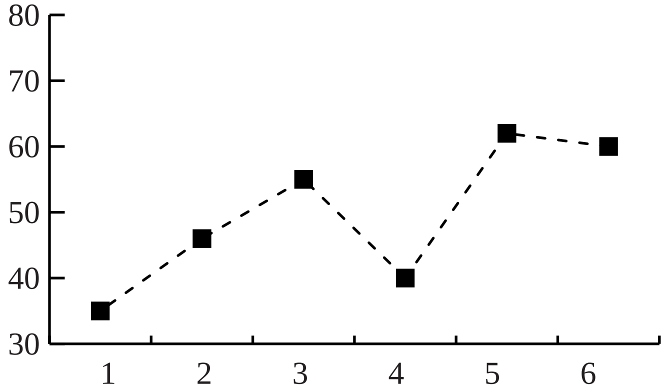


Figure 5.23

In this graph, the lines joining the points are broken, unlike in the example on temperature where the lines were smooth and continuous. The reason is that in this case the marks are discrete (that is, between them lie gaps where there are no test marks). The marks are obtained only once every week.

In Example 3, temperature is continuous. We can measure the temperature at any time in between the time intervals.

Practice Activity 5

1. Ask your pupils to carry out an experiment to observe the changes in the length of a shadow as the sun moves from morning to noon. Each child will fix a one metre stick in the football grounds and will measure the length of the stick's shadow at one-hour intervals, from 08:00 hours to 12:00 hours. Have them record their measurements in a table as shown below, and then plot the information on a line graph.

Time (h)	08:00	09:00	10:00	11:00	12:00
Length of shadow (cm)					

2. Formulate an activity for your class to carry out a survey and present the findings on a line graph.

Summary

In this unit you learned three ways to graph statistical information or data:

- pictographs
- bar graphs
- line graphs

You were also introduced to various practical activities that will help your pupils relate mathematics to real life situations.

Unit 5 Test

1. The table gives water use statistics for two villages in Mozambique.

	Village A	Village B
Drinking	1 500	4 000
Cooking	4 000	16 000
Washing Dishes	2 000	14 000
Bathing	4 000	10 000
Bathing Children	1 000	13 000
Washing Clothes	2 500	3 000
	15 000	60 000
Ratio:	1	4

   a) Draw a bar graph to represent the given information.
   b) From the given information, answer the following questions:
      (i) What was the total amount of water used per person in each village?
      (ii) What is the ratio of total water used per person in village A to village B?
      (iii) Which village do you think is closest to its water source?
      (iv) Which village do you think is healthier?

2. The table shows the readings from the under five clinic card for the weight of a baby from three to nine months of age.

Age in Months	3	4	5	6	7	8	9
Weight (kg)	5	$5\frac{1}{2}$	6	$6\frac{1}{2}$	7	$7\frac{1}{2}$	8

   Illustrate this information on a line graph.

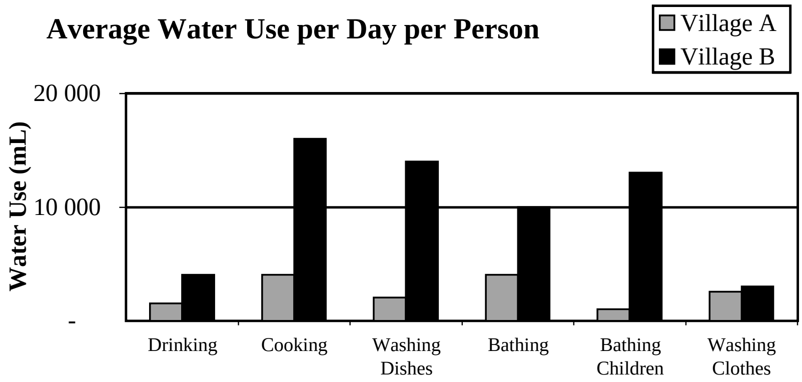
3. Mitti, a grade seven pupil at Kabanana Basic School in Lusaka, has been keeping a record of his performance in the final year of mathematics examinations from grade one to grade six. The table shows the marks.

Grade	1	2	3	4	5	6
Final exam score	70	68	85	90	80	88

   Illustrate the information on a line graph.

Answers to Unit 5 Test

1. (a)



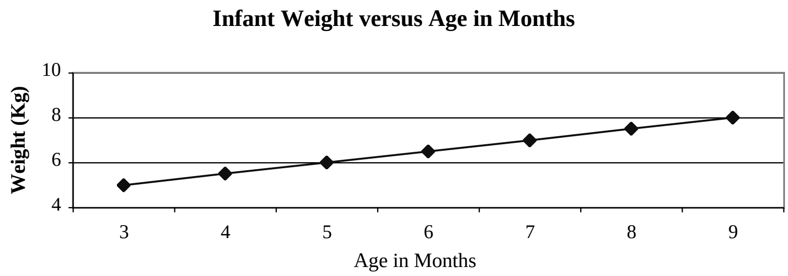
(b) (i) Village A used 15 000 mL of water per person
Village B used 60 000 mL of water per person

(ii) 1:4

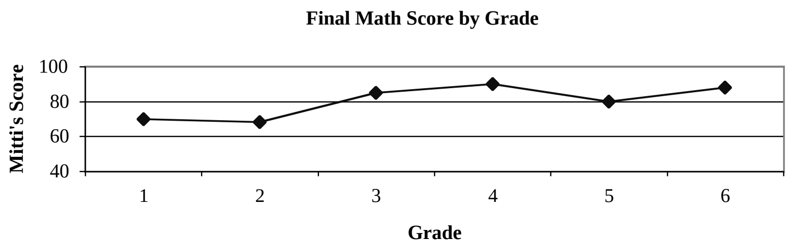
(iii) Village B

(iv) Village B

2.



3.



Unit 6: Statistics 2 – Averages

Introduction

In Unit 5 of this module you looked at two aspects of statistics: data collection and presentation. After data has been collected and presented in a more useful way, it needs to be processed and interpreted so that decisions can be made. Averages are an aspect of data processing and interpretation. In this unit you are going to learn more about averages.

It is important to point out that the word **average** refers to one type of average—the **arithmetic mean**. In this unit you will learn about three ways to determine the middle of a group of a set of numbers: the arithmetic mean, **mode, and median**. A set of numerical data is commonly referred to as scores. Each number in the set is a **score**. This unit will follow this convention, though you may prefer to teach without using the term "score".

Objectives

After working through this unit, you should be able to:

- explain the meaning of average
- calculate and use the three "averages" (mean, median, and mode)
- determine the most appropriate average to use in different situations

What is an "Average"?

Sometimes it is unnecessary to use graphical techniques to describe and analyse data. Often all that is needed is a single number that indicates the location of the centre of a set of data. Such a number is called an "average". The correct term for this number is the **measure of central tendency**, which means a score that is representative of all scores.

One common use of averages is to compare sets of data. There are three measures of central tendency:

- arithmetic mean
- median
- mode

Normally we refer to the arithmetic mean as simply the mean.

The purpose of finding the measure of central tendency is to produce a representative figure for a set of numerical data. Thus the average is a single score that is meant to represent a set of scores. Note that instead of using the word figure we can use **score**.

The measure of central tendency enables you to:

- Make comparisons between different sets of data, by comparing their means, medians, or modes.
- Make sense of individual scores in a set of scores by relating them to these averages.

Reflection

Table 6.1 shows three sets of mathematics marks for five pupils, each taken from three different classes, Grade 7A, Grade 7B, and Grade 7C. Each class claims it did better than the others. Which class scored better than the other two?

Grade 7A:	62, 94, 95, 98, 98
Grade 7B:	62, 62, 98, 99, 100
Grade 7C:	40, 62, 85, 99, 99

Table 6.1

Depending on how the scores are interpreted, all of the classes are correct in their claims.

Consider another example. *Table 6.2* shows Sandra's marks in five mathematics tests:

Test 1	Test 2	Test 3	Test 4	Test 5
69	71	78	82	73

Table 6.2 Sandra's marks

Sandra receives 69, 71, 78, 82, and 73 on her five mathematics tests. She gives her average mark as 74.6, but her friend Sipho claims Sandra's average mark is 73. Which is correct?

In this unit you will learn that both are correct. Sandra calculated the mean and Sipho used the median.

Mean

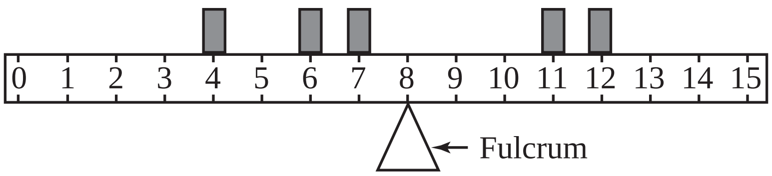


Figure 6.3

Figure 6.3 shows a **seesaw** consisting of a bar and a support called the **fulcrum**. The mean of 4, 6, 7, 11, and 12 is 8. If the fulcrum is at 8 and we place unit masses (weights) at 4, 6, 7, 11, and 12, the seesaw will balance. Can you explain why this happens?

What is the logic behind the process of finding the mean?

In the case of the seesaw, the distances of our five masses (weights) are 4, 6, 7, 11, and 12 units from 0 (the starting point). The sum of our distances are 4 + 6 + 7 + 11 + 12 = 40 units. The significance of the fulcrum balancing at 8 ($\frac{40}{5}$) is that 8 is the 'centre' of the distances.

Notice that the distances of the masses from the left side of the fulcrum balance with the distances on the right side. On the left side, 4 is 4 units, 6 is 2 units, and 7 is 1 unit away from the fulcrum, giving a total of 7 units (4 + 2 + 1). On the right, distances of the masses from the fulcrum also total 7 (3 + 4).

Example 1

Five workers in a factory earn $500, $300, $200, $400, and $600 per month. The factory owner decides that all workers will receive the same pay, but that the total amount he pays will remain the same. How much will each worker earn every month?

In this case, workers pool their earnings and the money is redistributed equally among them. To answer the question we first find the sum of 500 + 300 + 200 + 400 + 600 = 2000. The final step is to divide the sum (2000) by the number of workers (5). Each worker's pay will be:

$$2000 \div 5 = 400$$

The worker who was paid $500 has a loss of $100 per month ($500 - $400), while the worker who was paid $300 has a gain of $100 per month ($300 + 100 = 400). In this example, you used the arithmetic mean (average) to arrive at a solution.

Example 2

If Class A has a mean mark of 56% on a mathematics test, while Class B has a mean mark of 62% on the same test, which class performed better? Use the mean to compare the performance of the two classes. Class B performed better because of a higher mean.

As a teacher, many situations have probably required you to calculate the mean of a given set of your pupils' scores. Now go back to the data in *Table 6.1*. Use the two-step process to find the mean for each of the three classes—Grade 7A, Grade 7B, and Grade 7C. Which class has the highest mean? Which class had the best performance?

Grade 7A was the class with the best performance, with a mean of 89.4, compared to Grades 7B and 7C whose means are 84.2 and 77 respectively. Note—the mean has been used to compare the three sets of scores.

In general, if you have n scores $x_1, x_2, x_3,$ ——-, x_n

$$\text{Mean} = \frac{x_1 + x_2 + x_3 + \text{-----} + x_n}{n}$$

Unit Activity 1

1. Find the arithmetic mean of the following sets of scores:
   a) 8, 16, 4, 12, 10
   b) 25, 25, 25, 25, 30, 30, 30, 40, 40, 40, 40, 50
   c) 3, 6, 2, 6, 5, 6, 4, 1, 1
2. The mean score of a set of eight scores is 65. What is the sum of the eight scores?

Frequency Tables

Data can be handled better if it is well organized. Consider the marks for the Grade 7A class in *Table 6.1*. We can arrange the scores in form of a table as shown in *Table 6.3* below:

Score	62	94	95	98
Freq.	1	1	1	2

Table 6.3

Table 6.3 consists of two rows, one for the scores and the other for **frequency**. Frequency is the number of times a particular score appears in the set of data.

Instead of adding 62 + 94 + 95 + 98 + 98, you can write $52 \times 1 + 94 \times 1 + 95 \times 1 + 98 \times 2$ to find the sum. For example, 62 appears once, so you write 62×1. For 98, it appears twice so you write 98×2, i.e., 98 + 98. The sum is 447. Therefore, to calculate the mean, you would write:

$$\frac{62 \times 1 + 94 \times 1 + 95 \times 1 + 98 \times 2}{5} = 89.4$$

In Unit Activity 1, question 1(b) we have the scores: 25, 25, 25, 25, 30, 30, 30, 40, 40, 40, 40, 50. Arranged in a frequency table you will have:

Score	25	30	40	50
Freq.	4	3	4	1

For example, instead of writing 25 + 25 + 25 + 25 to find the sum you will write 25×4.

$\therefore$ Sum of scores = $25 \times 4 + 30 \times 3 + 40 \times 4 + 50 \times 1$

$$\text{The mean score} = \frac{25 \times 4 + 30 \times 3 + 40 \times 4 + 50 \times 1}{12}$$

Note—adding the frequencies gives the number of scores:

4 + 3 + 4 +1 = 12.

Unit Activity 2

1. Find the arithmetic mean of the following data:

(a)

Score	4	14	24	34
Freq.	2	8	20	10

(b)

Score	17	22	27	32	37
Freq.	4	7	5	2	2

What is Median?

The median is simply the middle number of a set of scores when the scores are arranged in numerical order. Finding this average is easier than calculating the mean.

Consider again the marks in *Table 6.1*. What is the median for each of these sets of data? Scores for Grade 7A class are:

62, 94, (95), 98, 98 (arranged in increasing order)
98, 98, (95), 94, 62 (arranged in decreasing order)

Clearly, 95 is the middle score. Hence median mark = 95.
Note—there are 5 scores and 95 is in the third position.

What is the median mark for Grade 7B and Grade 7C? (98, 85)
Using the median, Grade 7B class had the best performance.

Reflection

What is the median for this set of data?

64, 68, 70, 74, 82, 90

It appears there is no middle number, since there an even number of scores. In this case, the median is found by calculating the mean of the middle two scores. To find the median, we add the middle two scores and divide by two. What are the middle scores for the following data?

64, 68 $\underline{70, 74}$, 82, 90

70 and 74 are the middle two scores

$\therefore$ the mean is $\frac{70 + 74}{2} = 72$

If you have seven scores, what will be the position of the median? What will be the median position if you have ten scores?

Continues on next page

The following steps will help you find the median for a set of *n* scores:

Arrange the numbers in numerical order, from least to greatest.
(a) If *n is* odd, the median is the middle score.
(b) If *n* is even, the median is the mean of the two middle numbers.

What is Mode?

The mode is the score in the data set that occurs most frequently.

Find the mode for each of the set of scores in *Table 6.1*.

62, 94, 95, 98, 98	mode = 98
62, 62, 98, 99, 100	mode = 62
40, 62, 85, 99, 99	mode = 99

Reflection

What is the mode for these sets of scores?

a) 64, 79, 80, 82, 90 (has no mode or five modes)

b) 64, 75, 75, 82, 90, 90 (bimodal—two modes)

Unit Activity 3

Find the median and the mode for the following data sets:

(i) 60, 95, 100, 60, 95, 70

(ii) 25, 2, 5, 6, 5, 23, 7, 10, 22, 15, 21, 23

Mean, Mode, or Median?

Although the mean is the measure of central tendency most often used to represent a data set, it may not always be the best to use.

A manufacturing firm pays its director US$2000 per month, the deputy director is paid US$1500 per month, and eighteen other employees earn US$50 each per month. What is the mean salary for this company? The mean salary is US$220.

Is the mean salary of US$220 a good representative of all the workers' salary at this firm? The mean is affected by the extreme values US$2000 and US$1500. In this case, the mode and the median are both US$50, and better describe the typical salary. In most cases the median is not affected by extreme values (much larger or smaller than the rest).

The median, however, can be misleading. For example, nine pupils receive the following marks on a science test:

30, 35, 40, 40, 92, 92, 93, 98, 99.

When the scores are arranged in numerical order, you can see the median is 92. Is this score an accurate representation of this data? If you are told, without seeing the scores, that the median is 92, you might conclude that all individuals scored very well. However, 92 is not typical for the entire set of scores.

The mode, too, can be misleading. Suppose five people in South Africa are paid as indicated below:

R1000 R2000 R500 R8000 R8000

What is the modal pay? (R8000). R8000 is definitely not a representative score. Why? In most cases, if you have few scores, many frequently occurring scores, or when the range of values covered by the data is quite large, the mode will not be suitable.

A good use of the mode applies to a discussion of the ‘average’ family. In the SADC countries, for example, the modal number of children in a family is five, because more families have five children than any other number. Given that the mean is ‘5.3 children’, clearly the mode is more useful than the mean, since it is not possible for a family to actually have 5.3 children.

Self Assessment 1

1. Calculate the mean, mode, and median for each of the following sets of data.
   (a) 2, 8, 7, 8, 5, 8, 10, 5
   (b) 5, 5, 5, 5, 5, 10
   (c) 18, 22, 22, 17, 30, 18, 12
2. (a) If each of six pupils scored 80 on a test, find each of the following for the set of 6 scores:
   (i) mean (ii) median (iii) mode
   (b) Make up another set of six scores that are not at all the same but in which the mean, median, and mode are all 80.
3. (a) Give a set of scores where the mean will not be representative.
   (b) Give a set of scores where the median will not be representative.
   (c) Give a set of scores where the mode will not be representative.

Summary

This unit explored the idea of using an “average”, or a measure of central tendency, as a representative score for a set of data. Three measures of central tendency were covered—the mean, median, and mode—and we examined situations where their use is appropriate.

Unit 6 Test

1. Find the mean, median, and mode for the following data:
   (a) 4, 6, 6, 8, 10, 20
   (b) 7, 1, 3, 1, 4, 6, 5, 2
2. (a) Which is the most appropriate "average" to use for data in (a)? Why?
   (b) Why will the mode not be suitable for the data in (b)?
3. Make up a set of data with four or more scores, not all of which are equal, with each of the following characteristics:
   (a) The mean and median are equal.
   (b) The mean, median, and mode are equal.
4. The mean for a set of 28 scores is 80. Suppose two more pupils take the test and score 60 and 50. What is the new mean?
5. The names and ages for each person in a family of five follow:

Name	Radebe	Ramatsui	Banda	Matongo	Zandile
Age	40	36	8	6	2

   (a) What is the mean age?
   (b) Find the mean of the ages five years from now.
   (c) Find the mean ten years from now.
   (d) Describe the relationships among the means found in (a), (b), and (c).

Answers to Unit Activities

Unit Activity 1

1. (a) 10 (b) $33\frac{1}{3}$ (c) $3\frac{7}{9}$
2. 520

Unit Activity 2

1. (a) 23.5 (b) 24.75

Unit Activity 3

(a) Median = 82.5 The set of data is bimodal and has both 60 and 95 as modes.

(b) Median = 12.5
Mode = 5

Answers to Self Assessment 1

1. (a) 6.6 $7\frac{1}{2}$ 8
   (b) 5.8 5 5
   (c) 19.9 18 18 or 22
2. (a) (i) 80 (ii) 80 (iii) 80
   (b) 80, 78, 82, 80, 80, 80
3. (a) 6, 6, 6, 6, 100
   (b) 1, 1, 2, 3, 41, 50, 70
   (c) 4, 4, 50, 60, 90, 102

Answers to Unit 6 Test

1. (a) 9 7 6
   (b) 3.6 3.5 1
2. (a) 7 because the median because it is the most typical of the data set.
   (b) 1 represents an extreme score and is therefore not typical.
3. (a) 70, 60, 65, 85, 70
   (b) 70, 60, 65, 85, 70
4. 78.318
5. (a) 18.4 years or $18\frac{2}{5}$ years.
   (b) 23.4
   (c) 28.4
   (d) If the number of years are increased by 5, the mean increases by 5.
   $\therefore$ The mean in (b) is 5 more than that in (a).
   and mean in (c) is 10 more than that in (a).

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Appendix 1: Distance Chart for Towns in Zambia

	Chingola	Chipata	Chirundu	Choma	Kabwe	Kafue	Kapiri-Mposhi	Kasama	Kawamba	Kitwe	Landless Comer	Livingstone	Luanshya	Lundazi	Lusaka	Mansa	Mazabuka	Mbala	Mongu	Mpika	Mufulira	Mumbwa	Mwinilunga	Ndola	Nyimba	Solwezi
Chingola																										
Chipata	978																									
Chirundu	547	703																								
Choma	696	851	308																							
Kabwe	273	705	274	423																						
Kafue	455	612	91	241	183																					
Kapiri-Mposhi	205	773	342	491	68	251																				
Kasama	575	1419	988	1139	714	897	646																			
Kawamba	403	1296	865	1014	591	773	523	346																		
Kitwe	52	926	495	644	221	413	153	799	384																	
Landless Comer	343	635	204	353	70	113	138	784	661	291																
Livingstone	885	1040	498	188	611	429	679	1325	1202	883	541															
Luanshya	102	898	467	616	193	376	125	770	435	50	263	804														
Lundazi	1154	181	879	1027	881	788	949	1595	1472	1102	811	1216	1074													
Lusaka	411	567	136	284	138	44	206	852	729	359	68	473	331	746												
Mansa	235	1128	697	846	422	604	354	340	168	216	492	1034	267	1304	561											
Mazabuka	536	692	148	159	263	80	331	977	853	483	193	349	456	868	125	685										
Mbala	1014	1582	1151	1300	877	1060	809	164	510	962	947	1488	934	1759	1015	504	1140									
Mongu	897	1150	719	868	624	627	692	1338	1214	845	554	531	816	1326	583	1046	708	1501								
Mpika	639	1207	776	925	502	685	434	211	558	587	572	1113	559	1384	640	551	765	375	1126							
Mufulira	61	953	523	671	248	430	180	515	343	42	318	860	93	1129	386	175	511	678	871	614						
Mumbwa	458	719	288	437	185	196	253	899	775	406	115	625	378	895	152	607	277	1062	439	687	432					
Mwinilunga	462	1439	1008	1157	735	916	666	1037	864	513	804	1346	564	1615	872	696	997	1488	789	1100	522	919				
Ndola	111	888	457	607	183	366	115	761	407	59	253	792	32	1064	321	240	446	924	807	549	65	368	572			
Nyimba	747	231	472	620	474	380	542	1188	1065	695	404	809	667	410	336	897	461	1351	919	976	722	488	1208	657		
Solwezi	173	1151	719	869	446	628	378	747	576	225	516	1055	276	1327	584	408	706	1200	608	812	234	631	289	284	920	
Zambezi	690	1588	1157	1305	883	1066	815	1265	1093	662	953	1070	713	1764	1021	925	1146	1429	539	1249	750	1068	389	721	1357	517

Table 6.1: Distances between towns in Zambia

Appendix 2: Map of Zambia