Module 3

Upper Primary Mathematics

Measures

The Commonwealth of Learning

Science, Technology and Mathematics Modules

**for Upper Primary and Junior Secondary School Teachers
of Science, Technology and Mathematics by Distance
in the Southern African Development Community (SADC)**

**Developed by
The Southern African Development Community (SADC)**

Ministries of Education in:

- **Botswana**
- **Malawi**
- **Mozambique**
- **Namibia**
- **South Africa**
- **Tanzania**
- **Zambia**
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In partnership with The Commonwealth of Learning

Published jointly by The Commonwealth of Learning and the SADC Ministries of Education.

Residents of the eight countries listed above may obtain modules from their respective Ministries of Education. The Commonwealth of Learning will consider requests for modules from residents of other countries.

ISBN 1-895369-73-8

SCIENCE, TECHNOLOGY, AND MATHEMATICS MODULES

This module is one of a series prepared under the auspices of the participating Southern African Development Community (SADC) and The Commonwealth of Learning as part of the Training of Upper Primary and Junior Secondary Science, Technology and Mathematics Teachers in Africa by Distance. These modules enable teachers to enhance their professional skills through distance and open learning. Many individuals and groups have been involved in writing and producing these modules. We trust that they will benefit not only the teachers who use them, but also, ultimately, their students and the communities and nations in which they live.

The twenty-eight Science, Technology, and Mathematics modules are as follows:

Upper Primary Science
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Module 2: *Materials in my Environment*
Module 3: *My Health*
Module 4: *My Natural Environment*

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Module 4: *Social Arithmetic*
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Module 2: *Number Operations*
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Module 4: *Algebraic Processes*
Module 5: *Solving Equations*
Module 6: *Data Handling*

A MESSAGE FROM THE COMMONWEALTH OF LEARNING

The Commonwealth of Learning is grateful for the generous contribution of the participating Ministries of Education. The Permanent Secretaries for Education played an important role in facilitating the implementation of the 1998-2000 project work plan by releasing officers to take part in workshops and meetings and by funding some aspects of in-country and regional workshops. The Commonwealth of Learning is also grateful for the support that it received from the British Council (Botswana and Zambia offices), the Open University (UK), Northern College (Scotland), CfBT Education Services (UK), the Commonwealth Secretariat (London), the South Africa College for Teacher Education (South Africa), the Netherlands Government (Zimbabwe office), the British Department for International Development (DFID) (Zimbabwe office) and Grant MacEwan College (Canada).

The Commonwealth of Learning would like to acknowledge the excellent technical advice and management of the project provided by the strategic contact persons, the broad curriculum team leaders, the writing team leaders, the workshop development team leaders and the regional monitoring team members. The materials development would not have been possible without the commitment and dedication of all the course writers, the in-country reviewers and the secretaries who provided the support services for the in-country and regional workshops.

Finally, The Commonwealth of Learning is grateful for the instructional design and review carried out by teams and individual consultants as follows:

- Grant MacEwan College (Alberta, Canada):
  General Education Courses
- Open Learning Agency (British Columbia, Canada):
  Science, Technology and Mathematics
- Technology for Allcc. (Durban, South Africa):
  Upper Primary Technology
- Hands-on Management Services (British Columbia, Canada):
  Junior Secondary Technology

Dato' Professor Gajaraj Dhanarajan
President and Chief Executive Officer

ACKNOWLEDGEMENTS

The Mathematics Modules for Upper Primary and Junior Secondary Teachers in the Southern Africa Development Community (SADC) were written and reviewed by teams from the participating SADC Ministries of Education with the assistance of The Commonwealth of Learning.

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UPPER PRIMARY MATHEMATICS PROGRAMME

Introduction

Welcome to the programme in Teaching Upper Primary Mathematics. This series of five modules is designed to help you strengthen your knowledge of mathematics topics and acquire more instructional strategies for teaching mathematics in the classroom.

Each of the five modules in the mathematics series provides an opportunity to apply theory to practice. Learning about mathematics entails the development of practical skills as well as theoretical knowledge. Each topic includes examples of how mathematics is used in practice and suggestions for classroom activities that allow students to explore the maths for themselves.

Each module also explores several instructional strategies that can be used in the mathematics classroom and provides you with an opportunity to apply these strategies in practical classroom activities. Each module examines the reasons for using a particular strategy in the classroom and provides a guide for the best use of each strategy, given the topic, context, and goals.

The guiding principles of these modules are to help make the connection between theory and practice, to apply instructional theory to practice in the classroom situation, and to support you, as you, in turn, help your students to apply mathematics to practical classroom work.

Programme Goals

This programme is designed to help you:

- strengthen your understanding of mathematics topics
- expand the range of instructional strategies that you can use in the mathematics classroom

Programme Objectives

By the time you have completed this programme, you should be able to:

- guide students as they work in teams on practical projects in mathematics, and help them to work effectively as members of a group
- use questioning and explanation strategies to help students learn new concepts and to support students in their problem solving activities
- prepare your own portfolio of teaching activities

The relationship between this programme and the mathematics curriculum

The content presented in these modules includes some of the topics most commonly covered in the mathematics curricula in southern African countries. However, it is not intended to comprehensively cover all topics in any one country's mathematics curriculum. For this, you need to consult your national or regional curriculum guide. The curriculum content presented in these modules is intended to:

- provide an overview of the content in order to support the development of appropriate teaching strategies
- use selected parts of the curriculum as examples of the application of specific teaching strategies
- explain those elements of the curriculum that provide essential background knowledge, or that address particularly complex or specialised concepts
- provide directions to additional resources on the curriculum content

How to work on this programme

As is indicated in the goals and objectives, this programme requires you to participate actively in each module by applying instructional strategies when exploring mathematics with your students and by reflecting on that experience. There are several ways to do this.

Working on your own

You may be the only teacher of mathematics in your school, or you may choose to work on your own so you can accommodate this programme within your schedule. If this is the case, these are the recommended strategies for using this module:

1. Establish a schedule for working on the module. Choose a date by which you plan to complete the first module, taking into account that each unit will require between six and eight hours of study time and about two hours of classroom time to implement your lesson plan. For example, if you have two hours a week available for study, then each unit will take between three and four weeks to complete. If you have four hours a week for study, then each unit will take about two weeks to complete.
2. Choose a study space where you can work quietly without interruption, such as a space in your school where you can work after hours.
3. If possible, identify someone who is interested in mathematics or whose interests are relevant to it (for example, a science teacher in your school) with whom you can discuss the module and some of your ideas about teaching mathematics. Even the most independent learner benefits from good dialogue with others. It helps us to formulate our ideas—or as one learner commented, "How do I know what I'm thinking until I hear what I have to say?"

Working with colleagues

If there are other teachers of mathematics in your school or in your immediate area, then it may be possible for you to work together on this module. You may choose to do this informally, perhaps having a discussion group once a week or once every two weeks about a particular topic in one of the units. Or, you may choose to organise more formally, establishing a schedule so that everyone is working on the same units at the same time, and you can work in small groups or pairs on particular projects.

Your group may also have the opportunity to consult with a mentor, or with other groups, by teleconference, audioconference, letter mail, or e-mail. Check with the local coordinator of your programme about these possibilities so you can arrange a group schedule that is compatible with these provisions.

Colleagues as feedback/resource persons

Even if your colleagues are not participating directly in this programme, they may be interested in hearing about it and about some of your ideas as a result of taking part. Your head teacher or the local area specialist in mathematics may also be willing to take part in discussions with you about the programme.

Working with a mentor

As mentioned above, you may have the opportunity to work with a mentor, someone with expertise in maths education who can provide feedback about your work. If you are working on your own, communication with your mentor may be by letter mail, telephone, or e-mail. If you are working as a group, you may have occasional group meetings, teleconferences, or audioconferences with your mentor.

Resources available to you

Although these modules can be completed without referring to additional resource materials, your experience and that of your students can be enriched if you use other resources as well. A list of resource materials is provided at the end of each module. You might also find locally available resource material that will enhance the teaching/learning experience. These include:

- manipulatives, such as algebra tiles, geometry tiles, and fraction tiles
- magazines with articles about maths
- books and other resources about maths that are in your school or community library

ICONS

Throughout each module, you will find some or all of the following icons or symbols that alert you to a change in activity within the module.

Read the following explanations to discover what each icon prompts you to do.

	Introduction	Rationale or overview for this part of the course.
	Learning Objectives	What you should be able to do after completing this module or unit.
	Text or Reading Material	Course content for you to study.
	Important—Take Note!	Something to study carefully.
	Self-Marking Exercise	An exercise to demonstrate your own grasp of the content.
	Individual Activity	An exercise or project for you to try by yourself and demonstrate your own grasp of the content.
	Classroom Activity	An exercise or project for you to do with or assign to your students.
	Reflection	A question or project for yourself—for deeper understanding of this concept, or of your use of it when teaching.
	Summary	
	Unit or Module Assignment	Exercise to assess your understanding of all the unit or module topics.
	Suggested Answers to Activities	

CONTENTS

Module 3: Measures

Module 3
Measures

Introduction to the Module

Numbers form the basis for all other mathematics. In modules 1 and 2, we looked at numbers and the four operations of addition, subtraction, multiplication, and division. In this unit, we will discuss measures and for measures, we use numbers.

The common measures we use in everyday life are length, area, weight (usually confused with mass), volume and capacity, time, currency, and temperature. These are the measures you will use in this module.

When teaching measures, you should strive to help pupils to develop a conceptual knowledge of measuring as you teach them how to measure. Conceptual knowledge of measure means:

- Understanding the nature of the measure. This is achieved by making comparisons of *less than* and *more than*, without using standard units.
- Understanding how the use of standard units produces a measure. This is achieved by using physical models of measuring units.
- Understanding the way measuring instruments work. This is achieved by involving pupils in making the instruments and using them along with unit models.

In measuring, it is important to emphasise approximation because it is difficult to measure accurately. Estimation plays an important part in understanding measurement. You should, therefore, encourage pupils to estimate measurements before measuring. This helps them conceptualise the unit of measure, leading to familiarity with the unit and the process of measuring.

Aim of the Module

The module aims to enable you to:

- review your knowledge and understanding of the concepts of the stated measures
- deepen your knowledge of the subject matter
- sharpen your skills and improve your strategies for teaching the measures to your primary school pupils

Structure of the Module

This series of primary mathematics modules has been structured in a way that makes learning easier if you follow one after the other. However, adherence to the order will largely depend on your background knowledge of mathematics and your ability to structure the lessons you teach.

This module is made up of seven units. Unit 1 covers the measure of length and perimeter, followed by the measure of area in Unit 2. Unit 3 covers volume and capacity and Unit 4 looks at mass and weight. The concept of time as a measure is developed in Unit 5, Unit 6 looks at currency, and finally we discuss temperature in Unit 7.

Each unit provides a series of unit activities for you to work through to help you organise and consolidate your content knowledge. There is also a series of practice activities for you to practice with your pupils in class. The self assessment exercises will test your mastery of the subject content.

You are advised to create a file and keep a record of all the activities you do in this module. This will be required at a later stage by your supervisor.

Objectives of the Module

After working through this module, you should be able to:

- explain the meaning of each of these measures: length, area, volume and capacity, mass and weight, time, currency, and temperature
- distinguish between mass and weight
- distinguish between volume and capacity
- effectively teach primary school pupils the concepts of length, area, volume and capacity, mass and weight, time, currency, and temperature
- create activities that make your teaching of these concepts more practical and enjoyable for your pupils

Unit 1: Length and Perimeter

Introduction

In this unit you will deal with the meaning of length and perimeter, and develop skills to measure length. You will work through activities for developing the concept of length and perimeter, such as comparison and measuring using standard and non-standard units. Two types of measurement will be used. Estimation will form a major component of the activities. Pupils need to have a sound understanding of length before they learn about area and volume.

Objectives

After working through this unit you should be able to:

- devise activities to compare length and perimeter, using standard units and non-standard units of measure
- develop practical activities to demonstrate to your pupils the concept of measuring length and perimeter
- develop the formula for perimeter
- solve problems involving measuring length

Reflection

When given a broken ruler, Jane fails to measure the line segment (*Figure 1.1*) and asks for an unbroken ruler. Either Jane does not understand how to measure or she does not understand the meaning of measurement.

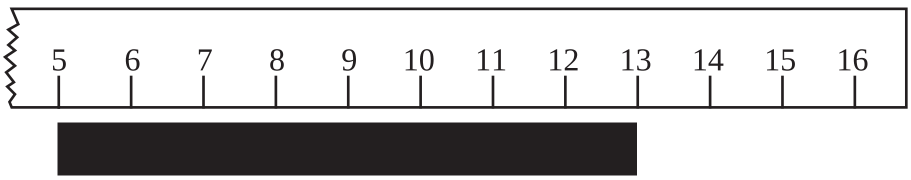


Figure 1.1: How long is this segment?

Many teachers teach pupils "how to measure" instead of "what it means to measure". Does this amaze you? What is meant here is that some teachers will emphasise the importance of putting the starting point of a ruler "0" at the beginning of the object being measured.

Measuring Length

To measure length, we pick a unit of measure that has an attribute of length. To help your pupils develop the concept of length, have them work through the activities on comparing and measuring using standard and non-standard units of measure.

Comparing Length

Comparing the length of two or more objects does not require a measurement. You may be aware of the two types of comparison—direct and indirect. At the upper primary level, we concentrate on indirect comparisons of length. Using a string, we will compare lengths that are straight with those that are not straight.

Before asking your pupils to make comparisons, let them first guess the order of lengths—from shortest to longest. After they make the actual comparison, have them compare that with their guesses.

By learning to make estimates and comparisons, your pupils will eventually understand that the length of an object remains unchanged, even if the object changes position and appears to be longer or shorter.

Reflection

Is it true that your height is about the length of your outstretched arms from thumb to thumb?

Unit Activity 1

- Identify parts of the body that have an attribute of length.
- Which parts of the body have the same length?
- Identify relationships between parts of the body that have the same length, twice the length, three times the length, and so on.

Practice Activity 1

Comparing parts of the body

Materials: Large squares of paper, big cubes, string

- Have pupils work together in co-operative learning groups.
- Let them identify parts of the body that have measurable properties of length.
- Let them compare, using indirect comparison, those parts of the body that are in different orientations so that a simple visual comparison is difficult.
- Have pupils identify the parts that are longer than others and those that are about the same length.
- Let pupils decide on a method of comparison.
- Let pupils report to the class how they arrived at their decisions.
- Did the pupils discover that, in each person, the length of the shoulders are about the same length as the circumference of the base of the neck?
- What discoveries have your pupils made?

Continues on next page

Comparing crooked lines

Materials: strings that are longer than the lines

- Make crooked lines and straight lines of the same length on the floor. (Hint: to make a crooked line, trace along a rope)
- Have pupils work in pairs. Ask them to compare the crooked line and the straight line.
- Observe the strategies pupils use to solve the problem.

Making a crooked path as long as a given straight line

Materials: string longer than the straight line

- Draw a straight line.
- Have pairs of pupils draw various crooked lines just as long as the straight line.
- Let them demonstrate to the class why they think their crooked lines are just as long as the straight line.

Optional Activity (Test for unchanged length)

Materials: two sticks of the same length

- Position two sticks of equal length in the same orientation, as shown in *Figure 1.2(a)*. Ask your pupils which stick is longer.
- Rearrange the sticks, as shown in *Figure 1.2(b)*. Ask your pupils which stick is longer and how they know.

Figure 1.2(a) *Figure 1.2(b)*

- Let your pupils compare the sticks and rearrange them in other positions. This will help them understand that an object's length does not change, regardless of position.

Non-Standard Unit of Measure

The first step in the measurement process is the choice of the unit of measure. Under non-standard units of measure of length, there are natural **units of measure** and **unit models**. Natural units of measure include body parts that have an attribute of length, such as thumb, hand, span (the greatest distance between the tips of the thumb and little finger), cubit (the length of the forearm, including the outstretched hand), arms, foot, and pace (the distance covered in a single step) (*Figure 1.3*). Unit models include footprints, measuring strips, measuring ropes, straws, and sticks. A string of straws can bridge the gap between a ruler and a measuring tape.

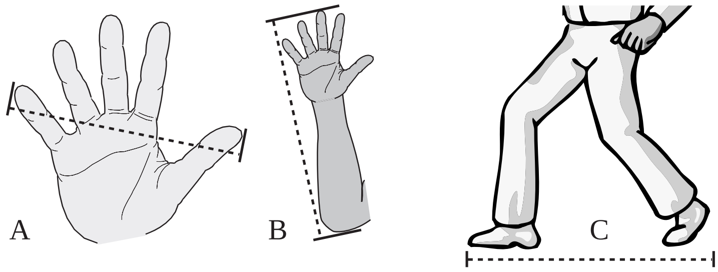


Figure 1.3: Examples of natural units of measure. (a) span (b) cubit (c) pace

Encourage pupils to invent their own units of measure. This will lead them to get different measurements for the same length and will demonstrate the need for a standard unit of measure.

Unit Activity 2

Investigation

Natural Unit of Measure

And there went out a champion out of the camp of the Philistines named Goliath of Gath, whose height was six cubits and a span. (1 Samuel 17.4)

- Ask one of your colleagues to work with you on this investigation.
- Using the cubit and span (*Figure 1.3*), cut a string that is as long as Goliath's height.
- Cut another piece of string, using your colleague's cubit and span.
- Compare the length of your string with that of your colleague. Are the lengths the same? If not, explain why.
- Explain the difficulties that could arise if a natural unit of measure was used for measuring in business.

Unit Models

- Once again, work with your partner.
- Together, decide to measure something that has an attribute of length.
- To measure the object, work independently to produce a measuring device using a unit of your choice.
- Work together to measure the object, using each unit. Did you have any difficulty communicating your measurements?
- Discuss the difficulties that might arise in the world if each nation had a local unit of measure of length different from others.

Practice Activity 2

1. Measuring using natural units of measure:
   - Ask your pupils to work in groups.
   - Have them make lists of things to measure in the classroom.
   - Let each group decide on the natural unit of measure they will use.

Continues on next page

- Encourage pupils to estimate before measuring.
- Have the groups take the measurements, using their natural units of measure, and record the results in a table.

Name of object	Guess	Measure
Height of door		

- Within each group, have pupils compare their estimates with the actual measurement.
- Have the groups compare their actual measurements. Did the groups have any difficulty communicating their measurements?
- Encourage your pupils to discuss the difficulties arising from the use of natural units of measure.

2. Estimation

   Materials: small unit, larger unit that is a multiple of the smaller one

   - Ask your pupils to estimate a distance before they measure it.
   - Have pupils measure a distance using the smaller unit.
   - Give pupils a larger unit of measure to measure the same distance.
   - Have them record their estimation and the actual measurement.
   - Ask pupils to explain how they arrived at their estimations.
   - Have your pupils estimate and measure different lengths using the same units of measure, and have them explain how they arrive at their estimation.
   - Are your pupils' estimations becoming more accurate?

3. Measuring with different units
   - Once again, have your pupils work in groups.
   - Ask each group to estimate and then measure the length and width of their desks. Each group should create its own unit of measure.
   - Have groups compare their actual measurements.
   - Have your pupils discuss the difficulties arising from each group using a different unit measure. Ask them to find a way to compromise or communicate effectively in terms of measuring length.

Standard Unit of Measure of Length

Your pupils may have realised on their own the need to have a common unit of measure for easy communication. This is what made nations come together and create standard units of measure of length. As stated earlier, we will use the metric system. The common standard units of measure in metric measurements are the millimetre, centimetre, metre, and kilometre, with the metre being the basic unit.

Develop the concept of these units of measure through estimation. Find objects that are about the same as a metre, centimetre, and kilometre. Several metric system devices are used to measure length, and some are shown in *Figure 1.4*. You should tell your pupils how each device is used, then let your pupils make 1 metre (m) and 1 centimetre (cm) measuring instruments.

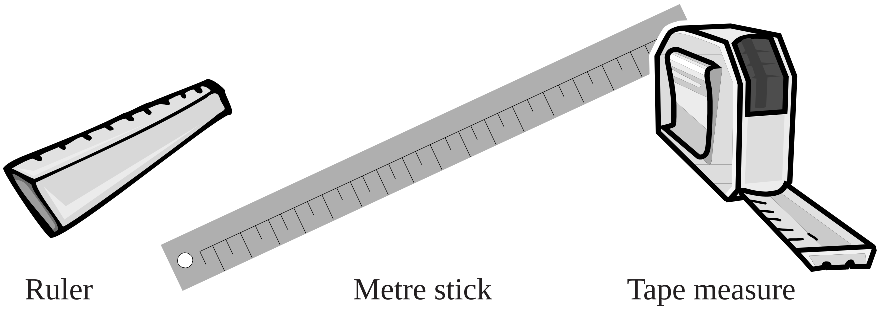
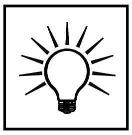


Figure 1.4

Reflection

When recording measurement, is it appropriate to record using mixed units, for instance 1 m 25 cm?



Unit Activity 3

Benchmarks

Materials: metric stick and centimetre ruler

- Find metric benchmarks on your body and in your classroom.
- Use your body benchmarks to measure short lengths and long distances such as length of your classroom block.
- Check your measurements with measures made with a centimetre ruler, a tape measure, or metric ruler.

Body Lengths

Materials: metric tape measure, metre stick, centimetre ruler

a) For each of the following, estimate and measure to the nearest indicated unit, enter the measurements in a table similar to *Figure 1.5*, then calculate the difference between the estimate and the actual measurement:
   - your waist measurement (cm)
   - the length of your arm (metres, use decimals)
   - the base of your neck (cm)
   - the length of your foot (cm)
   - your height (metres use decimals)
   - another body length of your choice

Continues on next page

b)

Object	Estimation	Measurement	Difference between estimates and measure

Figure 1.5

Length of Step

Materials: metre stick, trundle wheel

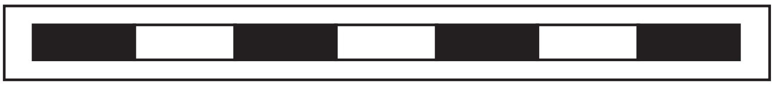
- Measure a distance of 10 metres.
- Walk the distance counting each step. Do this several times.
- Find the average number of steps required for you to walk 10 metres.
- Calculate the length of your average step.
- Walk a distance of your own choice. Take note of the number of steps. Calculate the distance using the knowledge of the length of your average step.
- Find out how many steps it takes to cover the distance from your classroom door to the Headmaster's office. Use the number of steps to estimate the distance.

Practice Activity 3

1. Making a ruler

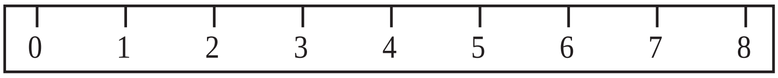
   Materials: strips of construction paper in two contrasting colours

   - Have your pupils use narrow, one-centimetre strips of construction (manila) papers in two contrasting colours to measure lengths of less than 10 cm. Then let them make rulers (*Figure 1.6*) by pasting 1 cm units of alternating colours along their ruler.



First rulers: Pupils count units

Second rulers: Numbers help to count. Numbers in centre of units



Standard rulers: Numbers are at end of units. Notice the position of 0

Figure 1.6

Continues on next page

2. Using the ruler

   Materials: rulers made by pupils

   - Have pupils estimate and measure lengths shorter than a ruler by using unit models.
   - Let pupils discuss how they measure in centimetres and identify lengths that are one centimetre long.
   - Now, using the rulers they made, have your pupils re-measure the lengths shorter than a ruler.
   - Have pupils find different ways to measure the same length using their ruler. They can start measuring from any point of the ruler, start from either end or at a point not at the end, or measure in sections and add the results.

3. Benchmarks

   Materials: sticks longer than 1 metre

   - Have your pupils make 1 metre sticks with 1 cm units of measure. Remind your pupils that 100 centimetres = 1 metre.
   - Have pupils find 1 metre and 1 centimetre benchmarks on their bodies (refer to unit activity).

4. Prepare an activity that requires your pupils to estimate and measure the lengths of parts of their bodies. Do they still remember which parts of the body have the same length?

5. Measuring in decimals

   Materials: standard rulers in cm and mm

   - Have your pupils work together in groups.
   - Demonstrate how to measure lengths in millimetres, and how to record the measurement using one unit (e.g. 12 mm).
   - Have pupils convert their measurement from millimetres to centimetres, using decimals (e.g. 1.2 cm).

6. Kilometre walk

   Materials: trundle wheel or metre stick

   - Measure with pupils a one kilometre length using a metre length (1000 m = 1 km)
   - Ask pupils to identify a landmark that is approximately one kilometre away.
   - Have the pupils walk the one-kilometre distance several times. Ask each one to record the number of steps it takes to walk a kilometre.

Continues on next page

- Help pupils make a note of the time it takes them to walk one kilometre.
- Have each pupil find out the distance from his/her home to school. You should also find out the distance from school to your house.

7. Metric Estimate Golf

Materials: metre stick, metric tape measure, and centimetre ruler

a) Have pupils play this game in pairs. Let pairs choose five objects in the classroom. These objects can include the length of parts of their body.
b) Have pupils make a scorecard to record their estimates and actual measurements of length of objects (*Figure 1.7*).

Object						
	Length of Tabo's feet	…	…	…	Width of door	Total
Player Tabo's Estimate						
Actual						
Score						
Player Moyo's Estimate						
Actual						
Score						

Fig. 1.7

- Have pupils record their estimates and the actual measurements.
- Their "score" is the difference between the estimate and the actual measure.
- Have each pair of pupils calculate their total score—the player with the lowest score wins.
- Then have pupils compare scores of the winning players from each pair. The player with the lowest score is the overall winner.

Perimeter

Another type of length is **perimeter**. The prefix "per" means around. So perimeter means distance around an object. Once the meaning is clear, pupils can compare perimeters using units that have attributes of length. Later, pupils can use the standard units of measure such as centimetre, metre, and kilometre.

Unit Activity 4

1. Perimeter of a foot
    a) Trace your foot, then estimate and measure, in centimetres, the perimeter of your foot. Compare your estimate with the actual measurement.
    b) Describe, in writing, how you measured the perimeter of your foot.
    c) Keep your drawing—you will need it when you cover area in the next unit.

2. Formula for the perimeter of a rectangle
    - Draw a rectangle. Estimate and measure its sides in centimetres.
    - Find the perimeter of the rectangle without using the formula for perimeter. For instance, x cm + y cm + x cm + y cm
    - Since addition is associative, group the addends as follows: (x cm + y cm) + (x cm + y cm)
    - How many times are you adding (x cm + y cm)? (twice)
    - Now make a multiplication sentence from (x cm + y cm) + (x cm + y cm)
    - If you wrote the multiplication sentence 2(x cm + y cm), you have the formula for the perimeter of a rectangle.

    x $\boxed{P = 2(x + y)}$

    y

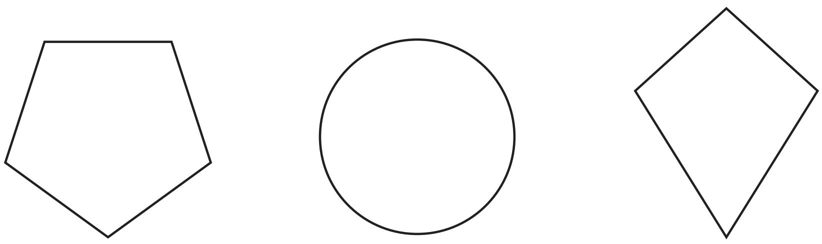
    - How accurate is your estimate?

Practice Activity 4

1. Worksheet
    - Design a worksheet that allows pupils to compare the perimeters of the shapes below. You can create a variation of this worksheet or devise your own.

    Name:

    ....



Pentagon Circle Kite

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- Measure around each shape with a string.
- Draw a line that is the same length as the distance around the shape.

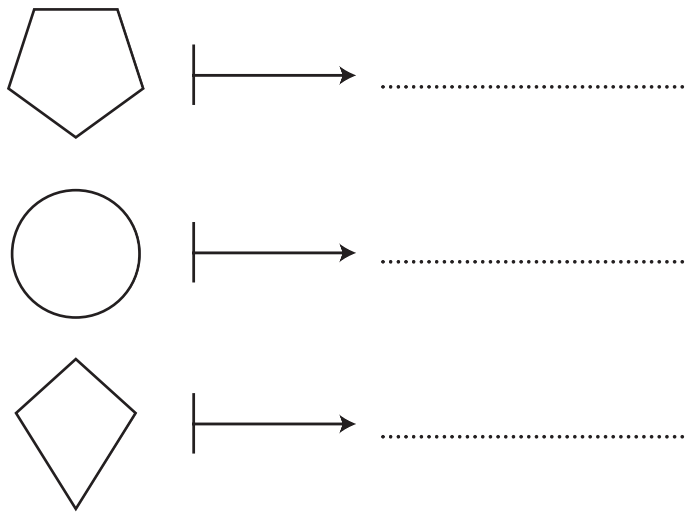


Figure 1.8

Making a straight path the same length as the crooked path.

2. Perimeter of a foot

   Materials: strings, rulers, A4 paper

   - Have pupils trace the outline of their feet.
   - Have pupils work independently to figure out the perimeter of their foot and record the perimeter on the paper with the drawing.
   - Observe the strategies pupils use to measure the perimeter.
   - Have each pupil write a description of how the measurement was made, including the unit of measure.
   - Collect the descriptions and read them to the class without mentioning the authors' names. Can everyone follow the explanations?
   - If the explanations are not clear, have pupils rewrite them.
   - Display the drawings of your pupils' feet, with their names, on the notice board. They will need these drawings for an activity in the next unit.

3. Formula for perimeter of rectangle
   - Help pupils determine the formula for measuring the perimeter of a rectangle.

Self Assessment

1. A man's shirt size corresponds to what measurement of the body? How do you know?
2. The width of the nail of the little finger is about how many centimetres?
3. Choose the most realistic measures of the following objects:
   a) Length of a foot – 27 mm, 27 cm, 27 m
   b) Height of a woman – 165 cm, 165 m, 165 km
   c) Height of a house – 35 cm, 3.5 m, 3.5 km

4. Convert to the stated unit of measure:
   a) 120 cm = ……………….m
   b) 1005 m = ……………….km
   c) 2.5 m = ……………..….mm

The perimeter of a rectangular field is 120 m. Its length is 40 m. What is its width?

Summary

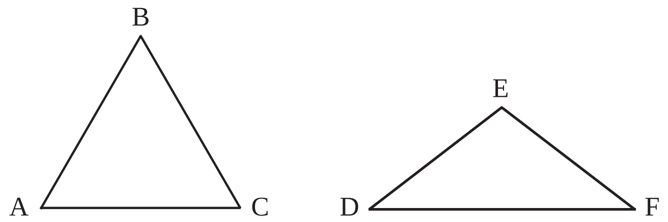
You may have noticed that it is difficult to define length. The indirect method is most effective in illustrating this concept, then pupils can be led through a series of comparison and estimation activities that will help them focus on and understand the concept of length.

To develop the process of measuring length, pupils should go through measuring activities involving the use of non-standard units of measure before embarking on standard units of measure. A combination of these activities will allow pupils to interact with their environment and provide a strong foundation in the measurement of length.

The order of events in the last unit is important: students *first* do measuring activities using non-standard but familiar units of measure, *then* learn to use standard units and concepts. Several recent studies have shown that upper primary students who learn the perimeter concepts this way have much greater success at using those perimeter concepts successfully when they leave the classroom. Early exposure to non-standard but practical activities improves students' *retention*. Paradoxically, if the order of teaching is reversed (teach the standard concepts first, then let students try other methods via activities), students' retention of the standard concepts is *hindered;* they fare no better outside the classroom than if they learned by rote.

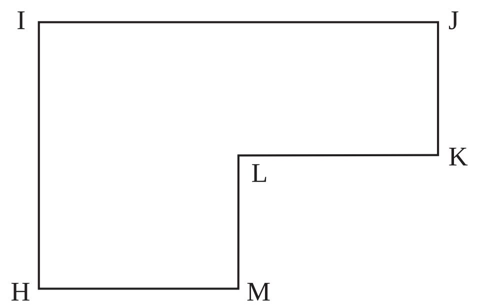
Unit 1 Test

1. Which of the two shapes below has a greater perimeter? How do you know?



2. (a) Use your little finger to estimate the length of the line AB.
   (b) Check your estimate using a ruler.

3. (a) Estimate the perimeter of Figure HIJKLM in cm.
   (b) Check your estimation using a ruler.



4. For each of the following, place a decimal point in the number to make the sentence reasonable:
   a) The table is 680 m
   b) A hip measurement is about 1050 m
5. Develop the formula for measuring the perimeter of a rectangle.
6. In a shop, a man put his forearm with an upright hand lengthwise in his new pair of trousers, as illustrated in *Figure 1.8*. Then the man smiled and asked the shopkeeper to pack the trousers for him. Why did he measure the waist of the pair of trousers in this way?

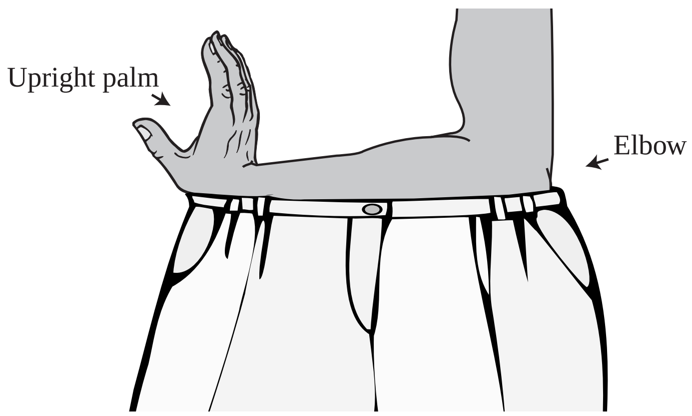


Figure 1.8

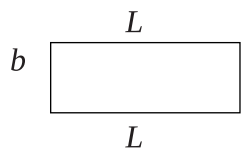
Answers to Self Assessment

1. The circumference of the base of the neck.
2. 1 cm
3. (a) 27 cm (b) 165 cm (c) 3.5 m
4. (a) 1.2 m (b) 1.005 km (c) 2500 mm
5. $$\begin{aligned} P &= 2(L + W) \\ P &= 120 \\ L &= 40 \\ 120 &= 2(40 + W) \\ 60 &= 40 + W \\ W &= 20 \end{aligned}$$

   $\therefore$ width of field is 20 m.

Answers to Unit 1 Test

1. They are of the same perimeter. Verify by measuring.
2. (b) actual measurement is about 10 cm
3. (b) actual measurement is 20 cm
4. (a) 0.68 m (b) 1.05 m
5. Let L & b be the length and with of rectangle respectively.



$$\begin{aligned} P &= L + b + L + b \\ &= (L + b) + (L + b) \\ &= 2(L + b) \end{aligned}$$

6. Maybe the length of his forearm is half of his waist size.

Unit 2: Measuring Area

Introduction

The previous unit dealt with length, which is a one-dimensional shape. In this unit we extend the concept of a one-dimensional shape to the two-dimensional area of an object. Practical work helps develop the concept of area and once it is understood, we will investigate the relationship between area and perimeter.

Objectives

After working through this unit, you should be able to:

- understand the concept of area and the process of measuring area
- help your pupils develop the skills to measure area
- estimate perimeter and area
- investigate the relationship between perimeter and area
- demonstrate how to develop formulas for area
- solve problems involving perimeter and area

Measuring Area

Area is the extent of the surface of an object or region, and it is measured in square units.

Comparison Activities

Comparison activities will help pupils understand the difference between area (size), length, width, perimeter, and shape.

Direct comparison with area may not be possible unless the shapes are similar. Besides comparing similar shapes, you can use activities that test for conservation of area.

This can be done by cutting out a shape once or more than once, and rearranging the parts to form another shape (a tessellation or mosaic). The intention here is to help pupils understand that area remains the same, even though the parts of the shape can be rearranged.

Unit Activity 1

Test for conservation of area

Materials: manila paper, straight edge, scissors

- Make a rectangle from manila paper.
- Cut diagonally (*Figure. 2.1*), making two identical triangles. What type of triangles are these? (right-angle triangles)

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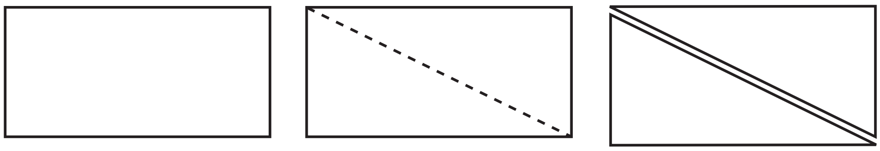

Figure 2.1

- Rearrange the two triangles to make different shapes. Only sides of the same length can be joined.
- How many shapes can you make, including the original shape? (five) Name the shapes (*Figure 2.2*).
- Do all five shapes have the same area? How do you know?
- What about the perimeters of the five shapes? Are they the same?

Practice Activity 1

1. Test for conservation of area

   Materials: rectangles of paper, scissors

   - Let pupils work in groups.
   - Have them fold the rectangles on one diagonal and cut them to make triangles.
   - Have each group rearrange two right-angle triangles into different shapes by joining sides of the triangles that are equal. Have groups find all the possible shapes (*Figure 2.1*).
   - Discuss the area and shape of the different shapes your pupils make. Focus on the amount of paper used in each shape. Is it same?
   - Help pupils conclude that although the shapes are different, the areas are the same (*Figure 2.2*).

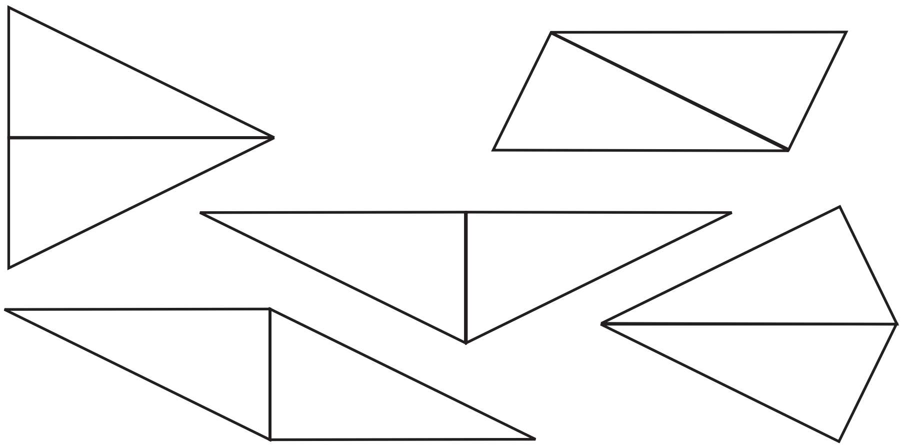

Figure 2.2

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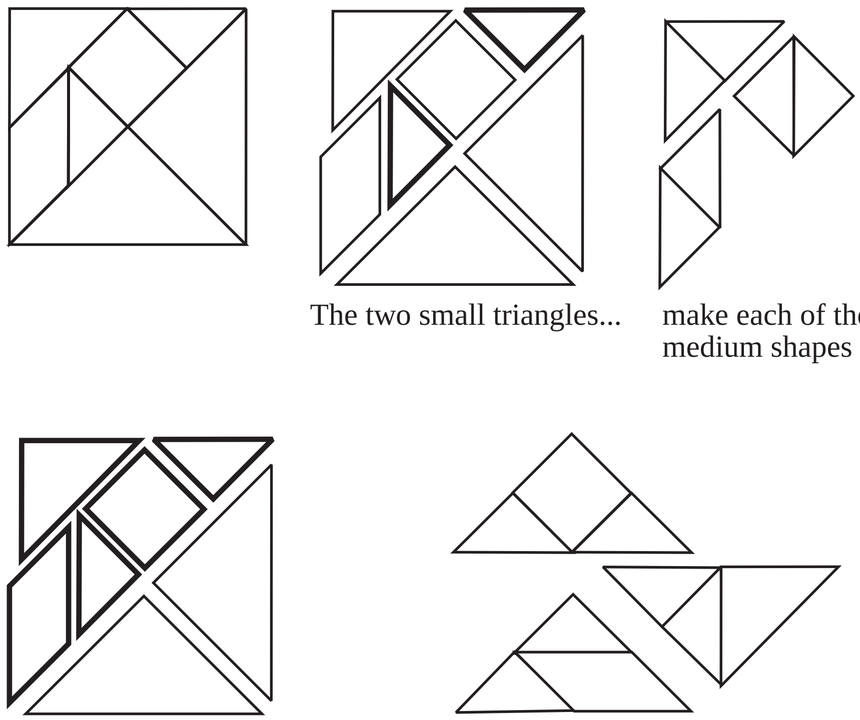
Materials: Set of tangram pieces

- The idea of rearrangement can be extended to tangrams, but the emphasis continues to be on area (*Figure 2.3*).
- Let pupils arrange the pieces in order, according to area.
- Have pupils make shapes that have the same area without using all the pieces of the tangram in each shape (*Figure 2.3*).

Seven tangram shapes



The two small triangles... make each of the medium shapes

The two small triangles with any of the medium pieces... will make a large triangle

Figure 2.3: Tangrams provide an opportunity to investigate the concepts of size and shape.

Using Square Units

To demonstrate that area is a measure of covering, we will use non-standard units of measure that have the attributes of area. These units can be tiles of any shape cut from cardboard or manila paper. For larger units of measure, you may need sheets of newspaper and poster board. Pupils can use these units to measure the surfaces (area) of books, desktops, and even the floor of a room.

Remember—have your pupils estimate a given area before measuring it.

Unit Activity 2

- Choose a non-standard unit of measure and use it to measure the area of an object of your own choice.
- Measure in two ways. First, make enough copies of your unit to cover the surface and measure its area. Then measure the entire surface using a single copy of your unit of measure. Which method did you find more difficult? Why?
- Try this activity with pupils. What is their opinion of the two methods?
- Which method do you think most effectively helps pupils understand the meaning of area? Explain.

Practice Activity 2

1. Determining the area of polygons

   Materials: different shapes of polygons

   - Have pupils predict the order of the size of the shapes, from the smallest to the largest.
   - Then have them determine the correct order by measuring the shapes, using units of their choice.
   - Encourage your pupils to discuss the results.

2. Measuring area

   Materials: construction paper

   - Have pupils work in groups.
   - Have each group use units of their choice to measure the area of their desks. Have them begin by estimating the area. Then have them use two methods to measure the surface, first by using enough copies to cover the surface, then by measuring with a single copy.
   - Prepare appropriate questions and discuss the results with your pupils. Which method did they prefer?
   - Have the groups compare their measurements. Are they able to communicate the measurements? If not, how do they explain their results to the other groups?

3. Measuring with the same unit of measure

   - Have the class decide on a single unit of measure for measuring their desks. Let each group prepare the agreed upon unit of measure.
   - Have pupils estimate and measure their desks again, using the prepared unit of measure.
   - Let them have a choice of how to measure.
   - Again, ask the groups to compare their measurements. Ask them if they have a problem communicating their measurements. Let them explain.
   - Have pupils compare their estimates from the previous activity with the actual measurements.

Standard Unit of Measure

To develop the concept of area, you had your pupils cover surfaces with units that had the attribute of area. Now you should extend the idea of unit models that have the attribute of area to grids made up of squares. Grids are one of the most convenient ways to measure area.

In the metric system, the square metre (m^2) is the basic unit for the measure of area. Other common units are the square centimetre (cm^2) and the hectare, which is equal to a square kilometre. Ask your pupils to name square objects with sides that are 1 cm, 1 m, or 1 km. They may need help visualising a square kilometre.

Conversion Table

1 sq. centimetre	=	10×10 sq. millimetres	=	100 sq. mm (mm^2)
1 sq. metre	=	100×100 sq. centimetres	=	10 000 sq. cm (cm^2)
1 sq. kilometre	=	1000×1000 sq. metres	=	1 000 000 sq. m^2
1 hectare	=	100×100 sq. metres	=	10 000 sq. m^2

Appendix One at the end of this module contains a sheet with a square centimetre grid. If you have access to a copier, you can make copies for your pupils and for your own use. The area of objects drawn or traced on these sheets can be measured.

The following Unit Activity contains instructions for making a transparent grid, which your pupils can also use to measure the area of irregularly-shaped objects.

Unit Activity 3

1. Making and using a transparent square-centimetre grid

   Materials: Manila paper or poster board, sheets of transparent plastic paper or overhead transparencies, a pen or marker that will write on plastic

   Make a square-centimetre grid on the plastic or transparency. Use the manila paper or poster board to make a frame for your grid (*Figure 2.4*). Place the transparent grid over an object and measure its area.

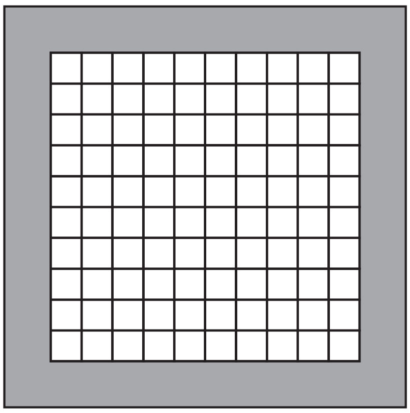


Figure 2.4: Square-centimetre grid with frame (actual size about 12 × 12 cm or 5" × 5")

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Remember—estimate the area of each object before measuring.

a) your thumb nail

b) the face of your watch

c) the cover of this module

d) an item that is about 1 square centimetre

Record your estimates and the actual measurements.

How close are your estimates?

2. Finding the area of an irregular shape

   Materials: sheet of 1 cm grid or your transparent grid from the previous activity, the drawing of your foot from the previous unit.

   - Estimate the area of your foot.
   - Measure the area of your foot by tracing it on a grid sheet or by placing your transparent grid over your drawing.
   - What did you do with squares that are not full squares?
   - How close was your estimate to your actual measurements?

3. Measuring a large area

   Materials: metre sticks

   - Estimate the area of your classroom in m^2.
   - Describe your process of estimation.
   - Use the metre sticks to make a square metre.
   - Find the actual measurement using your square metre.

Practice Activity 3

Once you have completed the above activities, try them with your pupils. When you feel that your pupils understand the concept of area and how to measure it, try the following activity with them.

1. Measuring a large area

   Material: trundle wheel

   - Using the trundle wheel, help your pupils locate a benchmark about one kilometre from your school.
   - Help them visualise one square kilometre.
   - Have them show you a square kilometre if there is enough clear space.
   - Tell your pupils that a hectare is another name for a square kilometre.

Reflection

When you measured area using unit models, what shapes did you use? Do you remember using circles? If not, why?

Developing Formulas

Now that your pupils understand the concept of area and how to measure it, they will appreciate the use of a formula. We are going to build the formula for area, using a square-centimetre grid.

The rectangles in *Figure 2.5(a)* and *2.5(b)* are covered with square grids.

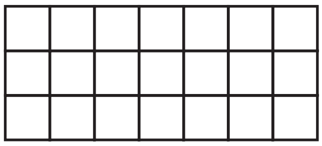

Figure 2.5(a)

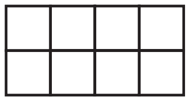

Figure 2.5(b)

How many squares are there in the rectangle in *Figure 2.5(a)*?

The rectangle in *Figure 2.5(a)* has three rows. In each row there are seven small squares. Find the answer using addition. The answer is $7 + 7 + 7 = 21$ squares. What is $7 + 7 + 7$ using multiplication? The answer is $7 \times 3 = 21$.

Study the grid in *Figure 2.5(b)*. How many small squares does it contain? Count each square or use addition to calculate the answer.

Figure 2.6 contains a rectangle. How many small squares will fill this rectangle? Calculate the number without counting or adding.

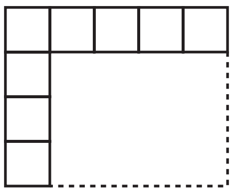

Figure 2.6

If you used multiplication to calculate your answer, you used the formula for finding the area of a rectangle. You can check your answer by filling in the rest of the rectangle with small squares.

If your answer is 4×5 small squares then you used the formula for area. This formula can generally be interpreted as $b \times l$. Since multiplication is commutative, it does not matter which side of the rectangle is labelled b and which is labelled l.

Unit Activity 4

Using formulas to calculate area

Materials: a trundle wheel

- Estimate and find the area of the football field in hectares (remember—one hectare = one square kilometre).
- Compare your estimate with your answer.

Every trapezoid can be divided into two triangles as shown in triangles I and II (*Figure 2.7a*). In both triangles, the heights (h) are equal. Use this information to prove:

$$\text{Area of trapezoid ABCD} = \text{Area } \Delta\text{K} + \text{Area } \Delta\text{II}$$
$$= \frac{1}{2} h\,(b_1 + b_2).$$

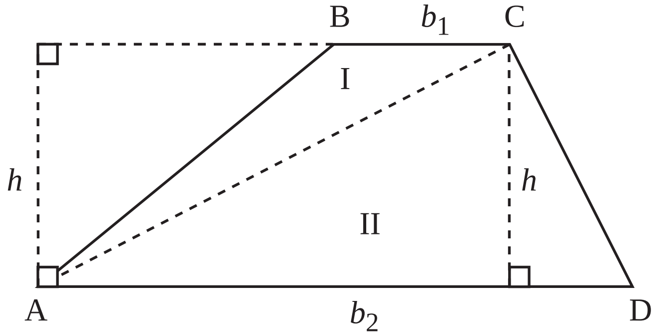

Figure 2.7a

Formula for calculating the area of a circle

- Cut a circle into narrow sectors, so narrow that when joined, they form an "approximate" rectangle. (*Figure 2.7b*)
- What is the length of the rectangle in terms of the circumference C of the circle? What is the width of the rectangle in terms of circumference C of the circle? What is the width of the rectangle in terms of radius r?
- What is the area of this rectangle in terms of C and r? Keep in mind that $C = 2\pi r$.

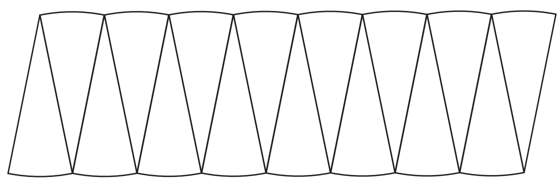

Figure 2.7b: A circle cut into 16 narrow sectors, and the sectors arranged to form an approximate rectangle.

The length of the approximate rectangle is half the circumference of the circle ($\frac{1}{2}C$) and the width of the rectangle is equal to the radius of the circle (r). We have already determined that the area of a rectangle is $l \times b$

$$\therefore\ l \times b = \frac{1}{2} C \times r = \frac{1}{2} \times 2\pi r \times r = \pi r^2$$

Practice Activity 4

1. Developing a formula for the area of a rectangle

   - Design an activity on developing the formula for the area of a rectangle.
   - Do not tell the pupils the formula, but help them find the formula.
   - Have pupils estimate and measure the area of the following items in square metres (let them determine how to find area):

   Materials: trundle wheel

   a) Netball field

   b) Football field

   Demonstrate how to convert square metres to hectares. Let pupils covert their answers in m^2 to hectares. Remind them about the size of a km^2.

2. Calculating the area of a parallelogram

   - Give each pupil a parallelogram cut from a piece of paper (*Figure 2.8a*).
   - Have pupils fold and cut off end of the parallelogram (*Figure 2.8b*).
   - Let them join the two parts of the parallelogram to form a rectangle (*Figure 2.8c*).
   - Let them estimate and measure the areas.
   - Have them measure and tell you the area of a rectangle. Since they have made a rectangle from the parallelogram, the area of the parallelogram is equal to area of the rectangle ($b \times h = A$) where b represents base, h height, and A area.

Figure 2.8a

Figure 2.8b

Figure 2.8c

3. Calculating the area of a triangle

   Materials: pairs of congruent triangles

   - Give each pair of pupils a pair of congruent triangles (*Figure 2.9a*).
   - Let them join a pair of triangles to make a parallelogram (*Figure 2.9b*).

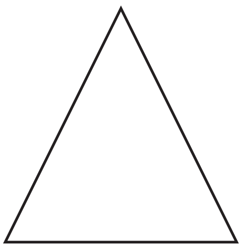

Figure 2.9a

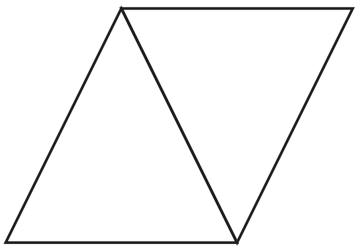

Figure 2.9b

Understanding the Relationship Between Area and Perimeter

Find different pairs of numbers that total 8, for instance 8 = 1 + 7, 8 = 2 + 6 and so on. This will give you the two dimensions of rectangles that have a perimeter 16 units (*Figure 2.10*).

7 sq. units   12 sq. units

Figure 2.10 Two rectangles with different areas—note that both rectangles have a perimeter of 16 units.

- There are four possible number pairs. Were you able to identify them? Do the rectangles have the same area? Which rectangle has the greatest area?
- Find all the rectangles with perimeters of 8 units. Which rectangle has the greatest area?
- Continue finding rectangles of the same perimeter and determine which has the greatest area.
- What is your conclusion? Is there a general relationship between perimeter and area?
- Now keep the area of the rectangle fixed and find the different perimeters.

  Hint: If the area of the rectangle is 16 square units, find multiplication factors for 16—1 and 16, 2 and 8, and so on. These factors give the dimensions (length and base) of the rectangle.
- Decide on the area and find the different perimeters for that area. Is the perimeter the same for all rectangles?
- If you had to build a fence around a given area of land, which rectangular shape would require the minimum amount of fencing?

Unit Activity 5

1. Determining “greatest” area

   Materials: Rectangular and circular geoboards, string

   A geoboard is a nailboard made by hammering nails partway into a board, in a particular pattern. If you do not have geoboards in your classroom, you can use the patterns in Appendix 2 and Appendix 3 to build your own. You can also copy these pages and have your pupils draw shapes.

   - Use the string to make a circle on a circular geoboard. Find the approximate area of the circle by counting square units.
   - Remember—always estimate before measuring.
   - Use a rectangular geoboard and string to make different rectangles with the same perimeter as the circumference of the circle.

Continues on next page

- Compare the areas of the rectangles and circle. Which shape has the greatest area? Why? The circle has the greatest area. The smaller the size of the side of the regular polygon (whose sides and interior angles are equal), the greater the area and the smaller the perimeter compared to polygons of the same area.

Do you agree? If not, explain why. As the size of the sides of a regular polygon become smaller and smaller, the polygon ceases to be a polygon and forms a circle.

Reflection

In some ancient civilisations, land was priced by counting the number of paces around the boundary, i.e. by perimeter (Haylock, p. 190). Was this a fair deal? Explain.

Practice Activity 5

1. Comparing area and perimeter

   Materials: Square geoboards and strings

   - Give groups of pupils each a geoboard with different pre-set rectangles.
   - Have each group work independently to draw and measure the perimeter and area of a given rectangle.
   - Using the string and geoboard, ask your pupils to make as many different rectangles with the same perimeter as possible. Then ask them to draw the rectangles, stating the fixed perimeter and the different area for each.
   - Have each group identify the rectangle with the greatest area.
   - Bring the groups together to discuss what happened to the area of their rectangles as the fixed perimeter changed shape. Which shape gave them the greatest area?

   Materials: paper, rulers, and scissors

   - Have each group of pupils make same-sized squares of paper, using their choice of units. The total number of same-sized squares can be left to individual groups.
   - Let groups make rectangular fields of different area, using all the squares in each rectangle. Each pupil should draw the rectangles made by the group and write the area and perimeter for each rectangle.
   - Have each group report to the class what happened to the perimeter as the fixed area changed shape. If they wanted to conserve material (wire fence), which rectangular field will they fence? Why? If they want to conserve wire fence material, are the pupils likely to choose a square field?

Self Assessment

The activities in this unit involved many practical exercises. This practical work carries over to the self-assessment exercises and unit test.

1. Try to discover a formula, called Pick's formula, for finding the area of a polygon drawn on dot paper. For this activity, you can also use a geoboard or the geoboard paper provided in Appendix Two. Check Pick's formula by counting the squares or using other formulas for area.

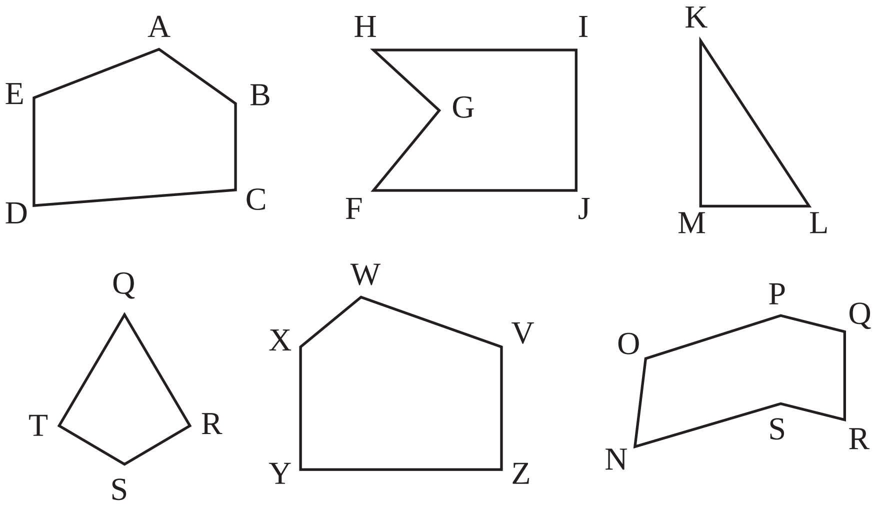

Figure 2.11

(i) Complete the following table, where b = number of boundary dots, i = number of interior dots (inside the polygon), and area = area of the polygon. The first row has been filled in for polygon ABCDE.

Polygon	b	i	$\frac{b}{2} + i$	Area
ABCDE	8	9	13	12
HIJFG				
QRST				

(ii) State Pick's Formula and use it to find the area of polygon KLM. Check the area of KLM using the formula for calculating the area of a triangle.

(iii) Now use Pick's formula to find the area of NOPQRS.

(iv) Use Pick's formula to find the area of polygons with holes, such as VWXYZ.

(v) Does Pick's formula work for the area of a circle?

2. *Figure 2.12* shows an isosceles trapezoid. By cutting and rearranging region p, show that the area of an isosceles trapezoid is $x \times y$. You can use scissors and construction paper.

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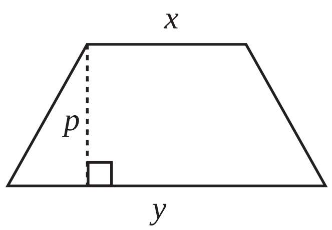


Figure 2.12

3. Estimate the area of the circle (*Figure 2.13*) in square centimetres. Then make square centimetre grids on plastic or tracing paper. Use your grid and formula to check your estimate.

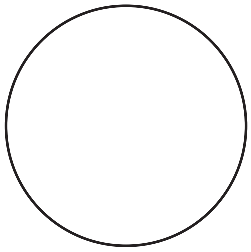


Figure 2.13

4. Choose the most appropriate metric unit (mm^2, cm^2, m^2, or km^2) for finding the area of each of the following:

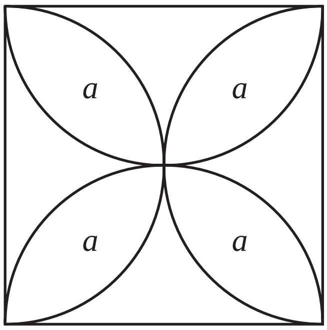
   (a) area of teacher's table

   (b) area of the cover of an exercise book

   (c) area of your thumb nail

   (d) area of your country

5. Arrange four semicircles in a 6 cm × 6 cm square, as illustrated in the following diagram.



   Find the area of the four petals (a) created by the overlapping parts of the semicircles by using:

   (a) estimation

   (b) a square-centimetre grid

   (c) a formula

Summary

Practical activities are the best way to understand the concept of area and to develop the skills for measuring area. You should work through all the activities in this unit before trying them with your pupils. Remember—activities for measuring area should include estimation, comparison, creating unit models, and taking metric measurements.

As mentioned in the Summary to unit 1, the order of events (*first* try practical activities, *then* learn the standard concepts) is important for students' retention of area concepts. For example, a student error after area concepts have been covered is to recall that the formula for area of a rectangle is "one-half base times height". This occurs because the student has confused, over time, the similar-sounding words "rectangle" and "triangle". If the student learned his area formulas by rote and by paper exercise, or if the rote learning was followed by hand-on activities, such errors are common enough. But if the hands-on activities were experienced first, such faulty retention occurs much less frequently.

Unit 2 Test

1. Use the square grid (*Figure 2.15*) to answer the following problems. If each small square on the grid represents a sewing needle stuck in a square board, then:
   (a) the board is covered with _____ needles
   (b) its area is approximately _____ mm^2
   (c) the board is 1cm on each side and has area of _____ cm^2
   (d) one square centimetre is _____ square metres

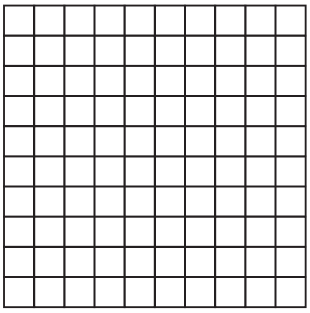
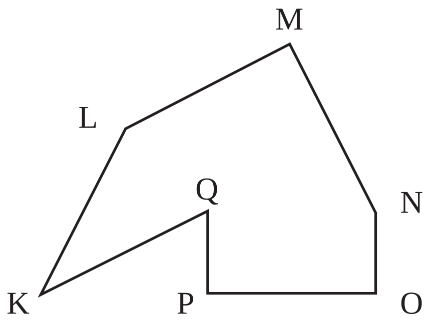


Figure 2.15

2. Use a geoboard to create the shape KLMNOPQ. Estimate the area of the shape and describe the process you used to arrive at your estimation.



3. Complete each statement below with the appropriate unit of measure.
   (a) The area of a sitting room is 32 ………….
   (b) The palm of the hand may have an area of 140 ………..
   (c) The area of the nail of the little finger is about 64 ………..
4. A field 485 m by 100 m has an area of ……….. hectares.
5. A rectangular garden is 470 cm long and 320 cm wide. Use two different methods to find the area of this garden in square metres.
   (a) Calculate the number of square centimetres and convert to square metres.
   (b) Convert the length and width measurements to metres, then calculate the area in square metres.
   (c) Which method do you prefer? Why?
6. A woman who enjoys flowers wants to use 16 metres of fence to make a rectangular flower bed with whole number dimensions. What should be the dimensions of the bed to take the least amount of the plot?

Answers to Self Assessment

1. (i)

Polygon	b	i	$\frac{b}{2} + i$	Area
ABCDE	8	9	13	12
HIJFG	16	5	13	12
QRST	8	5	9	8

(ii) Pick's formula is $\frac{b}{2} + i - 1$

(iii) Pick's formula is $\frac{b}{2} + i - 1 = \frac{12}{2} + 7 - 1 = 12$

$\therefore$ There are 12 sq. units.

Area formula is $\frac{1}{2}$ height $\times$ base $= \frac{1}{2} \times 4 = 12$

$\therefore$ There are 12 sq. units.

(iv) $\frac{b_1}{2} + i_1 - 1 - (\frac{b_2}{2} + i_1 - 1)$

$= \frac{b_1 - b_2}{2} + (i_1 - i_2) - 1 + 1$

$= \frac{b_1 - b_2}{2} + (i_1 - i_2)$

$= \frac{15 - 8}{2} + (18 - 1)$

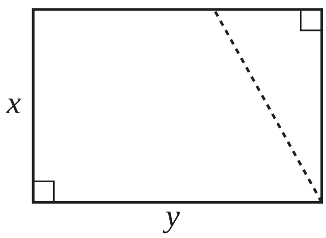
$= 3\frac{1}{2} + 17$

$= 20\frac{1}{2}$

$\therefore$ VWXYZ has $20\frac{1}{2}$ sq. units.

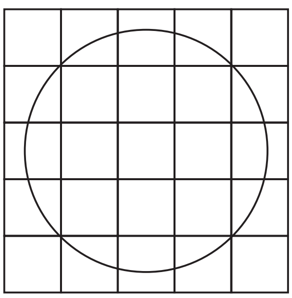
(v) Formula works perfectly well for a circle.

2.



Area = $x \times y$

3. (i) Grid

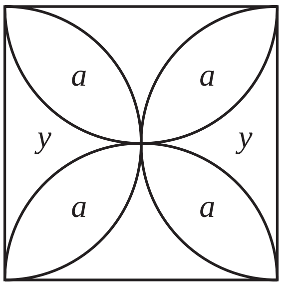


Area = 8 whole sq. centimetres + parts that make about 4 whole sq. cm.
= 12 sq. cm.

(ii) Formula
r is about 2 cm.
$A = \pi r^2 = \frac{22}{7} \times 2^2$ sq. cm = $12\frac{4}{7}$ sq. cm.

4. The most appropriate metric units are:
   (a) m^2
   (b) cm^2
   (c) mm^2
   (d) km^2

5. (a) estimate the area
   (b) measure the area using a grid
   For example:
   The area of one petal is approximately 5 sq. cm.
   $\therefore$ the area of four petals is about 4×5 sq. cm. = 20 sq. cm.
   (c)



The length of the square is l. The radius of each semicircle, r, is half of l. The four petals are the four areas "a". The four areas outside the petals are designated "y". Notice that $4a + 4y = l^2$.

Consider the two areas "y" in the above diagram. By inspection, they constitute the area of the square minus the area of two semicircles or one circle:

$y + y = l^2 - \pi r^2$

Now we add the other two similar areas, to double the above equation:

$$4y = 2(l^2 - \pi r^2)$$
$$= 2l^2 - \pi r^2$$

This is an expression for the area in the square that is outside all four petals. But we want the area inside the petals, so we subtract $4y$ from the area of the square:

$$\text{Area of petals} = A = l^2 - 4y$$
$$= l^2 - (2l^2 - 2\pi r^2)$$
$$= l^2 - 2l^2 + 2\pi r^2$$
$$= 2\pi r^2 - l^2$$

So the area A of the four petals is simply the area of two complete circles, less the area of the square.

In our case, $l = 6$ and $r = 3$, so the formula gives us

$A = 2\pi r^2 - l\ 2$

$= 2\pi(3)^2 - (6)^2$

$= 56.5487 - 36.000$

$= 20.55\ \text{cm}^2$, approximately.

Answers to Unit 2 Test

1. (a) 100 needles
   (b) 100 mm^2
   (c) 1 cm^2
   (d) 0.0001 m^2

2. (a) Using counting. There are 8 whole sq. units. Parts make about $4\frac{1}{2}$ sq. units. Together there are about $12\frac{1}{2}$ sq. units.

3. (a) 32 m^2
   (b) 140 cm^2
   (c) 64 mm^2

4. 485 hectares

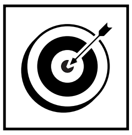
5. (a) $320 \times 470\ \text{cm}^2 = 150\,400\ \text{cm}^2$
   $= \frac{150\,400}{10\,000}$
   $= 15.04\ \text{m}^2$
   (b) $3.2\ \text{m} \times 4.7\ \text{m} = 15.04\ \text{m}^2$
   (c) Your preference is a matter of choice. The best method is always the one with the least possibility for error.

6. $P = 2(a + b)$
   $16 = 2(a + b)$
   $8 = a + b$
   $8 = 1 + 7$
   The rectangular bed should be 1 m by 7 m.

Unit 3: Volume and Capacity

Introduction

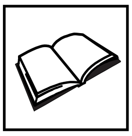
In the previous units, you measured length, perimeter, and area. This unit introduces the techniques and formulas for measuring three-dimensional objects. You will also explore ways to teach the concepts of volume and capacity to your pupils, so they will be able to apply them to everyday situations.



Objectives

After going through this unit you should be able to:

- distinguish between volume and capacity estimates
- measure capacity and volume
- do calculations involving capacity and volume
- develop concepts of volume and capacity with your pupils



Measuring the Volume of an Object

How would you measure the size or amount of space occupied by the objects below?

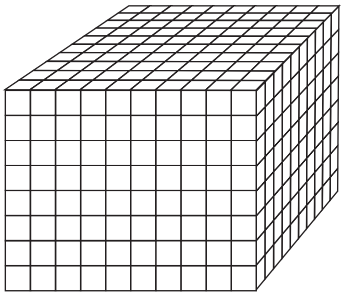


Figure 3.1a

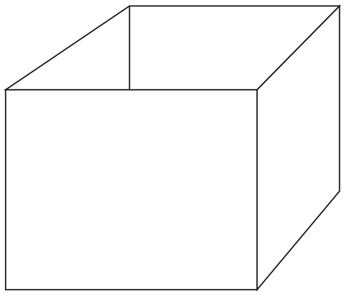


Figure 3.1b

First, try to estimate the size of the object. In *Figure 3.1a*, you can count the number of cubes that make up the box. Both the base length and width have ten cubes. Therefore the base ‘area’ is 10 cubes $\times$ 10 cubes = 100 cubes.

The box is eight cubes high, therefore 8 cubes $\times$ 100 cubes = 800 cubes.

Therefore, we can say the size of the object in *Figure 3.1a* is 800 cubes, or the space it occupies is 800 cubes. In this case, cubes are a non-standard unit of measure.

The formula for measuring this object is *area of the base* $\times$ *height*. We will come back to this formula later.

The attribute we are measuring is **volume**, which is the amount of space occupied by a three-dimensional object.

Figure 3.1b illustrates an empty, open container. How will you estimate the volume of this container?

One approach is to fill the container with water. For example, you can add water to the container, one tea cupful at a time, until the container is completely full of water. Before you do this, estimate the volume of the container by trying to guess how many cupfuls of water it will hold. Then fill the container with water, one cupful at a time, while you count the number of cups. This method uses non-standard units.

The amount of liquid the container holds, when filled to the brim, gives the 'size' of the container. The maximum amount of liquid the container holds is its **capacity**. Only containers have capacity.

Capacity is generally applied to containers that hold liquids, therefore capacity is measured in the same units as *liquid volume*. For example, a bottle of cooking oil holds 750 millilitres of oil when filled to the brim, therefore its capacity is 750 millilitres.

Both volume and capacity measures the space occupied by three-dimensional objects or material.

Reflection

From what you have read and experienced above, can you distinguish between volume and capacity?

Introducing volume and capacity to pupils should include direct comparison activities. Comparisons can be made by filling a container with something, then pouring the contents into another container. Does the second container hold more, less, or the same amount as the first container?

Start by using non-standard units to emphasise the idea that measurements need reference points, and that measurements are approximate. Errors in measurements are in many instances created by the measuring instruments, a point that pupils need to understand.

The activities below will help you introduce the concept of capacity to your pupils. They will most likely find these activities more fun than those dealing with length and area.

Practice Activity 1

For this activity find an assortment of cans, small boxes, plastic jars, and other containers. Also gather some plastic scoops, tea cups, and some cubes, or other small *measurers*. You will also need *fillers* to fill the containers (e.g., shelled maize, beans, sand, or water).

Sort the containers by volume, and select one container to use as the *standard*.

(a) Ask your pupils to pour fillers from the standard into the other containers. Let them determine which container holds:

   i) Less than the standard

   ii) More than the standard

   iii) About the same as the standard

Continues on next page

(b) Ask pupils to label each container with slips of paper on which they have written MORE, LESS, and ABOUT THE SAME.

Have them estimate the capacity of other containers. Let them measure these containers and indicate their results. Were their estimates close?

Practice Activity 2

Give your pupils five or six containers of different sizes and shapes. Ask them to order them from least to greatest volume.

The above activity emphasises estimation and measurement skills. This process will help pupils to make comparisons without the use of a measuring instrument. For example, someone might want to estimate whether or not they have enough mealie-meal to cook nshima for three people.

Think of other activities to help your pupils practice estimating and measuring volume and capacity.

Volume and Capacity Using Standard Units

In Unit 1 of this module, you were introduced to the internationally accepted system of metric units called SI units (System International).

This system specifies one base unit for each aspect of measurement. For volume, such as liquid volume or capacity, a litre or a millilitre is the standard unit. For solid volume, a cubic centimetre or cubic metre is the standard unit. The two base units are related.

If necessary, review the content at the beginning of this unit. Now study the cube in *Figure 3.2*. Each edge of this cube is 1 cm long.

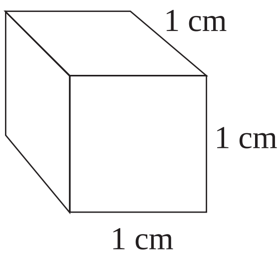


Figure 3.2

Therefore the volume of the solid is [1 cm × 1 cm] × [1 cm] = 1 cm^3. Read this as *one cubic centimetre*.

This cube has a volume of 1 cm^3 or one cubic centimetre.

This same cube will hold an amount of liquid equal to 1 millilitre. Therefore, 1 cm^3 = 1 millilitre, and 1000 cm^3 = 1 litre.

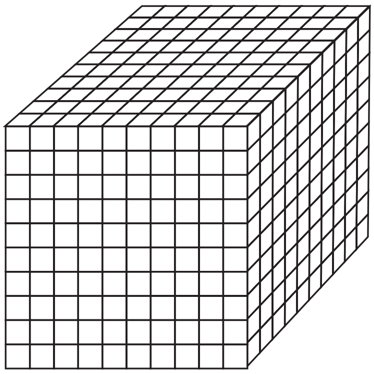


Figure 3.3

Figure 3.3 shows 1000 cubes, each with a volume of one cubic centimetre. The volume of this figure is 1000 cubic centimetres.

A litre is symbolised by L
1 litre is equal to 0.1 m^3

0.1 m^3 = 1000 cm^3,
it follows that 1 L = 1000 cm^3
and 0.001 litre = 1 cm^3

0.001 L = 1 mL, therefore 1 cm^3 = 1 mL

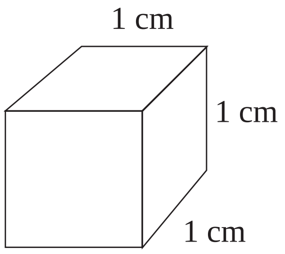


Figure 3.4

The table below shows the standard metric volume units, symbols, and their relationship to the litre. The prefixes used with the standard base units in the metric system define the units.

Unit	**Symbol**	**Relation to Litre**
millilitre	mL	0.001 litres
litre	L	1 litre
kilolitre	kL	1000 litres

Self Assessment 1

i) 75 cm^3 = ________L
ii) 1.5 L = ________cm^3
iii) 750 mL = ________cm^3
iv) 250 mL = ________cm^3

Reflection

It is important for you and your pupils to have benchmarks for estimating measurement. Your pupils will have an easier time understanding the concept of volume and capacity if they have a container that holds a specific unit (one litre, for example) and can use it to practice measuring.

Practice Activity 3

Provide your pupils with a standard volume unit, such as a one-litre container. Have them search for other containers that have about the same capacity as the standard unit.

Practice Activity 4

Let pupils go into their community and collect as many liquid containers as possible. These might include:

- Bottles for cooking oil
- Milk bottles
- Drink bottles

a) If they find transparent containers, they can fill them with water.

b) Using containers of different shapes, have your pupils pour a selected measure of water into each of the different shapes.

c) Let pupils play a game by putting a ‘known’ measurement into a container of a different shape and then asking other pupils to estimate the measurement.

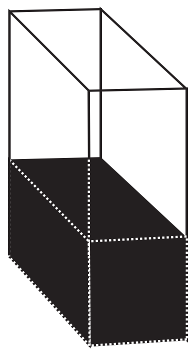


Figure 3.5

Which shape do they estimate has more liquid?

The principle of conservation is a fundamental idea in understanding measurement. Conservation of liquid volume can pose a challenge to some pupils. As they carry out the game described above, observe their estimates for cases such as the one above.

Practice Activity 5

Have pupils convert standard whole units to smaller or larger units. For example, 1 L = 1000 mL.

Design a worksheet with as many questions on conversion as possible.

Sample question: A bottle of medicine is labelled 100 mL. How many litres is this?

Self Assessment 2

Convert each of the following as indicated:

a) 27 L = ________mL
b) 3 mL = _______cm^3
c) 9 m^3 = ________L

Developing Formulas for the Volume of Solid Shapes

The concept of volume should be fully understood before you ask your pupils to consider formulas. The formula should be taught hand in hand with standard units and with making conversions from one unit to another.

Consider the shape in *Figure 3.6* below. For any rectangular right ‘prism’ with linear dimensions, standard cubes can be used to calculate volume.

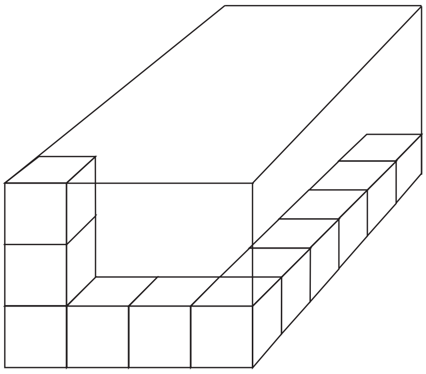


Figure 3.6

The base of the rectangular prism is $4 \times 7 = 28$ cubes.
The height is 3 cubes.
Therefore the volume of the prism is $28 \times 3 = 84$ cubes.

If each cube has a volume of 1 cm^3, the volume of the prism is 84 cm^3.

By demonstrating this to your pupils, using rectangular shapes of different sizes, you can establish that the area of a solid rectangular prism can be found with this formula:

$$\text{area of the base } (l \times b) \times \text{height } (h) \text{ of the prism} = \text{volume}$$
$$\therefore$$
$$\text{volume} = (l \times b) \times h$$

Reflection

The volume of a cube can be calculated by the formula ($l \times b \times h$) = v, but how is the volume of a cylinder measured? To determine this, we will examine the principle established by Bona Ventra Covaliveri (1598 – 1647). It states:

> "Given a plane and any two solids, if every plane parallel to the given plane intersects the two solids in cross sections that have the same area, then the solids have the same volume".

Picture the solid as being made up of many thin slices, as shown in *Figure 3.7(a)*.

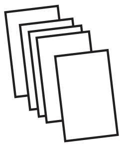


Figure 3.7(a)

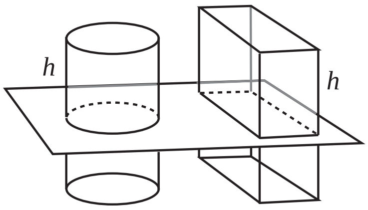


Figure 3.7(b) *Figure 3.7(c)*

If the area of the cross-section of the cylinder in *Figure 3.7(b)* is everywhere the same as the area of the cross-section of the prism in *Figure 3.7(c)*, then the volume of the cylinder and of the prism is the same.

Thus, a useful interpretation of the volume of a right rectangular prism formula is:

Volume = Area of base of the prism × height.

Coming back to the question posed earlier, the volumes of a cylinder and a cone can be approximated using prisms and pyramids with an increasing number of sides in their bases. For the example of a cylinder, the diagrams below illustrate the point:

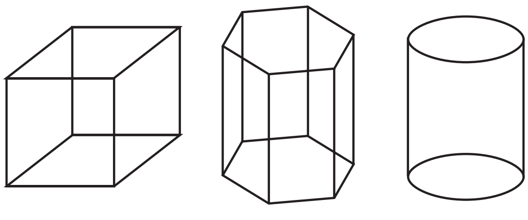


Figure 3.8

The volume of each prism is the product of the area of its base and its height. Note that the base of the cylinder is a circle with an area πr^2. Therefore, the volume V of a cylinder whose area is A and whose height is h is: $V = Ah$

Continues on next page

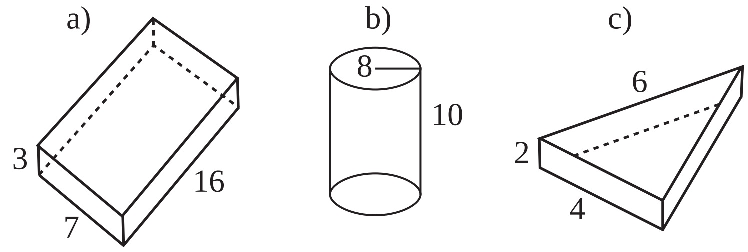
Find the volume of a cylinder with a diameter measuring 8 units and a height measuring 10 units.

Solution:

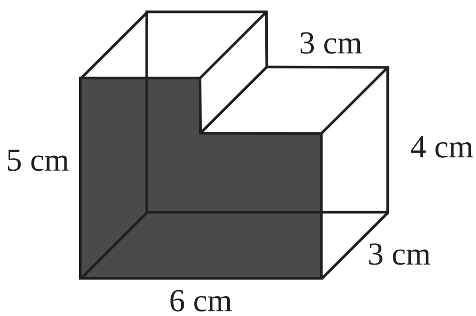
Area of the base (circle)	$= \pi r^2$
	$= 3.142 \times \frac{8}{2} \times \frac{8}{2}$
	$= 3.142 \times 4 \times 4$
	$= 50.272 \text{ unit}^2$
Therefore the volume of the cylinder	$=$ Area of base $\times$ height
	$= 50.272 \times 10$
	$= 502.72 \text{ unit}^3$

Self Assessment 3

1. Find the volume of each figure described below:
   a) a rectangular prism with $l = 4$, $w = 6$, $h = 5$
   b) a rectangular prism with $l = \frac{3}{4}$, $w = \frac{7}{8}$, $h = 3\frac{1}{2}$
2. Find the volume of the following objects:



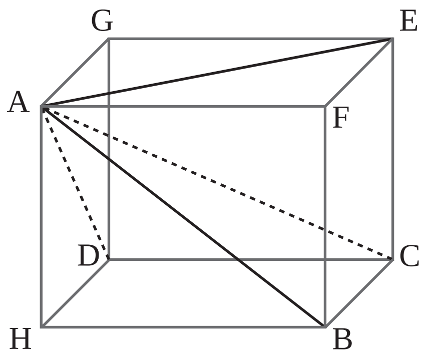
3. Convert each of the following to cubic centimetres:
   a) 36 m^3 b) 3200 mm^3
4. Which box in each of the following questions is larger?
   a) a box holding 12 m^3 or one holding $130\,000 \text{ cm}^3$
   b) a box holding $156\,000 \text{ mm}^3$ or one holding 25 cm^3
5. Find the volume of the figure below:



6. Estimate the volume of the following in cubic metres:
   a) a shoe box
   b) an average-sized bathroom

Unit Activity 1

If you have difficulty visualising the following explanation, proceed to the next section on Pyramids. This activity will likely be too advanced for pupils in Upper Primary Maths and is intended as background teaching material only.



Using the figure above:

a) Draw four diagonal lines from the vertex labelled 'A' to the other vertices, labelled 'B', 'C', 'D', and 'E'.

b) Taking three diagonals at a time, identify the three shapes that are formed.

c) What can you conclude about these three shapes in terms of:

(i) size?

(ii) shape?

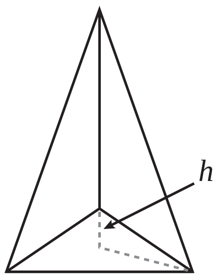
d) When you consider all three shapes within the cube, is there any space left in the cube?

e) What can you conclude about the volume of each shape in relation to the cube?

Pyramids

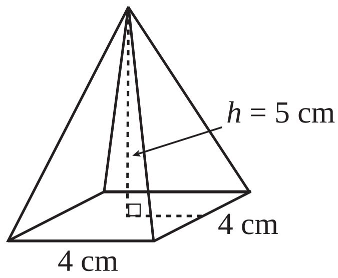
If you completed the activity above, you will have identified the three pyramids with square bases. They are identical in size and they fill the cube completely. Hence, the volume of each pyramid makes up one-third of the volume of the cube. This holds for pyramids with any base.

The volume of a pyramid V is given by the following formula:



$$\begin{aligned} V &= \text{one-third (area} \times \text{height)} \\ &= \frac{1}{3}Ah \\ &= \frac{1}{3}\text{Area of the base} \times \text{height} \end{aligned}$$

Find the volume of the right square pyramid below:



Solution;

Square base area = 4 cm × 4 cm = 16 cm^2

Height = 5 cm

Therefore, V $= \frac{1}{3}$ (Area of base) × (height)

$= \frac{1}{3}$ (4 cm × 4 cm) × 5 cm

$= \frac{1}{3} \times 16\text{ cm}^2 \times 5\text{ cm}^2$

$= \frac{80}{3}\text{ cm}^3$

$= 26.67\text{ cm}^3$

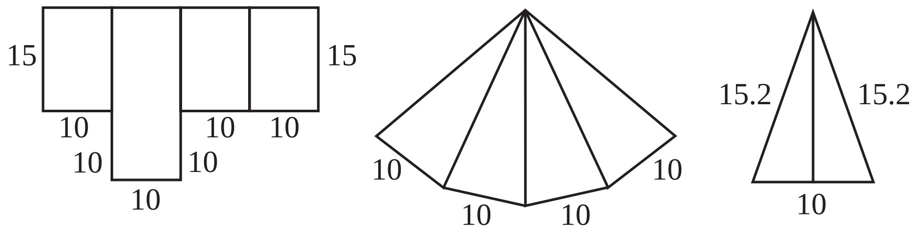
The relationship of the volume of a pyramid and a cone can be interpreted in the same way as the relationship between the cylinder and the right rectangular prism. An activity similar to that illustrated in *Figure 3.8* establishes that the volume of a cone is also one-third of the product of the area of its base and its height.

volume of a cone $V = \frac{1}{3} Ah$

Practice Activity 6

Materials: poster board, beans or rice

Give your pupils the following patterns and their dimensions:



Have your pupils make a three-dimensional prism and pyramid out of poster board, using the same height and base. Ask them to estimate the number of times the pyramid will fit into the prism.

Have them test their prediction by filling the pyramid with beans or rice and emptying it into the prism.

Ensure that pupils establish that exactly three pyramids fill a prism with the same base and height. This 3-to-1 volume ratio is true of all pyramids and cones with the same base and height, regardless of the shape of the base or the position of the vertex.

Volume of a Sphere

This is difficult to observe experimentally, but it can be established by using advanced methods. The volume of a sphere is given by:

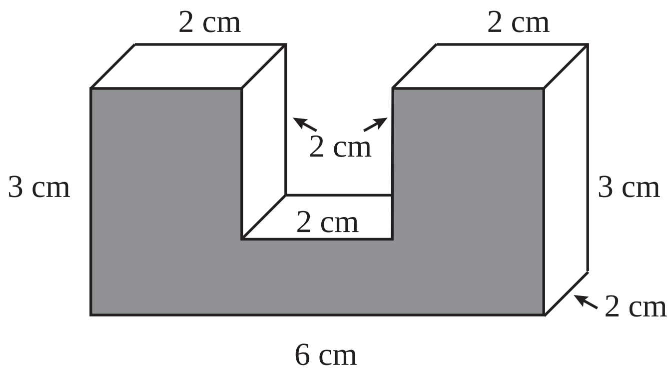
$V = \frac{4}{3}\pi r^3$

Example:

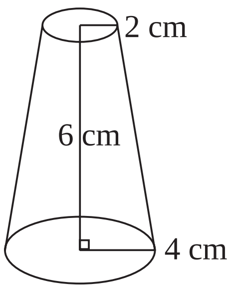
How many cubic centimetres of air can be pumped into a basketball if its maximum diameter is 20 cm?

Unit 3 Test

1. Find the volume of the figure below:



2. Compare a square prism with a base that has a perimeter of 40 cm with a cylinder that has a circumference of 40 cm. Both have a height of 20 cm. Which has the greatest volume and by how much?
3. Find the volume of the figure below:



4. A pyramid has a rectangular base 10 cm by 8 cm. The height is 14 cm. Find the volume of the pyramid.
5. How many litres of liquid are there in 20 millilitres?

Answers to Self Assessments

Self Assessment 1

i) 0.075 L

ii) 1500 cm^3

iii) 750 cm^3

iv) 250 cm^3

Self Assessment 2

a) 27 000 mL

b) 3 cm^3

c) 90 L

Self Assessment 3

1. a) 60 $units^3$

   b) $1\frac{19}{128}$

2. a) 336 $units^2$

   b) 2010.62 $units^3$

   c) Because volume equals $\frac{1}{2}bh \times l$, the figure's volume equals $2 \times 6 \sin\theta \times 2$ or $4(6 \sin\theta)$

3. a) 36 000 000 cm^3 or 3.6×10^7 cm^3

   b) 3.2 cm^3

4. a) 12 m^3

   b) 156 000 mm^3

5. 81 cm^3

6. Estimate your own answers.

Answers to Unit 3 Test

1. 28 cm^3
2. The cylinder has a greater volume by 1546.48 cm^3. The volume of the square prism is 1000 cm^3 and that of the cylinder is 2546.48 cm^3.
3. 175.93 cm^3 or 56π cm^3
4. 373.33 cm^3
5. 0.02 L

Unit 4: Mass and Weight

Introduction

In everyday life, the words mass and weight are generally used interchangeably. This unit demonstrates that, technically, there is a difference between the two. However, the concept of mass is explored more extensively than weight, as weight is normally covered at the upper secondary school level.

Operations involving quantities of mass are not covered in detail as it is felt that you can successfully do them yourself by applying the ideas discussed in previous units of this module.

Objectives

After working through this unit, you should be able to:

- define mass and weight
- explain the difference between mass and weight
- correctly and scientifically use the words mass and weight in a way that is distinct from everyday usage
- recall and use metric standard units and SI basic units of mass and weight
- measure the mass of objects using non-standard and standard units
- make and use estimates of masses of objects, both in and out of the school environment
- add and subtract masses that are given in grams and/or kilograms

Reflection

Upon visiting a doctor, Themba was requested to stand on a scale so his weight could be found. Themba's weight was 120 kg and the doctor thought Themba was overweight.

Can we replace every use of the word *weight* with *mass* in this example?

Meaning of Mass and Weight

Weight and mass are not the same. Strictly speaking, units such as grams and kilograms are units of mass, while units of pounds are weight. Are you puzzled by this? If you are puzzled, it is not surprising, given that the words mass and weight are used interchangeably in everyday life.

However, it is important for you to know that the **mass** of an object is a measure of the **amount of matter** that makes up the object. Note that this is not the same thing as the volume, or the amount of space occupied by the object.

When you hold an object in your hand, you cannot experience its mass. It is the **weight** of the object you feel and to which your muscles respond. So, weight is the **force exerted on the object by gravity**. The force of gravity varies, depending on the distance of the object from the centre of the earth. It is greater near the centre of the earth. Thus weight is a measure of force.

It might interest you to know that your weight on the moon is different from that on Earth. This is because the mass of the Moon is much smaller than the mass of the Earth and, therefore, the **gravitational pull** (i.e. the force) is weaker.

The metric unit of measurement of weight is not the kilogram but the **newton**. On the Earth's surface, a mass of 1 kg has a weight of approximately 10 newtons. In earlier units of this module, you have come across the SI (International System) version of metric units. You will recall that this is an internationally accepted convention used in trade and industry and in technological and scientific work. The basic SI unit of mass is the kilogram (kg), not the gram (g).

The mass of an average pupil on Earth remains the same as it would if that pupil could stand on the Moon. However, the pupil's weight on the Moon would be six times less because the force of gravity on the Moon is much less than on Earth.

	Mass of an individual	Weight of an individual
On Earth	36 kg	80 pounds
On Moon	36 kg	15 pounds

If the size of the pupil could be shrunk to one-sixth, the pupil's weight would change, as shown in the following table.

	Mass of a shrunken individual	Weight of a shrunken individual
On Earth	6 kg	15 pounds
On Moon	6 kg	2 pounds

Unit Activity 1

You will need a balance scale with two pans, and objects to weigh, such as books, small boxes, pencils, stones, sticks, wooden blocks, bags of sand, etc.

- Place one book on the side of the balance. What happens to the scale?
- Place small stones, one at time, on the other pan until the scale balances. How many small stones balance the book?
- Repeat this activity using the same book, but now try balancing it with sticks. How many sticks balance the book?
- Repeat the activity with a different set of objects, each time recording the number objects required to balance the scale.

Reflection

Remember, in the practice activity you balanced the same book with two different sets of objects. For example, you might have obtained the following results:

1 book	4 stones
1 book	2 sticks

Suppose you want to communicate the result of your activity to another teacher in another school. How would you do this?

Need for Standard Units

The British or imperial system uses the pound as a standard unit for measuring weight while the metric system uses the kilogram as the standard unit measuring mass. Fractions and multiples of these units are also commonly used. For example, 1 kg is equivalent to 1000 g and 1 tonne is equivalent to 1000 kg.

When we discussed length, we emphasised the use of a standard unit of measurement. Why is it important to use a standard unit? Compare your answer with your answer in the previous reflection.

Unit Activity 2

You will need a balance scale and the objects you used in Unit Activity 1.
You also need 1 g, 10 g, and 1 kg masses (weights).

Measure the mass of four different objects, one at a time, using the weights to balance the scale. Record the mass of each object in kilograms and grams.

Unit Activity 3

You will need a balance scale with 1 g, 10 g, and 1 kg masses and a number of different objects to weigh.

- Take one object and estimate its mass. Record your estimate in grams or kilograms.
- Now measure the mass of the object on the balance scale. Record the results.
- How close is your estimate to the actual mass of the object?
- Repeat the activity with different objects.

Self Assessment

1. Explain in your own words the difference between mass and weight.
2. What is the importance of using a standard unit for measurement?
3. When you carry a pile of books, do you feel its weight or its mass?
4. In the metric system, what is the standard unit of measurement of mass?
5. What is the basic SI unit of measurement of mass?

Note

Recall that we said it is incorrect from a scientific point of view to say that an object weighs two kilograms or the weight of an object is two kilograms. It is the mass of the object that is two kilograms. Unfortunately, for practical purposes, weight is used to mean mass. The idea of measuring weight in newtons is too sophisticated for primary school children and they will, understandably, continue to use the erroneous concept of weight for mass. However, your pupils will need to distinguish between weight and mass at higher levels of physics and mathematics lessons. The following suggestions might help your pupils begin to grasp the different meanings of the two words:

1. First, be consistent and always use the terms correctly. For example, you could have boxes of 1 g, 10 g, 100 g, and 1000 g masses in your classroom. Label them correctly.

2. Second, use a balance scale for the weighing activities described in this unit. After weighing an object, use the following language in your discussion of the results:

   > "The book weighs the same as the two hundred gram mass". The key phrase is 'weighs the same as'.

3. Finally, if an object weighs three hundred grams, we can also say this object has a mass of three hundred grams.

Summary

The mass of an object is a measure of the amount of matter in the object. Weight is the force exerted on the object by gravity.

The SI unit for mass is the kilogram and for weight, it is the newton.

The mass of an object remains the same, regardless of its distance from the centre of the earth. The weight of an object decreases as it moves away from the centre of the earth.

Unit 4 Test

1. On the earth's surface, a mass of 1 kg has a weight of approximately how many newtons?
2. Rewrite the following sentence to make it scientifically correct:

   The weight of this chair is 10 kg.
3. Define:
   a) Mass of an object
   b) Weight of an object
4. What is the difference between the mass and the volume of an object?
5. The SI unit for length is the metre. Why is it that the SI unit for mass is the kilogram and not the gram?
6. How many kilograms make two tonnes?
7. Do the following calculations, stating your answers in both kilograms and grams.

   a) $2\frac{1}{2}$ kg + 5 kg

   b) 0.1 kg + 500 g

   c) 4.8 kg - 0.25 g
8. From your experience, and your reading in other subjects such as Social Studies, where is the tonne normally used as the unit of measurement?
9. A load of coal has a mass of 50 kg at the bottom of a coal mine. The load is moved to the surface of the earth. Which of the following is correct about this load?
   a) The mass of the load is the same at the surface as at the bottom of a mine.
   b) The mass of the load is more at the surface than at the bottom of a mine.
   c) The mass of the load is less at the surface than at the bottom of a mine.
   d) The weight of the load is the same at the surface as at the bottom of a mine.
   e) The weight of the load is more at the surface than at the bottom of a mine.
   f) The weight of the load is less at the surface than at the bottom of a mine.

Answers to Self Assessment

1. Mass is the amount of matter in an object. Weight is the measure of the force or "pull" on the object by gravity.
2. to allow for comparison
3. weight
4. kilograms
5. grams

Answers to Unit 4 Test

1. 10 newtons
2. Mass of the chair is 10 kg.
3. (a) Mass is a measure of the amount of material in an object.
   (b) Weight is a measure of the force of gravity exerted on an object.
4. Volume of an object measures the space occupied by the object while mass measures the amount of material in an object.
5. By convention
6. 2000
7. (a) $7\frac{1}{2}$ kg; 7500 g
   (b) 0.6 kg; 600 g
   (c) 4.80025 kg; 4800.25 g
8. Crop produce, minerals, etc.
9. (a) and (f) are correct.

Unit 5: Time

Introduction

So far in this module, you have dealt with measurements relating to length, area, volume, and mass. In each case, you used both standard and non-standard units of measure. In this unit, you will:

- explore the concept of time
- use standard units for determining intervals of time
- deal with recorded time as measured by the clock

Objectives

After working through this unit, you should be able to:

- teach the concept of time, using standard and non-standard units
- increase your pupils' understanding of the two aspects of time—time intervals and time on the clock
- be able to demonstrate the main principles of time measurement
- develop activities to enhance your pupils' understanding of time

Measuring Time

There are two aspects of time that pupils must understand. They need to know time in terms of an interval—that is, the duration of an event from its start to finish. Time intervals are measured with standard units, such as seconds, minutes, hours, days, weeks, months, years, decades, centuries, and millennia. The interval may also be measured in non-standard units, such as the steady drip of water or the phases of the moon.

Your pupils also need to understand the concept of 'recorded time', for example, the time of the day, the time classes begin, and the time of the year when the weather is very cold.

A Sense of Time

The following two practice activities will help your pupils develop a sense of time intervals. The activities relate to both short and long intervals of time.

Practice Activity 1

For this activity, you need round objects (such as small balls), a watch, a table, and piles of books or similar objects for raising one end of the table.

Have your pupils work in small groups. Raise one end of the table by placing books under the legs, as illustrated in *Figure 5.1*.

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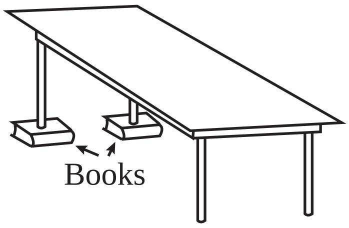


Figure 5.1

Take note of the time. Ask one pupil to release a ball at the raised end of the table. Record the time, in seconds, it takes the ball to roll from one end of the table to the other. Do this several times and have your pupils agree on the time (duration) it takes the ball to roll from one end of the table to the other.

Repeat this activity with different heights at the raised end of the table. Record the results. Discuss the results with your pupils and try to determine if there is a connection between the height of the table and the time it takes the ball to roll from one end to the other.

Involve your pupils in a discussion about the possibility for errors in this experiment and what the errors might be.

- What are the implications of the possible errors in interpretation of results?
- What would happen if the table was level?
- What would happen if the table was vertical?

Have your pupils plot a graph of the results of this experiment.

- For heights that were not experimented with, can they use the graph to determine the time it would take the ball to roll the length of the table?
- What other activities can pupils do that take the same time interval as rolling the ball down the table?

This activity is meant to give pupils a sense of short time intervals.

Practice Activity 2

Work with your pupils to make a calendar for the upcoming year. Mark the following events on the calendar:

- the opening and closing dates of your school
- public holidays
- the length of each school term
- the actual number of learning days
- the length of holidays

Ask your pupils to suggest other important events and mark them on the calendar. This activity is intended to help your pupils develop a sense of long time intervals.

Teaching Suggestions

To consolidate a sense of time, compare time intervals of both short and long duration with various other activities the class is familiar with. For example, what other activity takes the same time as:

- walking to school?
- preparing a meal?
- one school period?
- a person's lifetime?
- the duration from planting to harvest?

Unit Activity 1

Order the following events according to the length of time they take. Where necessary, provide an explanation for your answer.

(a) one lesson, break, school assembly
(b) cold season, hot season, rain season
(c) century, decade, millennium

Reflection

Read the following article:

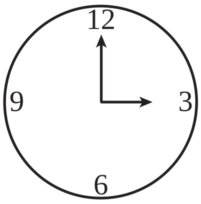
Haylock, D. (1995). "What principles are central to teaching measurement in the primary age range?" *Mathematics Explained for Primary Teachers*, London, Paul Chapman, 162.

Consider how the measurement of time relates to the following:

(a) comparison and ordering
(b) transitivity
(c) non-standard and standard units of measure
(d) approximation
(e) the meaning of zero

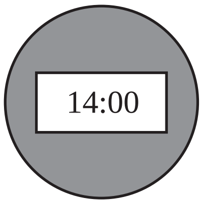
Clock Reading

We return to the second aspect of time—recorded time and clock reading. Being able to tell the time indicated on the clock is not the same as measuring the duration of time. The two types of clocks, the analogue clock with "hands" and the digital clock, are shown in *Figures 5.2(a)* and *5.2(b)*.



Analogue clock

Figure 5.2 (a)



Digital clock

Figure 5.2(b)

The number of different notations for telling time, e.g., 08 15, quarter past 8, 8 15, 08:15, can make teaching clock reading difficult. Another difficulty arises from the use of both analogue clocks (with 'hands') and digital clocks. Although a day is 24 hours long, an analogue clock only represents 12 hours. Digital clocks are easier to read.

Unit Activity 3

Prepare a list of the advantages of an analogue clock over a digital clock, and the advantages of a digital clock over an analogue clock.

Reflection

Between the analogue clock and the digital clock, which is better for introducing recorded time? Some points for and against analogue and digital clocks might include:

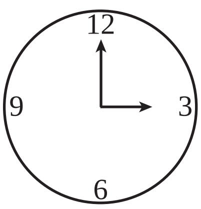
- a digital clock is more likely to have a 24 hour clock
- it is easier to relate time on the analogue clock (e.g., the fact that 06 58 is nearly seven o'clock)

It is important that your pupils understand both analogue and digital clocks.

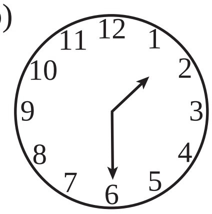
Self Assessment

1. Give two possible 'times' for the way a 24-hour digital clock would display the following:

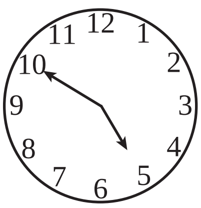
a)



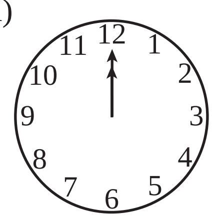
b)



c)



d)



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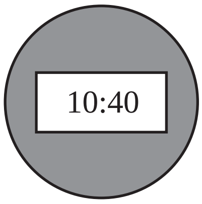
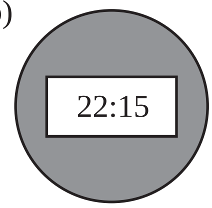
2. Give the following ‘times’ in terms of ____ o’clock, or minutes to or past the hour. Note—the following digital clocks are 24-hour clocks.

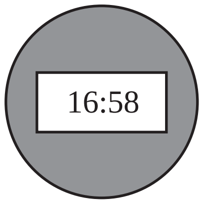
a)



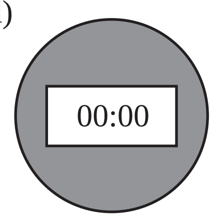
b)



c)



d)



Approaches to Teaching Time

Van de Walle (1990) says the standard approach to clock reading does not take into consideration the distinctly different actions and functions of the hour and minute hands of the clock. He suggests the following approach:

1. Begin with a one-handed clock. A clock with only an hour hand can be read with reasonable accuracy. When reading clocks with only an hour hand, use approximations. For example, it is about seven o’clock, or it is almost four o’clock, or it is halfway between two o’clock and three o’clock.
2. With a two-handed clock, discuss what happens to the larger minute hand as the small hour hand goes from one hour to the next. When the larger hand is at 12, the hour hand is pointing directly at a number. If the hour hand is at little past or before an hour (10 to 15 minutes), where will the minute hand be?
3. Use two analogue clocks, one with only an hour hand and one with two hands. Cover the two-handed clock. Periodically during the day, direct attention to the hour hand. Discuss the time in ‘approximate’ language. Have pupils predict where the minute hand should be, then uncover the other clock and check.
4. Teach ‘time after the hour’ in five-minute intervals. After you begin the activity in Step 3 above, count around the clock by fives. Instead of predicting that the minute hand is pointing at the 4, encourage pupils to say it is about 20 minutes after. As their skills develop, suggest that pupils look first at the little or hour hand to learn what time it is, then focus on the minute hand for precision.
5. Predict a digital clock reading when shown an analogue clock, and vice versa. Set an analogue clock when shown a digital clock. This can be done with both one-handed and two-handed clocks.

Practice Activity 3

Try Van de Walle's suggested approach to teaching your class to read time on analogue clocks. Does his method improve your pupils' understanding of time?

Managing Time

It is one thing to understand the mathematics of time, but quite another to manage time. One difficulty with managing time can be the poor estimation of the duration of events. The next activity is intended to help you improve your time management skills.

Unit Activity 4

Plan your mathematics lessons for the week so that each stage of every lesson is strictly timed. Inform the class that you intend to time the stages (e.g., introducing the main activity of the lesson, practical activity, discussion, exercise, and marking). Give the class the intended duration of each stage. Ask pupils to time you and signal when it is time to end that stage. At the end of the lesson, compare the intended time to the time actually used. Account for each of the following events:

- the actual time used was less than the intended time
- the actual time used was equal to the intended time
- the actual time used was more than the intended time

Explain each situation. At the end of the week, identify the key aspects of planning and managing your use of time and decide where you need to make changes or improvements.

Reflection

You probably found that thorough planning has a lot to do with effective time management. Managing time without proper planning is like spending money without a budget—before your realise it, the money is gone!

Adhering to a time frame in class will help you achieve the immediate objectives for the lesson. Keep in mind, however, that the main goals of teaching mathematics are fulfilled by interactions in and out of class, and go beyond the achievement of individual objectives. These options should not be lost because of the rush to a single objective.

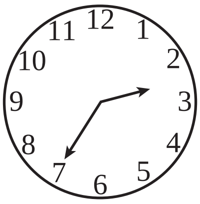
Summary

Pupils should develop their skills and understanding of time by using practical activities and events they are familiar with. They should develop a sense of estimating the length of time of both short-duration and long-duration events.

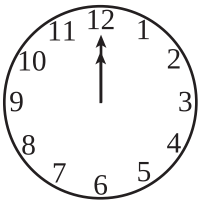
In the early stages, teaching time should include a comparison of duration and ordering. Pupils should be able to tell time using both analogue and digital clocks, and should understand the different notations for expressing time.

Unit 5 Test

1. Give two possible ‘times’ on the 24-hour digital clock for the time shown on the analogue clock below:

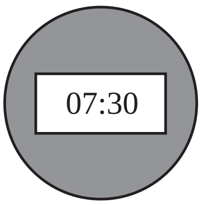


2. Give three possible ‘times’ on the 24-hour digital clock for the time shown on the analogue clock below:

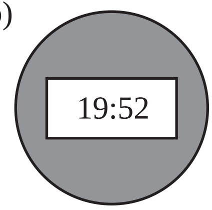


3. Give the following ‘times’ in terms of minutes to or past the hour.

a)

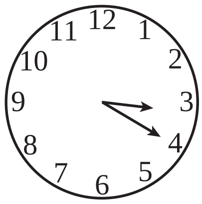


b)



4. An athlete starts a marathon at 10:52 hours and finishes at 13:12 hours. How long did it take the athlete to complete the race?

5. On one day, a teacher finished teaching at the time indicated on the analogue clock shown below:



a) Using the 24-hour digital clock notation, write the time the teacher finished teaching.

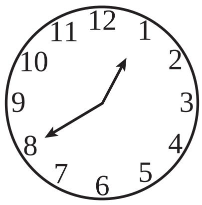
b) If the teacher had been teaching for 2 hours and 40 minutes, draw a diagram of an analogue clock, indicating the time the teacher started teaching.

Answers to Self Assessment

1. (a) 09 00 hours and 21 00 hours
   (b) 06 30 hours and 18 30 hours
   (c) 04 50 hours and 16 50 hours
   (d) 12 00 hours and 00 00 hours
2. (a) 20 minutes to 11
   (b) quarter past 10
   (c) 2 minutes to 7
   (d) 12 o'clock

Answers to Unit 5 Test

1. 02 35 and 14 35 hours
2. 00 00, 12 00, and 24 00 hours
3. a) half past seven
   b) 8 minutes to 8
4. 2 hours 20 minutes
5 a) 15:20 hours
   b)



Unit 6: Currency

Introduction

In this unit we will discuss currency as a measure for money. Due to the importance of money in our lives, other matters relating to the subject of money will be discussed in Module 4.

Objectives

After working through this unit, you should be able to:

- teach currency to primary pupils
- explain the meaning of foreign currency
- convert the currency of one country to the currency of another country

Reflection

At one time or another, you might have given something to someone in exchange for another item. If you have not done this yourself, you have probably seen people doing this type of exchange. Would you call this **transaction** buying and selling?

The Barter System

The transaction described above is called **barter**. In the barter system, people buy and sell by exchanging one commodity for another. For example, a farmer who grows enough maize for consumption and has surplus to sell, might exchange the maize for a cow.

Unit Activity 1

Carry out a simple research to find out whether there are places within your country where some form of barter system still exists. If there is, what sort of commodities are exchanged? How is the price or exchange value determined?

Money

As trading developed over time, people found it increasingly difficult to deal with commodities as a means of exchange. Not only was it difficult to carry around such items as elephant tusks, cattle, bales of tobacco, or salt to be used in buying and selling transactions, it was difficult to determine prices or value for exchange. Precious metals began to take over other commodities as a medium of exchange, and became common because they were:

- easy to carry
- easy to share out
- durable, that is they last a long time

- easy to represent in a standard measure
- widely recognised and accepted

In addition to these reasons, the value of precious metals was relatively stable and did not change over time.

Currency

A standard unit of measure for money facilitates buying and selling commodities that come in different sizes and amounts. When a standard unit of measure has been determined, other values of that currency are expressed as either fractions or multiples of the standard unit. Among international currencies, the standard units of measure are:

- Japanese yen
- United States dollar
- French franc
- German deutsche mark
- British pound sterling

Therefore, currency is the name of the money used by a given country.

Each country has its own standard unit of measure. The standard unit of measure of a currency is further divided into smaller and larger units called denominations. Smaller denominations are fractions of the standard unit, while the bigger denominations are multiples of the standard unit.

Practice Activity 1

Give your class a small research exercise.

1. Ask your pupils to find out from their parents if they know of a barter system, the kind of commodities involved, and the way prices of the commodities are/were determined.
2. In a class discussion, ask pupils to report on their findings.
3. Explain the currency of your country, giving all the denominations available. Show your pupils examples of these denominations.

Foreign Exchange

Each country has its own currency that is used for transactions that take place within that country. However, trading knows no boundaries and takes place within a country and between countries, including those overseas. Therefore, when you buy from another country, you need to change your money to the currency of that country.

To make international transactions easier, the US dollar has been established as the standard international currency. The US dollar (US$1) is the standard unit, and the smallest denomination of the dollar is the cent (100 cents = US$1). On a daily basis, the banks of a particular country provide current exchange rates for the currencies of the countries with which it does trade, as well as for other selected international currencies.

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The rates for exchanging the Zambian kwatcha to other currencies are given in *Table 6.1*. These rates were for September, 1999. This information is provided as an example only—you should check with a bank for current exchange rates.

Currency	Buying ZK	Selling ZK
1 US dollar	2360.00	2420.00
1 UK pound sterling	3818.00	3923.00
1 SA rand	387.87	403.70
1 Zim dollar	62.11	63.69
1 Bot pula	511.88	524.90
1 Malawi kwatcha	54.50	55.89

Table 6.1

Banks usually give exchange rates in two categories, the **buying rates** and the **selling rates**. For example, if you go to a bank in your home country to exchange your currency for US dollars, the bank will sell US dollars to you. Therefore, you will exchange at the selling rate.

If you take US dollars to the bank and exchange them for your currency, the bank will buy the dollars from you. Therefore, you will exchange at the buying rate.

Example

Ms. Motlotle works in Canada for the Commonwealth of Learning. She wants to go to her home country Botswana for the Christmas holidays but before she starts her holiday, she has to pass through Zambia to attend a seminar.

She calculates that she will require US$45 for out-of-pocket expenses in Zambia.

a) How much will she spend in Zambian kwatcha?

b) At the end of her two-day stay in Zambia, Ms. Motlotle finds that she has K60 000 remaining from her out-of-pocket expenses. She will not need the Zambian kwatcha in Botswana, so she decides to exchange the kwatcha for pula. How much pula does she get?

Answer:

a) In exchange for the Zambian kwatcha, the bank will buy US dollars from Ms. Motlotle. She will therefore exchange at the buying rate of ZK2360 to US$1

$$\text{US\$45} = \text{ZK}2360 \times 45$$
$$= \text{ZK}106\ 200.00$$

b) Ms. Motlotle wants to exchange kwatcha for pulas, therefore the bank will sell the pulas to her at the selling rate of ZK524.90 to P1.

$$\text{ZK}60\ 000 = 60\ 000 \text{ pula} \div 524.90$$
$$= \text{P}11.40$$

Foreign exchange banks only deal with notes or paper money, not coins, so Ms. Motlotle will receive P11.

Unit Activity 2

1. If you want to exchange US dollars for pulas in Botswana, the bank of Botswana will offer the exchange rate of US$1 = 4.7 pula. How many pulas will you get for US$60.
2. Using the exchange rates in Table 6.1, how much kwatcha do you get for US$60?
3. How many pulas do you get from US$60 by first changing to kwatcha, then to pulas?
4. Which is better, changing US dollars directly to pulas, or first changing US dollars to kwatcha, then to pulas?

Practice Activity 2

1. Find out the current rate for exchanging your currency to other currencies.
2. Based on these rates, formulate ten questions on foreign exchange.
3. Work out the answers before you assign these questions to your pupils.

Self Assessment

Use the exchange rates given in Table 6.1 to answer the following questions:

1. How much of each of the following currencies will you receive in exchange for ZK105 000.00?
   (a) US dollars
   (b) UK pounds
   (c) SA rand
   (d) Zimbabwean dollars
   (e) Botswana pulas
   (f) Malawi kwatcha
2. How much kwatcha will you receive in exchange for each of the following currencies?
   (a) US$52 (52 US dollars)
   (b) £70 (70 UK pounds)
   (c) R120 (120 SA rand)
   (d) Z$250 (250 Zimbabwean dollars)
   (e) P520 (520 Botswana pulas)
   (f) MK25 (25 Malawi kwatcha)

It is important to emphasise this essential social aspect of mathematics to your pupils. They will begin to understand the economics of their country and realise the importance of hard work.

Summary

- Money is a medium of exchange.
- Each country has its own money that has a name and a standard unit of measure.
- The name and the unit of measure of the money for a country is called currency.
- Foreign exchange is the exchange of currencies between countries.
- The US dollar has assumed an international status above all other currencies in international trade.

Reflection

Now that you have completed the unit on using currency as a measure:

- Are you satisfied with the way the topic was covered?
- Are you able to confidently impart this knowledge to your pupils?
- Are there any areas of the topic that you need to clarify?
- If you had difficulties, please discuss them with your colleagues or your supervisor.

Remember—the subject of money and currency is complicated, requiring the skills of an economist to be fully explored.

Unit 6 Test

Use the exchange rate chart in *Table 6.1* to answer the following questions:

1. Mutinta has just returned from studies abroad. She has US$630 that she wants to change to Zambian kwatcha. How much kwatcha does she get?
2. Milupi is travelling abroad. He wants to change ZK2 670 000 to US dollars. How many dollars will he receive?
3. Dube is a Zimbabwean national who works in Zambia. He wants to visit home during Christmas. He has ZK3 750 000. He wants to spend $\frac{2}{5}$ of this amount for shopping in South Africa before he proceeds home.
   (a) How many rands does he need for shopping in South Africa?
   (b) How much money remains to spend at home in Zimbabwean dollars?
4. Mrs Banda is travelling from Lusaka to Lilongwe on a business trip. She has ZK750 000 that she wants to change into Malawian kwatcha. How much Malawi kwatcha does she get?
5. How many UK pounds sterling do you get from ZK750 000?

Answers to Self Assessment

1. (a) US$43
   (b) £26
   (c) R260
   (d) Z$1648
   (e) P200
   (f) MK1878
2. (a) ZK122 720
   (b) ZK267 260
   (c) ZK46 540
   (d) ZK15 527
   (e) ZK266 177
   (f) ZK1362

Answers to Unit 6 Test

1. ZK1 486 800
2. ZK1103
3. (a) ZK3715
   (b) Z$35 327
4. MK134 192
5. £191

Unit 7: Temperature

Introduction

Temperature is an important measure in our daily lives. The measures introduced in the previous units—length, area, volume, weight, and currency—either have dimension or can easily be quantified in one form or another. In this unit, you will learn about temperature as a measure.

Objectives

After working through this unit you should be able to:

- explain temperature as a measure
- describe the Fahrenheit and Celsius (formerly Centigrade) scales as standard units for measuring temperature
- devise strategies to teach temperature to upper primary school pupils

Reflection

Think about the following questions before you continue.

- Based on your present knowledge, how would you define temperature?
- What are the non-standard units of measure of temperature?
- What are the standard units of measure of temperature?
- How were the standard units developed?

Temperature

It is beyond the context of this unit to give a detailed description of temperature in scientific terms. The opportunity might arise for you to teach those details in a science lesson. For the purposes of this unit, think of temperature in relation to "hotness". (Heat is measured in joules, but "hotness" is measured in units of temperature. Using temperature with a classroom vocabulary of "cold" and "warmth" and "hotness" keeps the science accurate.)

What are some activities that generate heat?

- burning fire
- doing strenuous work or physical exercise, such as running, playing football, etc.
- rubbing one object against another

You can probably list many other examples.

How can you detect or measure temperature using non-standard units of measure? There are two common ways to measure temperature using non-standard units:

- touch—for example, feeling the body of a sick person
- observation—for example, boiling water or heating a metal

Temperature is the degree of hotness of a body or substance.

Teaching Note

Compared to measures of length or area, temperature can be a difficult concept for pupils to understand. It can be made easier by using such words as hot, warm, cool, and cold. These levels of hotness can then be put in relative order to create the non-standard measuring scale shown below.

Very hot
Hot
Warm
Cool
Cold
Very cold

Figure 7.1

Initiate a class discussion to introduce temperature through questions, such as those in the discussion given above. Ask pupils to come up with their own definition of temperature before you give them yours.

Practice Activity 1

This activity is intended to show pupils how scales for measuring instruments, such as rulers and thermometers, were created. Pupils will need to understand number relations, including the concept of ‘less than’ and ‘greater than’.

Materials: water, a heat source, a pot

Caution—The following activity involves both a heat source and boiling water. To avoid a severe burns, you and your pupils must be extremely careful! This activity can be done by groups of pupils, if you are confident they can carry it out safely. You can also set up a demonstration so you handle the heat source and boiling water, and your pupils record their observations.

1. In this activity, pupils will:
   - find a way to record the levels of hotness of a pot of water, without using a thermometer
   - create a scale, using non-standard units, to measure temperature
2. Before you begin to heat the water, have pupils prepare the following scale:

5	Water is boiling continually	______ minutes
4		______ minutes
3		______ minutes
2		______ minutes
1		______ minutes
0	Water at room temperature	______ begin heating water

Figure 7.2

Continues on next page

Pupils can allocate numbers of their choice to the vertical scale. Observations can be made in five-minute increments, however you should note that time is recorded on a **separate scale**. It is important for pupils to understand that the measure of temperature, or the degree of hotness, is not time-dependent.

3. Place a pot of water over a heat source and have pupils begin recording their observations.
4. What detection methods can they use to measure the temperature of the water?
   - Before heating begins, they can use touch to determine that the temperature of the water is about the same as room temperature.
   - After heating begins, they can use observation. For example, small bubbles begin to form and rise to the surface of the water, steam begins to rise from the surface of the water, etc.

The Thermometer

Have you observed milk when it boils? What happens? Milk expands and if not watched, you will find some of the milk spilling out of the pot. The expansion of some fluids when heated probably gave scientists the idea to invent an instrument for measuring temperature.

Alcohol was the first liquid used in a thermometer because alcohol in a glass tube will expand and rise when heated. In 1714, a German physicist named Daniel Gabriel Fahrenheit (1686–1736) replaced alcohol with mercury—a silver-coloured metallic liquid that also expands when heated. The thermometer, an instrument used for measuring temperature, consists of mercury in a sealed glass tube. The tube is graduated (marked) with a scale. The number on the scale indicated by the upper level of the mercury is the temperature.

The Fahrenheit Scale

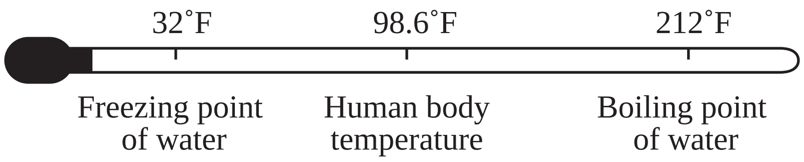

Figure 7.3

Fahrenheit chose the coldest temperature obtainable with a water-ice-salt mixture as his zero point, and he used the temperature of the human body as his second fixed point. The freezing and boiling temperatures of pure water were identified as 32°F and 212°F respectively. Temperatures below 0°F were marked with negative numbers.

The Celsius Scale

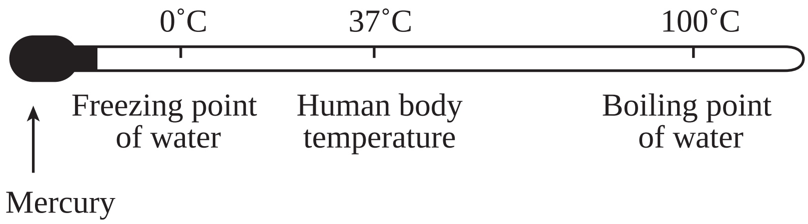

Figure 7.4

In 1743, a Swedish astronomer by the name of Anders Celsius (1701–1744) devised another thermometer scale, called the centigrade scale because it used a 100-degree scale. This scale is now called the Celsius scale, named after the inventor.

Celsius marked his 100 degree scale with 0°C as the freezing point of water and 100°C as the boiling point of water. He marked temperatures below 0°C with negative numbers.

Practice Activity 2

Have your pupils make a Celsius scale for measuring temperature. Their models can be drawn on paper or made of wood. Give your pupils a variety of temperatures (boiling water, a child with a fever, the current atmospheric temperature, etc.) and have them mark the temperatures on their models.

If your school has a thermometer, design activities for your pupils to practice measuring temperatures, reading the thermometer scale, and recording their observations.

Converting Celsius (°C) values to Fahrenheit (°F)

To convert Celsius (°C) to Fahrenheit (°F) use the formula:

$F = \frac{9}{5}C + 32$, where C is the temperature in Celsius and F is the temperature in Fahrenheit.

Example 1

The normal temperature of the human body, in Celsius, is 37°C. What is it in °F?

$F = \frac{9}{5}C + 32$ $\qquad$ $C = 37$

Body temperature $= \frac{9}{5} \times 37 + 32$

$= 1.8 \times 37 + 32$

$= 66.6 + 32$

$= 98.6°F$

To convert Fahrenheit (°F) to Celsius (°C) use the formula:

$C = \frac{5}{9}(F - 32)$; where F is the temperature in Fahrenheit and C is the temperature in Celsius.

Example 2
Convert 98.6°C to °C

$$
\begin{aligned}
C &= \frac{5}{9}(F - 32);\ F = 98.6°C \\
&= \frac{5}{9}(98.6 - 32) \\
&= \frac{5}{9} \times 66.6 \\
&= 5 \times 7.4 \\
&= 37.0 \quad = 37°C
\end{aligned}
$$

Convert ten Celsius (°C) temperatures to Fahrenheit (°F). Plot these temperatures on a graph and draw a line through the points. How could your pupils use this graph?

Self Assessment

1. Convert the following temperatures given in °F to °C.

   (a) 212°F (b) 32°F (c) 15°F (d) 85°F (e) $^{-}$54°F

2. Convert the following temperatures given in °C to °F.

   (a) 100°C (b) 0°C (c) 26°C (d) $^{-}$63°C (e) 63°C

Reflection

Now that you have completed the unit on measuring temperature, how well do you understand this subject? Do you have any difficulties with this subject? Try to discuss these difficulties with your colleagues or supervisor. How well can you teach the topic of temperature as a measure?

Summary

Temperature is a measure of the degree of hotness of a substance. In this unit, you learned how scientists developed the thermometer scale for measuring temperature. Two scales for measuring temperature were discussed—the Fahrenheit and Celsius scales.

Unit 7 Test

1. Who invented the Fahrenheit scale for measuring temperature and in which year? What was his nationality?
2. Who invented the Celsius scale for measuring temperature and in which year? What was his nationality?
3. Convert the following temperatures given in °C to Fahrenheit:

   (a) 27°C (b) $^{-}27$°C
4. Convert the following temperature given in °F to Celsius:

   (a) 27°F (b) $^{-}27$°F

Answers for Self Assessment

1. (a) 100°C
   (b) 0°C
   (c) $^-$9.4°C
   (d) 29.4°C
   (e) $^-$47.78°C

2. (a) 212°F
   (b) 32°F
   (c) 78.8°F
   (d) $^-$81.4°F
   (e) 145.4°F

Answers to Unit 7 Test

1. Gabriel Daniel Fahrenheit; 1714; German
2. Anders Celsius; 1742; Swedish
3. (a) 80.6°F
   (b) $^-$16.6°F
4. (a) $^-$2.78°C
   (b) $^-$32.78°C

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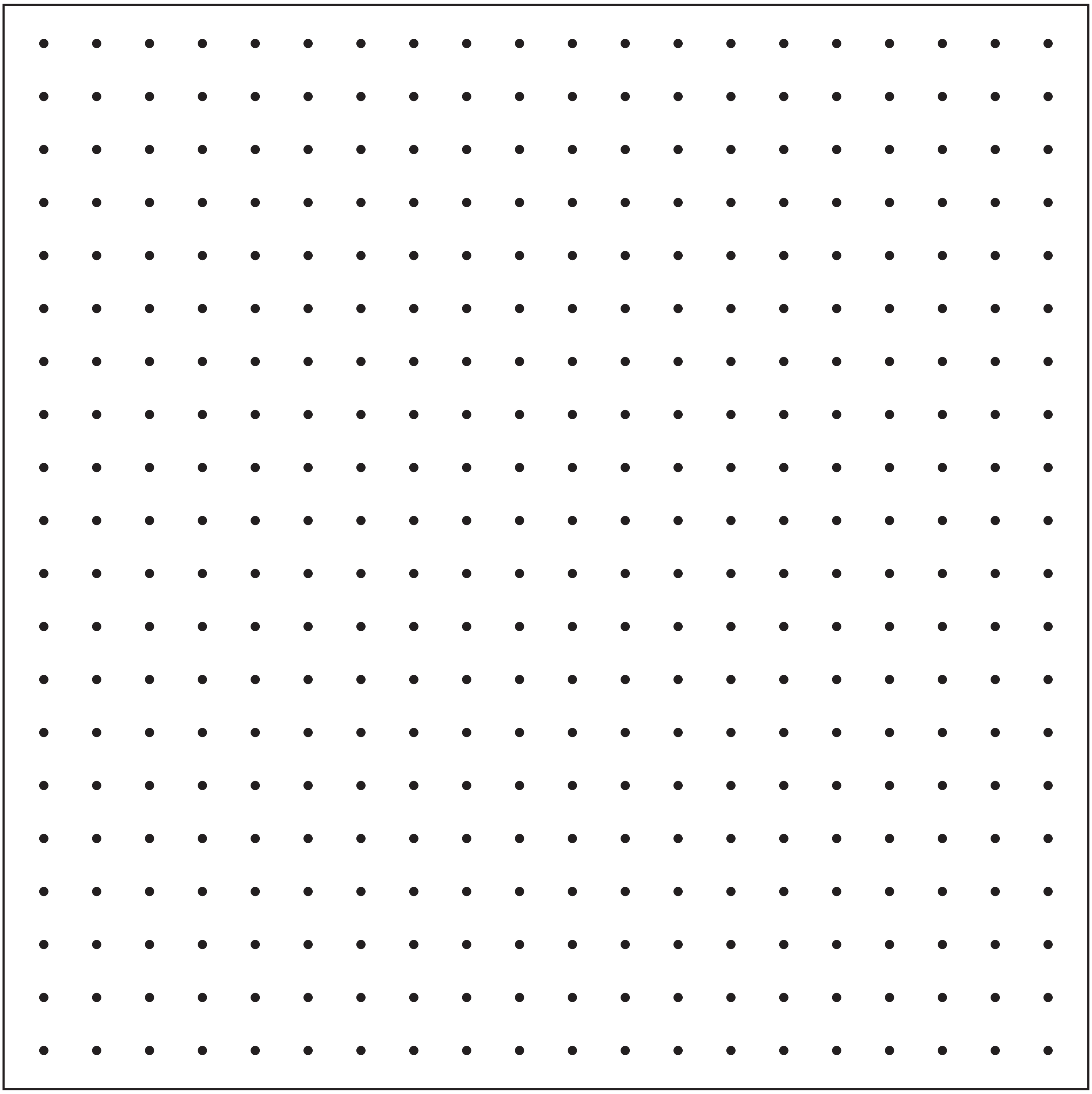
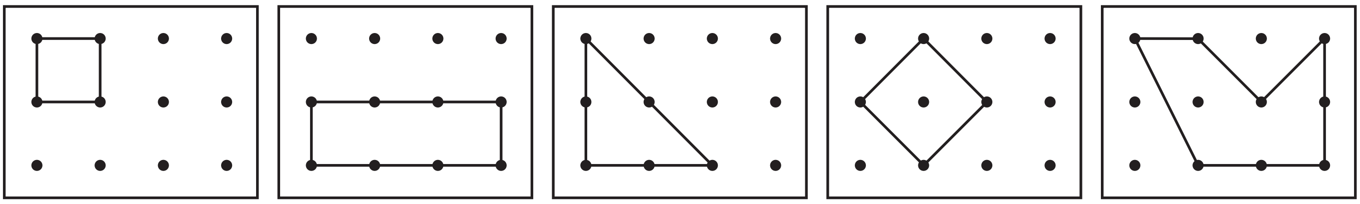
Appendix 1: 1 cm squares

Appendix 2: Square Geoboard Paper





Appendix 3: Circular Geoboard