Module 5

Upper Primary Mathematics

Geometry

THE COMMONWEALTH of LEARNING

Science, Technology and Mathematics Modules

for Upper Primary and Junior Secondary School Teachers of Science, Technology and Mathematics by Distance in the Southern African Development Community (SADC)

Developed by The Southern African Development Community (SADC)

Ministries of Education in:

- **Botswana**
- **Malawi**
- **Mozambique**
- **Namibia**
- **South Africa**
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SCIENCE, TECHNOLOGY, AND MATHEMATICS MODULES

This module is one of a series prepared under the auspices of the participating Southern African Development Community (SADC) and The Commonwealth of Learning as part of the Training of Upper Primary and Junior Secondary Science, Technology and Mathematics Teachers in Africa by Distance. These modules enable teachers to enhance their professional skills through distance and open learning. Many individuals and groups have been involved in writing and producing these modules. We trust that they will benefit not only the teachers who use them, but also, ultimately, their students and the communities and nations in which they live.

The twenty-eight Science, Technology, and Mathematics modules are as follows:

Upper Primary Science
Module 1: *My Built Environment*
Module 2: *Materials in my Environment*
Module 3: *My Health*
Module 4: *My Natural Environment*

Upper Primary Technology
Module 1: *Teaching Technology in the Primary School*
Module 2: *Making Things Move*
Module 3: *Structures*
Module 4: *Materials*
Module 5: *Processing*

Upper Primary Mathematics
Module 1: *Number and Numeration*
Module 2: *Fractions*
Module 3: *Measures*
Module 4: *Social Arithmetic*
Module 5: *Geometry*

Junior Secondary Science
Module 1: *Energy and Energy Transfer*
Module 2: *Energy Use in Electronic Communication*
Module 3: *Living Organisms' Environment and Resources*
Module 4: *Scientific Processes*

Junior Secondary Technology
Module 1: *Introduction to Teaching Technology*
Module 2: *Systems and Controls*
Module 3: *Tools and Materials*
Module 4: *Structures*

Junior Secondary Mathematics
Module 1: *Number Systems*
Module 2: *Number Operations*
Module 3: *Shapes and Sizes*
Module 4: *Algebraic Processes*
Module 5: *Solving Equations*
Module 6: *Data Handling*

A MESSAGE FROM THE COMMONWEALTH OF LEARNING

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Dato' Professor Gajaraj Dhanarajan
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UPPER PRIMARY MATHEMATICS PROGRAMME

Introduction

Welcome to the programme in Teaching Upper Primary Mathematics. This series of five modules is designed to help you strengthen your knowledge of mathematics topics and acquire more instructional strategies for teaching mathematics in the classroom.

Each of the five modules in the mathematics series provides an opportunity to apply theory to practice. Learning about mathematics entails the development of practical skills as well as theoretical knowledge. Each topic includes examples of how mathematics is used in practice and suggestions for classroom activities that allow students to explore the maths for themselves.

Each module also explores several instructional strategies that can be used in the mathematics classroom and provides you with an opportunity to apply these strategies in practical classroom activities. Each module examines the reasons for using a particular strategy in the classroom and provides a guide for the best use of each strategy, given the topic, context, and goals.

The guiding principles of these modules are to help make the connection between theory and practice, to apply instructional theory to practice in the classroom situation, and to support you, as you, in turn, help your students to apply mathematics to practical classroom work.

Programme Goals

This programme is designed to help you:

- strengthen your understanding of mathematics topics
- expand the range of instructional strategies that you can use in the mathematics classroom

Programme Objectives

By the time you have completed this programme, you should be able to:

- guide students as they work in teams on practical projects in mathematics, and help them to work effectively as a members of a group
- use questioning and explanation strategies to help students learn new concepts and to support students in their problem solving activities
- prepare your own portfolio of teaching activities

The relationship between this programme and the mathematics curriculum

The content presented in these modules includes some of the topics most commonly covered in the mathematics curricula in southern African countries. However, it is not intended to comprehensively cover all topics in any one country's mathematics curriculum. For this, you need to consult your national or regional curriculum guide. The curriculum content presented in these modules is intended to:

- provide an overview of the content in order to support the development of appropriate teaching strategies
- use selected parts of the curriculum as examples of the application of specific teaching strategies
- explain those elements of the curriculum that provide essential background knowledge, or that address particularly complex or specialised concepts
- provide directions to additional resources on the curriculum content

How to work on this programme

As is indicated in the goals and objectives, this programme requires you to participate actively in each module by applying instructional strategies when exploring mathematics with your students and by reflecting on that experience. There are several ways to do this.

Working on your own

You may be the only teacher of mathematics in your school, or you may choose to work on your own so you can accommodate this programme within your schedule. If this is the case, these are the recommended strategies for using this module:

1. Establish a schedule for working on the module. Choose a date by which you plan to complete the first module, taking into account that each unit will require between six and eight hours of study time and about two hours of classroom time to implement your lesson plan. For example, if you have two hours a week available for study, then each unit will take between three and four weeks to complete. If you have four hours a week for study, then each unit will take about two weeks to complete.
2. Choose a study space where you can work quietly without interruption, such as a space in your school where you can work after hours.
3. If possible, identify someone who is interested in mathematics or whose interests are relevant to it (for example, a science teacher in your school) with whom you can discuss the module and some of your ideas about teaching mathematics. Even the most independent learner benefits from good dialogue with others. It helps us to formulate our ideas—or as one learner commented, "How do I know what I'm thinking until I hear what I have to say?"

Working with colleagues

If there are other teachers of mathematics in your school or in your immediate area, then it may be possible for you to work together on this module. You may choose to do this informally, perhaps having a discussion group once a week or once every two weeks about a particular topic in one of the units. Or, you may choose to organise more formally, establishing a schedule so that everyone is working on the same units at the same time, and you can work in small groups or pairs on particular projects.

Your group may also have the opportunity to consult with a mentor, or with other groups, by teleconference, audioconference, letter mail, or e-mail. Check with the local coordinator of your programme about these possibilities so you can arrange a group schedule that is compatible with these provisions.

Working with a mentor

As mentioned above, you may have the opportunity to work with a mentor, someone with expertise in maths education who can provide feedback about your work. If you are working on your own, communication with your mentor may be by letter mail, telephone, or e-mail. If you are working as a group, you may have occasional group meetings, teleconferences, or audioconferences with your mentor.

Resources available to you

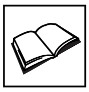
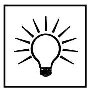
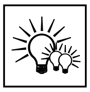
Although these modules can be completed without referring to additional resource materials, your experience and that of your students can be enriched if you use other resources as well. A list of resource materials is provided at the end of each module. You might also find locally available resource material that will enhance the teaching/learning experience. These include:

- manipulatives, such as algebra tiles, geometry tiles, and fraction tiles
- magazines with articles about maths
- books and other resources about maths that are in your school or community library

ICONS

Throughout each module, you will find some or all of the following icons or symbols that alert you to a change in activity within the module.

Read the following explanations to discover what each icon prompts you to do.

	**Introduction**	Rationale or overview for this part of the course.
	**Learning Objectives**	What you should be able to do after completing this module or unit.
	**Text or Reading Material**	Course content for you to study.
	**Important—Take Note!**	Something to study carefully.
	**Self-Marking Exercise**	An exercise to demonstrate your own grasp of the content.
	**Individual Activity**	An exercise or project for you to try by yourself and demonstrate your own grasp of the content.
	**Classroom Activity**	An exercise or project for you to do with or assign to your students.
	**Reflection**	A question or project for yourself—for deeper understanding of this concept, or of your use of it when teaching.
	**Summary**	
	**Unit or Module Assignment**	Exercise to assess your understanding of all the unit or module topics.
	**Suggested Answers to Activities**	

CONTENTS

Module 5: Geometry

Module 5
Geometry

Introduction to the Module

The world around us is all about shapes. Shapes are a cornerstone of geometry, just as they are for building construction, and are probably second only to numbers in terms of the hierarchy of mathematics topics that pertain to everyday life.

This module looks at shapes as a geometry topic for upper primary school and covers the parts of the primary school syllabus that are generally considered to be the basics of primary geometry.

Aim of the Module

Like all other modules in the Upper Primary Mathematics series, this module is intended to help primary school teachers sharpen and update their teaching skills and improve their knowledge of the mathematics topics covered in this module.

Structure of the Module

The module is divided into units and presents a series of **unit activities** to help you consolidate your knowledge of mathematics. It also presents **practice activities** for you to try with your pupils. These practice activities are intended to improve the methods you use to teach mathematics. The self assessment exercises will help you test your knowledge of the content of this module.

The module topics are divided into seven units, covering angles, the properties of angles, similar figures, constructions of two dimensional shapes, constructions of three dimensional shapes, transformations, and scale drawing and direction.

Objectives of the Module

After working through this module, you should have a good understanding of the mathematics content for upper primary in the following topics:

- angles
- similar figures
- constructions in both two and three dimensions
- transformations
- scale drawing and directions

In addition to acquiring the mathematics content, you will be equipped with practical and engaging teaching strategies.

Unit 1: Angles

Introduction

Angles, as taught to upper primary students, are essentially about measurement. When we talk about the angle between two lines, we do not dwell on the shape formed by the two lines nor on the point where they meet nor on the space between the lines, but rather on a particular kind of measurement.

Primary-grade students often have difficulty stating exactly what an angle is, but they tend to have little difficulty recognising angles and measuring them with a protractor. This unit begins by defining what an angle is, then moves quickly to drawing and measuring angles. Memorising a definition can hinder pupils' understanding of angles, so we suggest you use an informal definition, such as "a place where two lines meet." Then have your pupils begin cutting out angles and applying the concept to concrete figures.

We begin by classifying angles—acute, right, obtuse, straight, and reflex angles— then move to triangles, so angles are seen in a more concrete context. These classifications and uses of angles are appropriate starting points for your students' learning.

The word 'line' will always refer to a straight line. You will need a protractor for this unit, as you will be called upon to use it.

Objectives

After working through this unit you should be able to:

- define an angle using both dynamic and static views
- measure angles in non-standard units
- measure angles with a protractor
- classify angles by size
- apply knowledge of different types of angles to carry out mathematical and non-mathematical 'tasks'
- use properties of angles of a triangle to investigate types of polygons other than the triangle

Meaning of angle

Euclid's definition, circa 300 BC, stated that "A plane angle is the inclination to one another of two lines in a plane which meet one another and do not lie in a straight line" (Elements, Book 1, Definition 7).

Musser and Burger (1988: 436) define an angle as the "union of two line segments with a common end point…." The common endpoint is called the **vertex** of the angle. The plural of vertex is **vertices**.

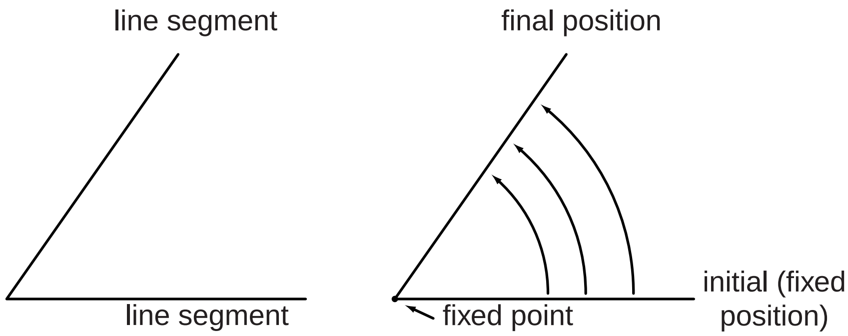
Billstein and others (1990:508) say the more recent definition of an angle is the union of two rays with a common end point.

Reflection

Can you make sense of these definitions? Write down your “working” definition of an angle and explain how it is consistent with the classical definitions.

Definition

Whatever your definition of an angle, consider the following alternatives. We can view an angle as the amount of opening between two intersecting line segments. We will refer to this view as the **static** view. Also, we can view an angle as the “amount of turning” of a line segment from a fixed or starting position, about a fixed point. We will refer to this as the **dynamic** view. Both views are illustrated below and will be used in this unit.



a) Static View   b) Dynamic View

Figure 1.1

You may be uncomfortable with Euclid’s definition because it is written in old English. In the definitions by Musser and Burger, and Billstein and others, the word ‘union’ may pose problems.

Naming angles

The line segments of an angle are sometimes called the arms of an angle, although students don’t need to learn this term. The intersection point is the vertex, and they *do* need to know this term. An angle can be named from three points: the vertex and a point on each arm, with the vertex always listed between the other two points. Thus, the angle below (*Figure 1.2*) may be named Angle CBA or Angle ABC.

Note—we can use the symbol ∠ for the word angle, and write ∠CBA or ∠ABC.

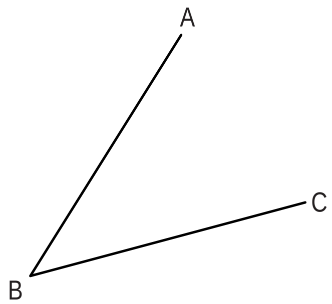


Figure 1.2

When there is no risk of confusion, it is customary to name an angle simply by its vertex or with a number. The angle QPR in *Figure 1.3*(a) can be named ∠P.

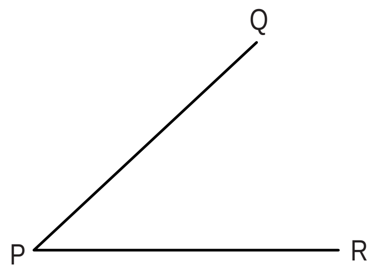


Figure 1.3(a)

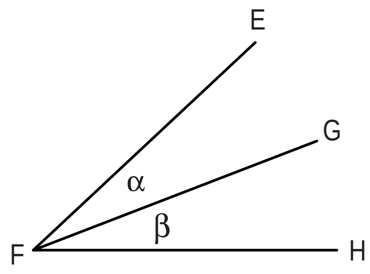


Figure 1.3(b)

However, more than one angle in *Figure 1.3*(b) has a vertex F (∠EFG, ∠GFH, and ∠EFH), so the angle EFG in *Figure 1.3*(b) is still named ∠EFG.

Note the following points about the diagram in *Figure 1.3*(b):

1. The notation ∠F is inadequate for naming any one of the angles.
2. Greek letters α and β have been used to label two angles. This is common, especially in older textbooks, but Greek symbols are unnecessary for upper primary students. We suggest you avoid them, unless you have other reasons for introducing Greek symbols to these grades.
3. Many pupils will look at *Figure 1.3*(b) and logically infer this:

   $$\angle GFH + \angle EFH = \angle EFG$$

   This inference, that adjacent angles can be added together, should be a starting point for teaching classical Euclidean geometry. Encourage the intuitive notion that angles can be added together; it looks perfectly obvious and eases student learning. However, look at the classical definitions: angles are not numerical values, so you cannot add them. Rather, the **measures** of the angles can be added. If “m” refers to the measure of an angle in degrees, then:

   $$m\angle GFH + m\angle EFH = m\angle EFG$$

   Note—the distinction between an angle and its measure is not made in this module and it is not necessary for you to teach this in the upper primary classroom. We avoid all m∠EFG notation and simply add angles as if they were numbers.

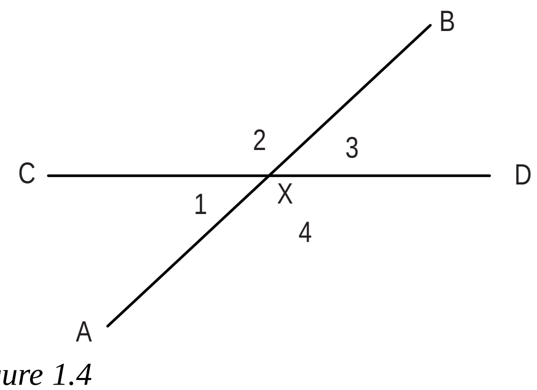


Figure 1.4

In the previous diagram, lines AB and CD intersect at X. So X is the vertex, and “4” refers to one of the angles.

Self Assessment 1

1. Angle AXC can be written as _______ or _______
2. Angle 3 can be written as _______
3. Angle BXD can be referred to as ∠X but this is not advisable. Why?

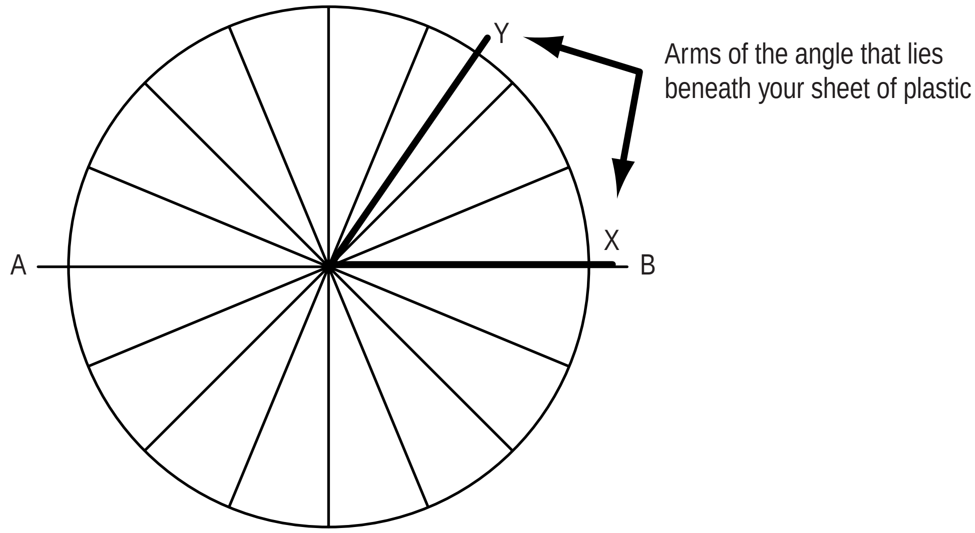
Unit Activity 1

This exercise can also serve as a Practice Activity for your students, early in their study of angles.

You will need:

- a sheet of transparent plastic and a pen that can write on it
- a pair of compasses, or some means of drawing a neat circle (i.e., a jar lid or the base of a tin can)
- paper

1. Draw a circle with a radius of about 5 cm on the plastic sheet.
2. Divide the circle into sixteen equal parts as shown below (draw lines but do not cut).
3. Draw line AB to extend outside the circle as shown in the diagram.
4. On a sheet of paper, draw four angles of different sizes using a ruler.
5. Now measure the angles you have drawn using the circle on the plastic. Place the plastic on one of your angles so the centre of the circle coincides with the vertex and line AB lies on one of the arms of your angle, as shown below:



For example: ∠XPY = 2.5 parts

- How many parts (approximately) of the circle make your angle?
- Record your results.
- Repeat activity with the other angles you drew.
- Now subdivide your measuring circle into thirty-two equal parts, measure the four angles, and record the results.

Reflection

Suppose you want to communicate your results to a fellow teacher in another school. How will you do it?

Measuring using standard units

Your reflection on the above section will have demonstrated the need for a **standard unit** to measure angles.

Now return to the measuring instrument you used in the practice activity. Suppose you divided it into 360 equal parts, and called each part a **degree**. The whole circle is made up of how many degrees? One complete revolution of the circle is 360 degrees.

Self Assessment 2

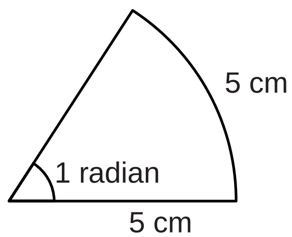
1. Half of the revolution of the circle is…………….degrees.
2. A quarter of a revolution is ……………degrees.
3. ……………..of a revolution is 45 degrees.
4. ……………..of a revolution is 60 degrees.
5. One and a half revolutions is …………….degrees.
6. One degree is ……………of a revolution.

Historical note on angle measurement

The use of 360 degrees to measure angles seems to date to the Babylonian culture (4000 to 3000 BC). A **degree** is further divided into sixty equal parts called **minutes**, and a minute is further divided into sixty equal parts called **seconds**. How many seconds in one complete revolution around a circle? The words minute and second come from Latin, and ultimately from Arabic translations of Babylonian sexagesimal (base 60) fractions.

We write a measurement of 29 degrees and 40 minutes as 29° 40’. That of 29 degrees 40 minutes and 17 seconds is written 29° 40’ 17’’. Also note 29° 30’ can be written as 29.5° (decimal notation for the fractions of a degree). Decimals are gradually displacing minutes and seconds for fractional degrees, since decimals are more readily manipulated with the aid of calculators.

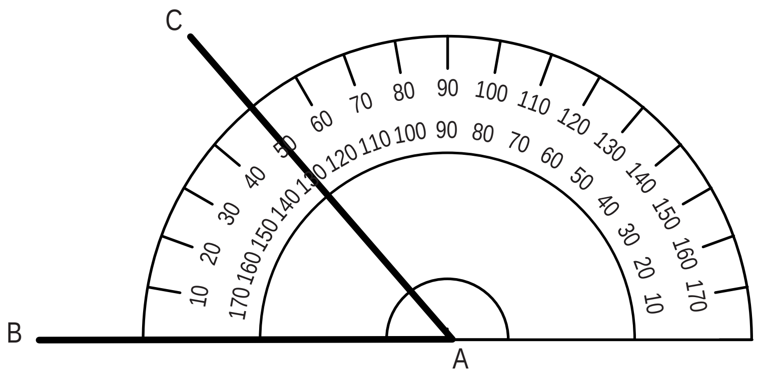
While the common unit for measuring angles is the degree, angles can also be measured in **radians**. Imagine drawing an arc of a circle so that the length of the arc is the same as the length of the radius as shown below: the angle it forms is equal to one radian.



360 degrees, one full revolution around a circle, equals exactly 2π radians. This follows from the formula for circumference: $C = 2\pi r$. One radian is thus equal to 360/2π degrees, or roughly 57.29578°.

Radian measure has the advantage of being dimensionless, since it is not based on an arbitrary division of the circle into sixteen compass points or 360 degrees or 400 percentage points. One often sees radian measure, without dimensions, given in senior textbooks. For example, ∠BAC = 3π/2.

In this module, we measure only in positive integer (whole) degrees. Fractional degrees and radians are not used. To measure angles in degrees, students use a protractor.

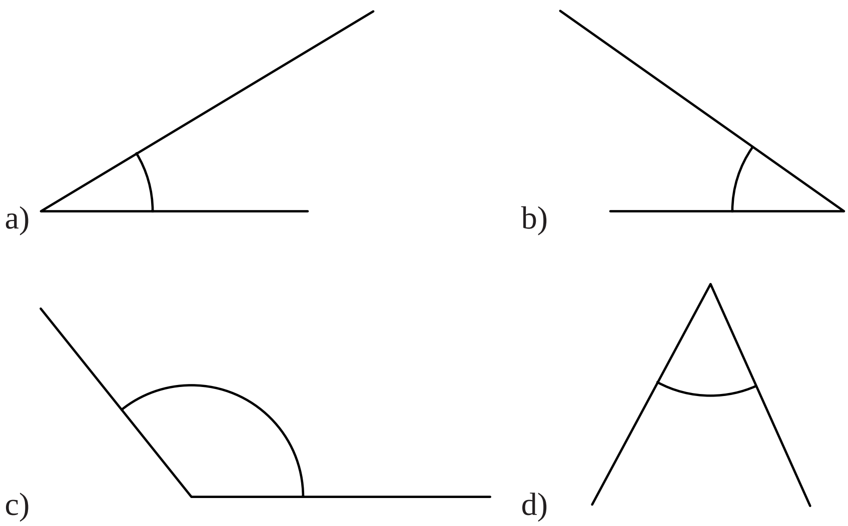


The angle ∠BAC in the figure above is being measured with a protractor.

What is the size of ∠BAC?

Unit Activity 2

You will need a protractor for this activity. Measure the following angles using your protractor.

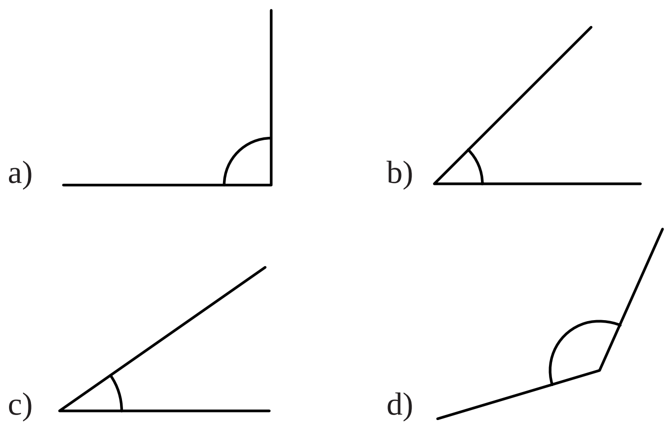


Reflection

You should have noticed that your protractor has two scales, one going clockwise and the other counter-clockwise. Which scale did you use to measure the angles in the last practice exercise? For each angle in the last exercise, try measuring the same angle using the other scale. What do you notice? Did you get the same answer?

Unit Activity 3

1. Estimate the size of each of the following angles. Follow by actual measuring:



Reflection

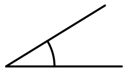
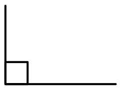
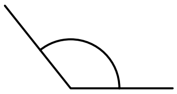
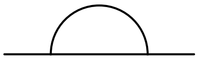
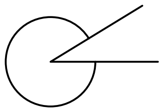
How close are your estimates to the correct answer? Do not worry too much if your estimates were off. As with linear measure, accuracy comes with practice.

Unit Activity 4

You will again need a protractor. Draw angles of the following sizes:

50, 20, 150, 5, 114, 88, 60.5, 39

Classifying angles according to size:

Angle Name	**Diagram**	**Description of Angle**
Acute Angle		More than 0 but less than 90
Right Angle		Angle size equal to 90
Obtuse Angle		More than 90 but less than 180
Straight Line Angle		Angle size is equal to 180
Reflex Angle		More than 180 but less than 360

Unit Activity 5

You will need a ruler for this activity. Use the ruler (not a protractor) to draw:

a) an acute angle that is more than 30 but less than 60

b) a right angle

c) an obtuse angle that is more than 120 but less than 150

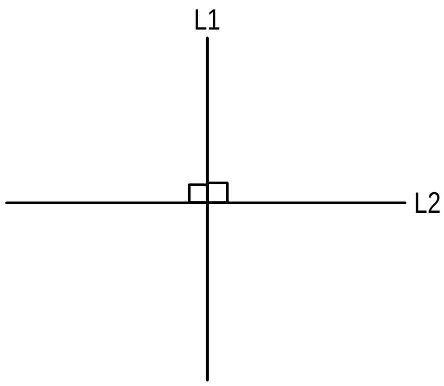
Notes about the classified angles

We usually indicate a 90-degree right angle in diagrams as shown in the preceding table.

The reflex angle was described as more than 180 but less than 360. Here, we use the static view of an angle; the assumption being that the biggest angle size is 360. However, if we use the dynamic view, the amount of turning can continue past 360. For example, we can make two revolutions that are equivalent to 720. Thus a reflex angle, using the dynamic view, is any angle that is more than 180.

Lastly, it is a matter of choice to use a curved line in an angle diagram to indicate which static angle is being labelled. A curved arrow indicates a dynamically defined angle. Using a curved line is optional when there is only one angle at a vertex, but it becomes important when two or more angles share the vertex. In this case, shorter and longer curved lines can differentiate the overlapping angles.

Perpendicular Lines



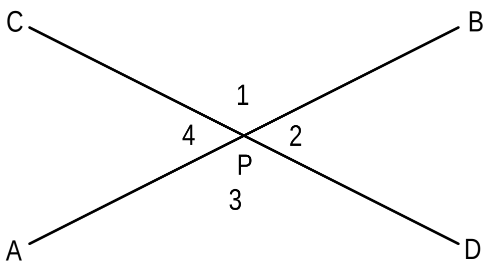
When two lines intersect so that the angles formed are at right angles, as shown in the diagram above, the lines are perpendicular. In the diagram, lines L1 and L2 are perpendicular, so we write L1 ⊥ L2.

Reflection

Remember—the symbol ⌉ is used to indicate a right angle. If a horizontal is drawn perpendicular to a vertical line and the lines intersect, what sizes are the angles at the intersection point?

Type of angles

Vertically Opposite Angles



If two intersecting lines are drawn, then vertically opposite angles are created. $\angle 1$ and $\angle 3$ are vertically opposite and $\angle 2$ and $\angle 4$ are vertically opposite.

Are vertically opposite angles equal? Measure and find out!

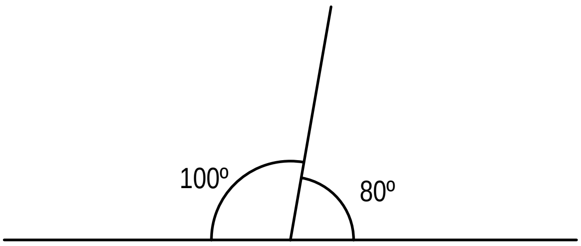
Supplementary angles

Given that $\angle X + \angle Y = 180$ degrees, state a pair of possible values for $\angle X$ and $\angle Y$. $\angle X$ and $\angle Y$ are called supplementary angles. We say $\angle X$ is the supplement of $\angle Y$, and also $\angle Y$ is the supplement of $\angle X$. The sum of supplementary angles is always 180 degrees.

For example, the supplement of 100 is 80 because 100 + 80 =180

What is the supplement of 60?
If you add 60 to its supplement, do you get 180?
Remember—60 + (supplement of 60) =180.

Now, accurately draw and cut out angles of 100 and 80. Place them side by side as shown below:



Do the two angles form a straight line? You should recall what was said about the angle of a straight line.

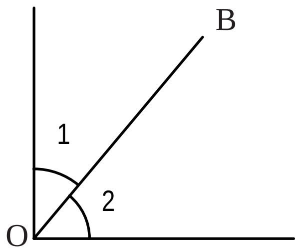
Complementary Angles

Given that: $\angle X + \angle Y = 90$, what are the possible values of $\angle X$ and $\angle Y$? If $\angle X$ is 30, then $\angle Y$ is 60˚. $\angle X$ and $\angle Y$ are called complementary angles. Each is a complement of the other.

What is the complement of 45? Add 45 to its complement. You should get 90, thus 45 is its own complement.

Adjacent Angles

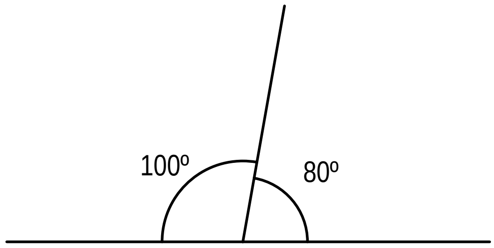
In the following illustration, $\angle 1$ and $\angle 2$ are adjacent, meaning they are next to each other.



What is common to both $\angle 1$ and $\angle 2$?

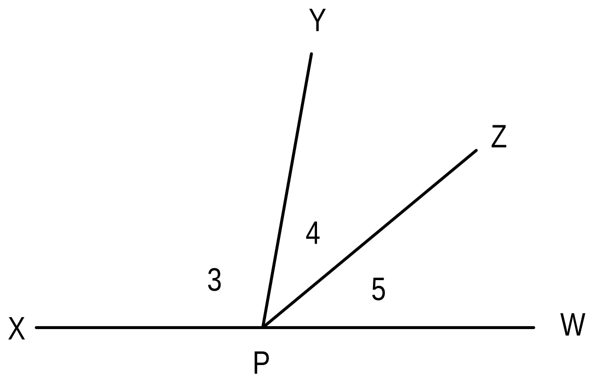
If you said they have a common vertex (O) and a common arm (OB), you are correct. Therefore, we can say adjacent angles share a common arm and vertex.

We said 100 and 80 are supplementary and also that they form a straight line (see below):



Are these two angles adjacent? Why? What is common to both of them? Yes, they are a special case of adjacent angles in that they form a straight line.

Consider the diagram below:

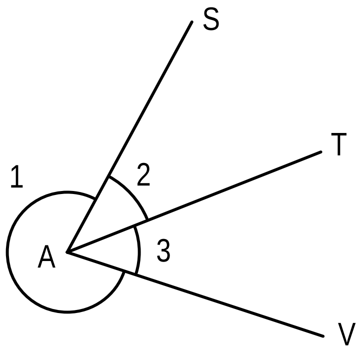


∠XPW is a straight line. What is the sum of ∠3, ∠4, and ∠5?

∠3 + ∠4 + ∠5 = ?

Remember what we said about angles on a straight line—they add up to 180.

Angles at a point

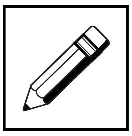


Consider the above diagram.

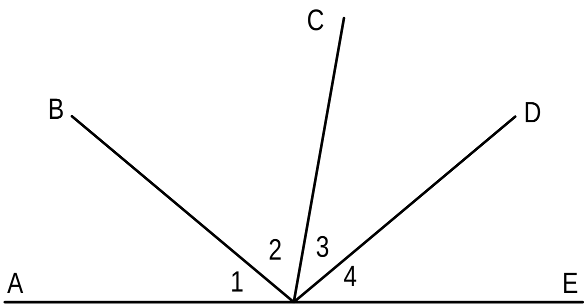
What is the sum of ∠1, ∠2, and ∠3? Remember—one revolution is 360.
Now, what is common to all three angles?
What is the vertex of ∠1?
What is the vertex of ∠2?
What is the vertex of ∠3?

The common vertex of these angles is A.

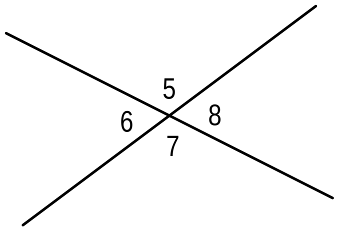
Angles having a common vertex at the centre of a circle, as in the above diagram, are sometimes referred to as angles at a point and their sum is 360.



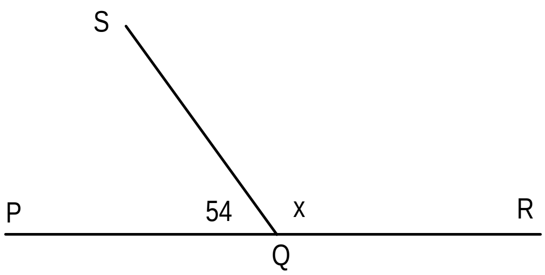
Self Assessment 3



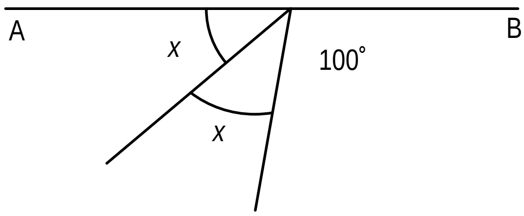
1. Use the diagram above to:
   a) Name all angles that are adjacent.
   b) Name all pairs of adjacent angles on a straight line.
2. Use the following diagram to:



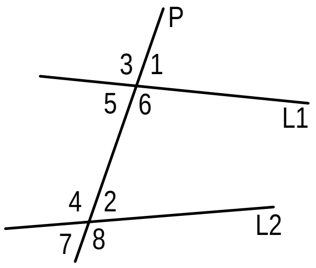
   a) Name all pairs of vertically opposite angles.
   b) Name all pairs of adjacent angles on a straight line.
   c) Name the angles at the point.
3. What are supplementary angles?
4. What is the supplement of:
   (a) 130 (b) 40 (c) 102 (d) 10 ?
5. What are complementary angles?
6. What is the complement of:
   (a) 30 (b) 70 (c) 10 (d) 50 ?
7. In the diagram below, line PQR is a straight line. What is the value of x?



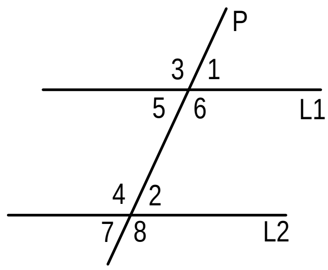
8. AB below is a straight line. What is the value of x?



Angles formed by parallel lines and a transversal



a) L_1 is not parallel to L_2



b) L_1 is parallel to L_2

In both diagrams above, the line P intersects line L_1 and L_2. Any line that intersects a pair of lines is called a **transversal** of those lines. Therefore, line P is a transversal of lines L_1 and L_2.

Angles formed by these lines are named according to their position in relation to the transversal and the two given lines. Various types of angles, together with examples from the above figure, are listed:

- Interior angles: $\angle 2$, $\angle 4$, $\angle 5$, $\angle 6$
- Exterior angles: $\angle 3$, $\angle 1$, $\angle 7$, $\angle 8$
- Alternative interior angles: $\angle 5$ and $\angle 2$, $\angle 4$ and $\angle 6$
- Alternative exterior angles: $\angle 1$ and $\angle 7$, $\angle 3$ and $\angle 8$
- Corresponding angles: $\angle 3$ and $\angle 4$, $\angle 5$ and $\angle 7$, $\angle 1$ and $\angle 2$, $\angle 6$ and $\angle 8$

Unit Activity 6

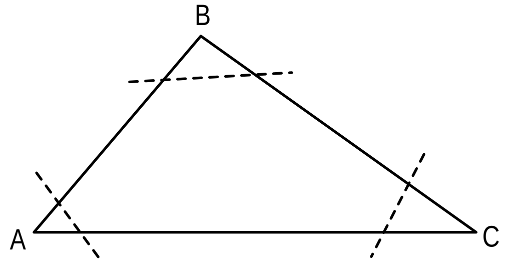
Focus now on the angles formed by parallel lines and a transversal, as in diagram b) above:

- Measure $\angle 1$ and $\angle 2$
- What do you notice?
- What type of angles are $\angle 1$ and $\angle 2$?
- Repeat activity with $\angle 3$ and $\angle 4$, $\angle 5$ and $\angle 7$, $\angle 6$ and $\angle 8$
- You should notice equality in these pairs of angles, and that they are corresponding.
- Repeat activity with $\angle 2$ and $\angle 5$, $\angle 4$ and $\angle 6$
- You likely found that the angles are equal and that they are alternate interior angles.
- Repeat activity with $\angle 3$ and $\angle 8$
- Repeat activity with $\angle 1$ and $\angle 7$
- You likely found that the angles are equal and that they are alternate exterior angles.
- Measure $\angle 2$ and $\angle 6$, then find their sum, i.e., $\angle 2 + \angle 6$
- Measure $\angle 4$ and $\angle 5$, and find their sum.
- Did you get 180?

These angles are called **allied** or co-interior angles, meaning both are inside angles. Thus $\angle 2$ and $\angle 6$ are allied, as are $\angle 4$ and $\angle 5$, and they add up to 180.

Unit Activity 7

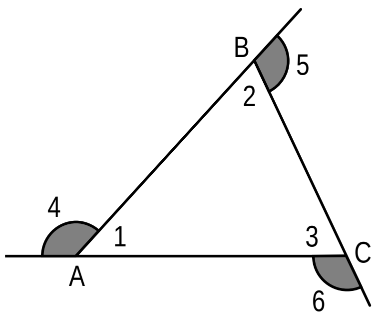
- Draw any triangle.
- Cut it out.
- Follow it by cutting the angles as shown below.



- Now draw a straight line.
- Place your cut-out angles side by side so the vertices coincide.
- What do you observe?
- What is the sum of ∠A, ∠B, and ∠C?
- Did you get a straight line when you placed the angles side by side?
- Repeat this activity with a different type of triangle.

Interior angles

We referred to ∠A, ∠B, and ∠C in the last figure as interior angles of triangle ABC. Therefore, interior angles of a triangle add up to 180 (supplementary). Remember this important result because you will make use of it later.



If you begin at A facing B (in the above diagram), “walk” all the way around the triangle, and end up in the same position and pointed in the same direction as you started, you will have moved through the shaded angles. These shaded angles are called the **exterior** angles of triangle ABC. What is the total amount of turning in degrees that you have done? Remember, a complete revolution is 360. Thus $\angle 4 + \angle 5 + \angle 6 = 360^\circ$.

Using the result that sum of interior angles in 180, we can show that
$\angle 4 + \angle 5 + \angle 6 = 360$ in the following manner:
$\angle 2 + \angle 5 = 180$ (Why?)
$\angle 4 + \angle 1 = 180$
$\angle 3 + \angle 6 = 180$

If you add $\angle 1$ through $\angle 6$, you get:
$\angle 2 + \angle 5 + \angle 4 + \angle 1 + \angle 3 + \angle 6 = 180 + 180 + 180$
$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 = 540$ (rearranged)
but $\angle 1 + \angle 2 + \angle 3 = ?$

Check $\angle 1$, $\angle 2$, and $\angle 3$ in the diagram.
They are interior angles of a triangle, and must add up to 180.
Replace $\angle 1 + \angle 2 + \angle 3$ with 180
Now you have $180 + \angle 4 + \angle 5 + \angle 6 = 540$
$\angle 4 + \angle 5 + \angle 6 = 540 - 180$ (subtracting 180)
$= 360$ (providing the result)

Note— $\angle 4$, $\angle 5$, and $\angle 6$ are the exterior angles.

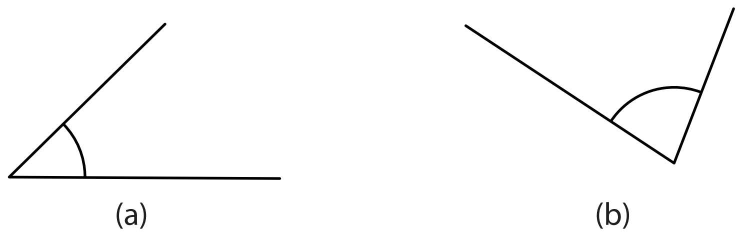
Summary

While the concept of the angle was developed without much reference to its everyday use, it is felt that this won’t jeopardise pupils’ ability to apply the concepts to everyday situations. Architecture and Mechanical and Civil Engineering are some of the specialized areas in which angles are applied. Angle measures also appear in commonplace occupations, such as house construction.

Although presented without an everyday context, angle concepts were developed using practical tools, including the protractor, and real shapes like triangles. Drawing and measuring of angles and exploring different types of angles should have enhanced your geometrical knowledge, and should have guided you towards similar activities for teaching the concepts in your upper primary classroom.

Unit Test

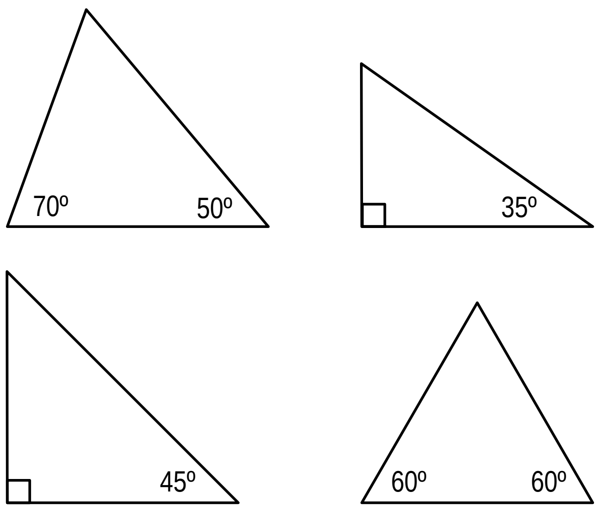
1. What are the sizes of the following angles? (measure)



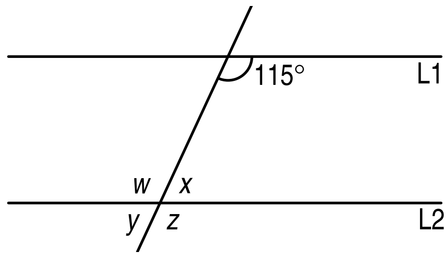
2. What is the size, in degrees, of a right angle?
3. What is:

   (a) an acute angle

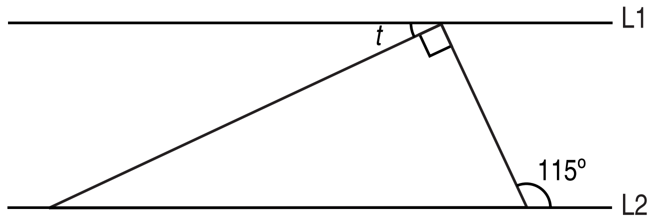
   (b) an obtuse angle?
4. What are supplementary angles?
5. Find the size of the missing angle in each of the following:



6. L_1 is parallel to L_2. Find the values of *w*, *x*, *y*, and *z*.



7. Find the value of *t* in the diagram below. L_1 is parallel to L_2.



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8. What is the size of an angle whose measure is twice that of its complement?
9. If two angles of a triangle are complementary, what is the size of the third angle?

Answers to Self Assessments

Self Assessment 1

1. $\angle$AXC
2. $\angle$3
3. $\angle$X can refer to other angles like $\angle$AXC

Self Assessment 2

1. 180
2. 90
3. 1/8
4. 1/10
5. 540
6. 1/360

Self Assessment 3

1. (a) $\angle$1 and $\angle$2, $\angle$2 and $\angle$3, $\angle$3 and $\angle$4; $\angle$AOB and $\angle$BOE, $\angle$AOC and $\angle$COE, $\angle$AOD and $\angle$DOE

   (b)$\angle$AOB and $\angle$BOE, $\angle$AOC and $\angle$COE, $\angle$AOD and $\angle$DOE
2. (a) $\angle$6 and $\angle$8, $\angle$5 and $\angle$7

   (b) $\angle$5 and $\angle$6, $\angle$7 and $\angle$8

   (c) $\angle$5, $\angle$6, $\angle$7, and $\angle$8
3. Their sum is 180.
4. (a) 50 (b) 140 (c) 78 (d) 170
5. Their sum is 90.
6. (a) 60 (b)20 (c) 80 (d) 40
7. 126
8. 40

Selected Answers to Unit 1 Test

5. Sizes of missing angles: 60°, 55°, 45°, 60°
6. $w = z = 115°$; $x = y = 65°$
7. Angle t plus the right angle must equal 115°. Therefore, $t = 25°$
8. $x + 0.5 = 90°$. Therefore, $1.5x = 90$ and $x = 60°$. (Did you find $x = 30°$? That is the angle whose measure is *half* its complement, not *twice* its complement.)
9. 90º

Unit 2: Properties of Shapes

Introduction

You may already be familiar with one of the desirable outcomes of successful teaching and learning—the ability to "say what you mean and mean what you say." (Cockcroft, 1982). Geometry uses special language to classify shapes into categories. Classification is an important intellectual process that helps us make sense of our experiences.

In this unit, you will work with your geometry **vocabulary**. You will also work on ways to help your pupils recognise geometrical features and properties, and use these to name and classify shapes. The angle concepts from Unit 1 will be used extensively.

This unit assumes that you know how to draw Venn diagrams, based on a written description in which a group of objects is a subset of another group.

Objectives

After working through this unit, you should be able to:

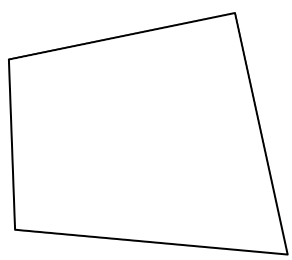
- give pupils an opportunity to explore the properties of various shapes
- demonstrate an understanding of basic topological concepts
- use the language needed to identify and discuss shapes
- explain the importance of classification as a process of making sense of shapes in real life

Quadrilaterals

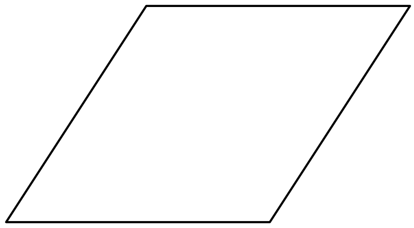
Consider the following two cases:
Set P contains parallelograms, Set Q contains quadrilaterals, Set R contains rectangles, and Set S contains squares. What Venn diagram illustrates the relationship between sets P, Q, R, and S?

Your Venn diagram should have P as a subset of Q, R as a subset of P, and S as a subset of R. That is, all squares are rectangles, all rectangles are parallelograms, and all parallelograms are quadrilaterals. You can put this in more mathematical terms as follows:

Quadrilateral—a figure with four sides. For example:



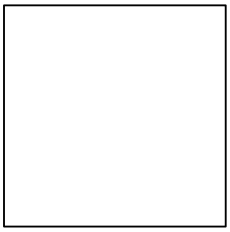
Parallelogram—a figure with four (quadrilateral) opposite sides that are equal and parallel (or opposite angles are equal). For example:



Rectangle—a figure with four sides. Opposite sides are equal and parallel (parallelogram); the size of the (opposite) angles is 90. For example:

In other words, a rectangle is a parallelogram in which, in addition to equal opposite sides, has four equal (90˚) angles.

Square—all sides are equal and parallel. For example:



A square is a rectangle which, in addition to having equal opposite sides, has four equal sides.

You also need to know the following figures since they are special types of quadrilaterals.

- A **rhombus** is a quadrilateral in which all four sides are equal in length. Of necessity, both pairs of opposite sides are parallel.
- A **trapezium** has one pair of opposite parallel sides.
- A **kite** has two pairs of adjacent sides equal in length, though not necessarily parallel.

Unit Activity 1

1. What is the nature of diagonals in each of the following: parallelogram, rectangle, square, rhombus, kite, and trapezium?
2. Five sets are defined as follows:

   H: Rhombus

   K: Kites

   P: Parallelograms

   S: Squares

   T: Trapeziums

   Draw a Venn diagram to show how the following relate to each other. Where there is an intersection between two sets, state the type of quadrilaterals in the intersection set.

   a) Rectangles and Rhombus
   b) Parallelograms and Trapeziums
   c) Kites and Rhombuses
   d) Kites, Rectangles, and Trapeziums
   e) Parallelograms, Rhombuses, and Squares

Reflection

When setting the plan of a rectangular building on the ground, is it adequate to ensure that the opposite sides measure the same? If not, what else should be measured? Why?

Need for classification of shapes

Now that we have looked at the properties of various shapes, let us refocus on classification. The ability to classify shapes gives students a greater understanding of shapes. Classification begins with recognising an attribute common to certain shapes (e.g., the four vertices in every quadrilateral) and then making the classification with a precise definition. This process of classifying and naming leads to greater confidence in handling shapes and a better awareness of the shapes in our environment.

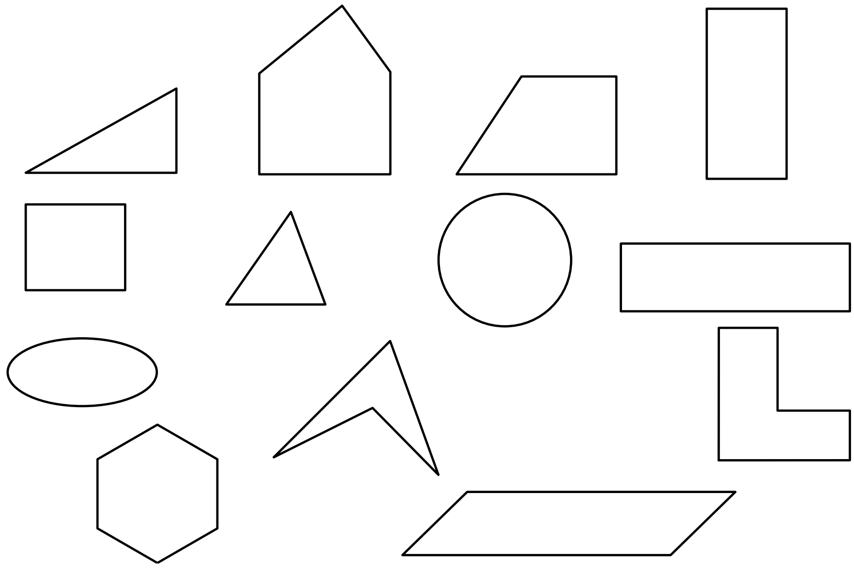
Basic classification of plain figures

Separate figures with:

1. straight edges from those with curved edges, e.g., circles and ellipses
2. the same number of sides, e.g., triangles and quadrilaterals
3. polygons that are regular and those that are not (regular polygons are those in which all sides and all angles are equal)

Practice Activity 1

Do this activity with your class. Cut a variety of shapes, such as the following, from manila paper or card board paper:



Display the shapes on a table. Decide on a set of shapes that can be classified into one set, say quadrilaterals. Let pupils guess what set you have in mind by suggesting a property to which you will say ‘yes’ or ‘no’. From the analysis of the properties you have said yes or no to, they should figure out the rule.

Let a pupil take the role of deciding on a set, then repeat the process.

Tessellation

Another way to classify shapes is to distinguish between those that **tessellate** and those that do not. A shape tessellates if it can be used to make a tiling pattern (**tessellation**) that fills a plane surface. That is, the shapes fit together without leaving any gaps and can be used over and over again to cover a flat surface. Figure 2.1 shows examples of tessellations.

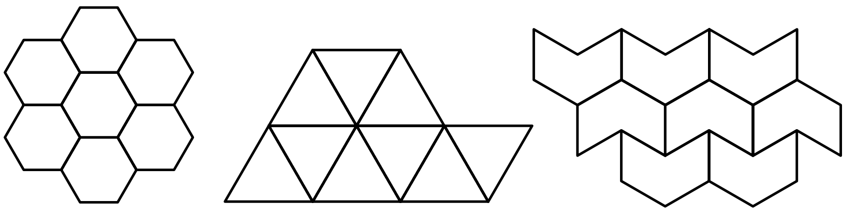


Figure 2.1. Tessellations

Shapes are allowed to assume different orientations as they tessellate, but they cannot change size. If both small and large squares are used in a tessellation, it is considered a tessellation employing two *different* shapes, just like a tessellation that uses squares and large octagons.

Unit Activity 2

1. Name three situations where tessellation occurs.
2. Do all triangles tessellate? Explain your answer.
3. Do all quadrilaterals tessellate? Explain your answer.
4. What are the conditions for a shape to tessellate?

Three-dimensional shapes

In classifying two-dimensional figures, we use features such as whether the shape has straight or curved sides, a certain number of sides, and an overall shape that is regular or irregular.

Reflection

For three-dimensional shapes (i.e., solids), what do you think could be the categories for classification? Give examples of each category.

Did you include the following in your classification?

- Those with curved surfaces (cylinders and spheres), those with a flat planar surface (prisms), and those with a mix of curved and flat surfaces (cones).
- Regular solids (cubes) and irregular solids (cuboids).

Terminology for describing three-dimensional shapes (solids) includes **faces**, **edges** (where two faces meet), and **vertices** (singular **vertex**) where edges meet. Your students will recognise the vertex from their study of angles.

Have you encountered a general name for all three-dimensional shapes with flat surfaces? Have you encountered any regular three dimensional shapes?

Solids with all sides flat are polyhedral (noun: **polyhedron**). Below are names of the five **regular polyhedra**, so called because all the faces, edges, and vertices are equivalent. Incidentally, there are no other regular polyhedra other than these five.

- Tetrahedron: four faces, each of which is a triangle
- Hexahedron: six faces, each of which is a square
- Octahedron: eight faces, each of which is a triangle
- Dodecahedron: twelve faces, each of which is a pentagon
- Icosahedron: twenty faces, each of which is a triangle

Some of the above shapes are illustrated in *Figure 2.2* below.

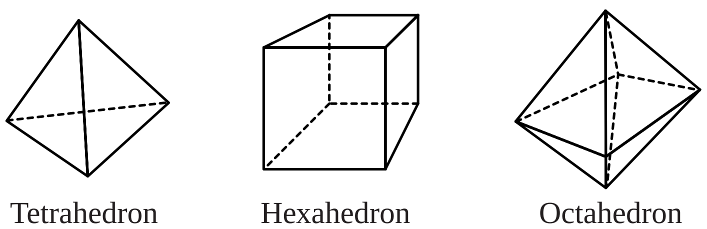


Figure 2.2: Some regular polyhedra

Self Assessment

Complete the following table:

Name of each face	No. of faces	No. of edges	No. of vertices
Tetrahedron			
Hexahedron			
Octahedron			
Dodecahedron			
Icosahedron			

Practice Activity 2

Make a collection of the following shapes and display them in your class:

- Regular Polyhedra (if not available, work with your pupils to make them)
- Prisms (shapes made up of identical polygons at opposite ends) as in *Figure 2.3* below

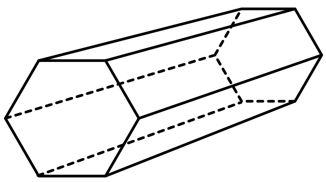


Figure 2.3: Prism

- Pyramids (polygon base and meeting at a point called apex) as in *Figure 2.4* below

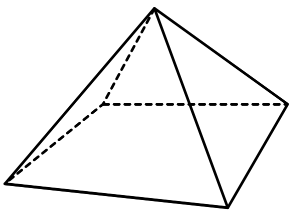


Figure 2.4: Pyramid

- Cones ('pyramids' with the circle as a base)

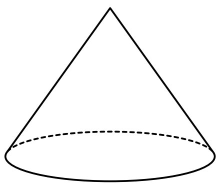


Figure 2.3: Cone

Practice Activity 3

This activity involves a tour of your town or village. Show students a set of shapes like squares, pentagons, right triangles, circles, etc. (You can add more). Make copies of these shapes so that each pupil has a copy. If this is not possible, draw each of them on a large sheet of manila paper that every pupil is able to see. Let the pupils explore their local community, searching for examples of the shapes illustrated.

Discuss with your pupils the shapes that appear most often and those that do not. Discuss the reasons for this situation.

Reflection

To classify shapes, is it enough to measure the sides? Angles should be confirmed to ensure they are 90˚. Diagonals should also be measured to confirm they make a rectangle.

Unit 3: Similar Figures

Introduction

You have studied shapes and their properties in Unit 2. A concept closely related to shape is **similarity**. This unit deals with similarity from two main perspectives: having the same geometrical shape but a different size, and being **topologically equivalent**. Similarity, from a geometric transformation point of view, is further developed in Unit 6 of this module.

Objectives

After working through this unit, you should be able to:

- demonstrate an understanding of geometric similarity
- demonstrate an understanding of basic topological concepts
- argue why, when, and how to teach the ideas of similarity
- identify the "real word" application of similarity

Congruence

Study the shapes in *Figure 3.1* below. What do the shapes have in common?

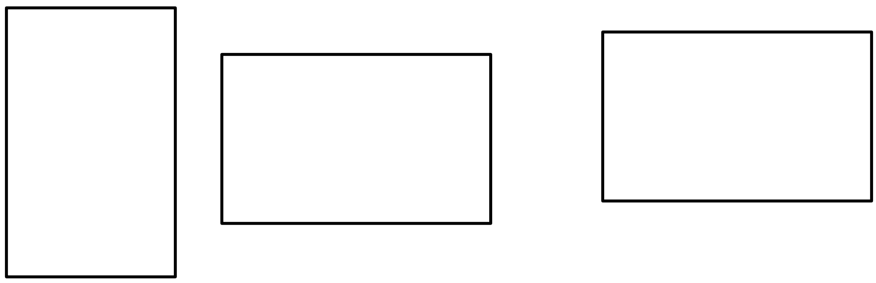


Figure 3.1: A set of rectangular shapes

The shapes in *Figure 3.1* are exactly the same shape *and* same size. They fit each other's outline exactly. Such shapes are called **congruent** shapes.

Practice Activity 1

Ask your pupils to bring to class shapes that are congruent. Display these on notice boards in your classroom. One can often find dry seed pods that are congruent, for example, even if the trees they came from are merely similar.

Similarity

Study the shapes in *Figure 3.2* below.

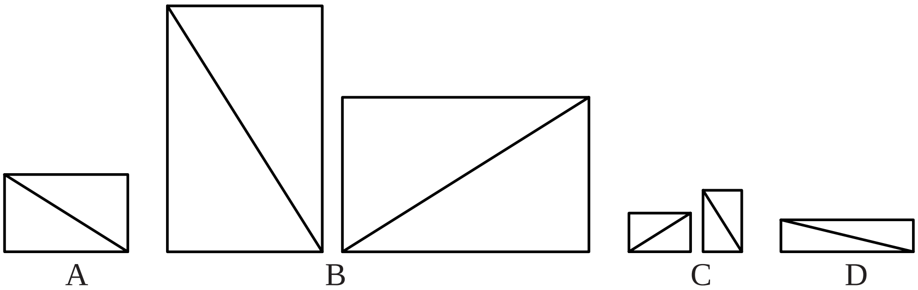


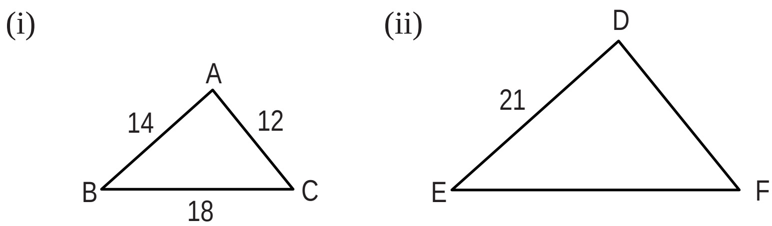
Figure 3.2: A set of rectangular shapes.

The shapes in *Figure 3.2* 'look alike' in many respects. They are all rectangles and they all have a diagonal drawn through them, but they are not all geometrically similar. Which shapes in *Figure 3.2* are geometrically similar?

Two figures are similar when one is an enlargement of the other. In *Figure 3.2*, shapes A, B, and C are similar. The nature of the enlargement is that:
All the lengths increase by the same factor.
All the angles remain the same size.

In *Figure 3.2* for example, **all** lengths in shape A have been multiplied by a scale factor of two to get shape B. **All** lengths in shape A have been multiplied by a scale factor of $^1/_2$ to get shape C. There is no single factor by which you can multiply the lengths in shape A to get shape D.

Reflection



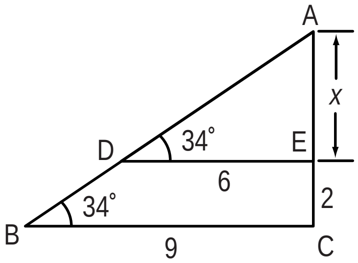
The diagram shows two triangles ABC and DEF that are similar. What are the lengths of DF and DE? Since in similar figures, lengths are multiplied by a scale factor, corresponding sides will be in the same ratio.

Side AB in triangle (i) corresponds to sides DE in triangle (ii).
Ratio of sides is 14:21 = 1:15
DF = 1.5 × AC = 1.5 × 18 = 27
EF = 1.5 × BC = 1.5 × 12 = 18

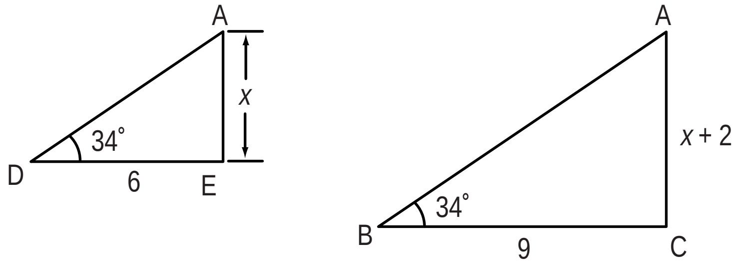
Practice Activity 2

With your class, discuss real life situations in which similarity either occurs or is applied. Now look at the following two examples of working with similar figures.

Example 1
Find *x* in the following figure:



First notice that *x* is a side in triangle ADE. Triangle ADE is similar to triangle ABC, since the two triangles have the same angles (draw the triangles separately to see better how the sides relate to each other).



Now, corresponding sides will be in the same ratio.

In triangle ADE $\qquad 6/8 = \dfrac{x}{X + 3}$

In triangle ABC

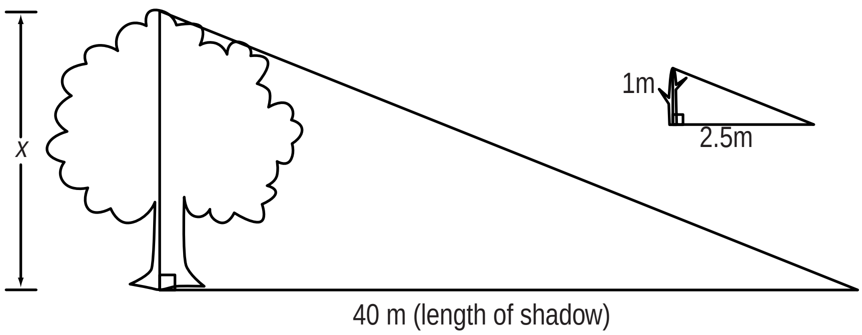
We have $8x = 6x + 18$

$2x = 18$

$x = 9$

Example 2
A tree casts a shadow of 40 m, while at the same time a stick with a height of 1 m casts a shadow of 2.5 m. How tall is the tree?
Hint—refer to problem solving strategies in Unit 8 of Module 1.

Sketch a diagram to illustrate the problem.



The two triangles are similar because the angles are the same. Both are at right angles to the ground, and the angles made by the sun's rays are the same. When two pairs of angles in two triangles are the same, the third pair of angles will also be the same.

$\frac{x}{1} = \frac{4.0}{2.5}$ (estimate the answer before dividing)

$x = \frac{4.0}{2.5}$

$x = 16$

The tree is 16 m tall.

Unit Activity 1

When two geometric figures are **equiangular** (have the same angles), does it mean they are similar? If not, give a example of equiangular figures that are not similar.
When two geometric figures have corresponding sides in the same ratio, does it mean they are similar? If not, give an example of figures where corresponding sides have the same ratio, but are not similar.

In the first part of the activity, rectangles illustrate how two figures can be equiangular but not similar, as is the case in *Figure 3.3*.

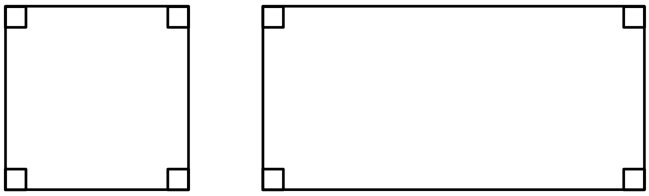


Figure 3.3: Equiangular figures that are not similar.

For the second part of the activity, rhombuses illustrate how two figures can have sides in the same ratio yet not be similar, as shown in *Figure 3.4*.

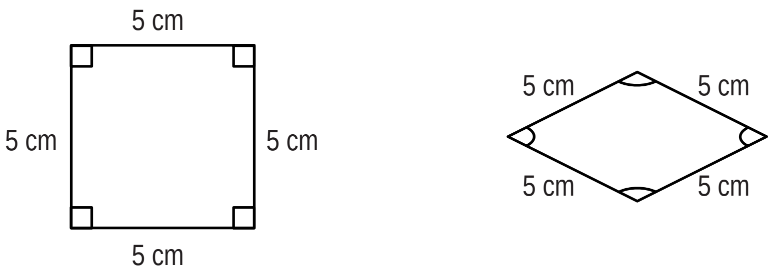


Figure 3.4: Shapes with equal sides that are not geometrically similar

Figures 3.3 and *3.4* illustrate that shapes must have two properties to be similar:

- Equiangular
- Corresponding sides in the same ratio

For triangles, when one property holds, the second property also holds.
To show that triangles are similiar, it is enough to establish that one of the properties holds. For shapes in general, this is not the case.

Equivalent Shapes

When we study changes in shapes (**transformations** are further developed in Unit 6), we focus on measurable changes such as lengths of sides or sizes of angles. With similar polygons, for example, lengths change but the shape (the number of vertices) remains the same. However, certain shapes have geometric features that can only be described with numbers. For example, a circle has no geometric property that does not use a number to describe it.

Study *Figure 3.5* below. The property that there is no starting point (or end point) remains true as the circle is distorted in various forms up to a triangle.

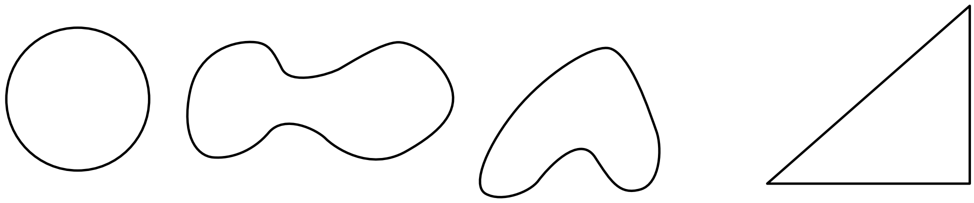


Figure 3.5: Different forms of the circle preserving the "connectedness" property.

Do you feel we are deviating from mathematics? Not at all! Keep working through the unit.

Topology

Figure 3.6 below shows four shapes that are "topologically" the same.

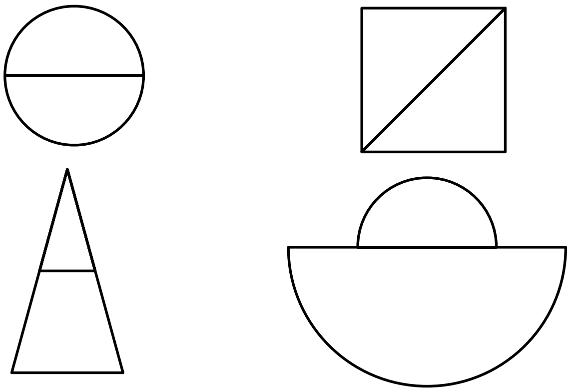


Figure 3.6: Four "same" shapes

What is the "same" about the shapes in *Figure 3.6*?

You will notice that the shapes in *Figure 3.6* are all simply networks of three paths from one junction to a second and two regions.

What we have been discussing so far is an aspect of geometry called **topology**. "Topology is often described as the study of properties of curves and surfaces that are maintained as the curves and surfaces are stretched or distorted" (O'Daffer and Clemens, 1992). Another way to look at topology is to say that it looks at properties of connectedness and continuity of lines and surfaces as shapes undergo change. We will look at further examples.

Study *Figure 3.7* below. The shapes show a pipe that undergoes distortion until it is in the shape of a washer (a piece of metal used in tightening nuts and bolts). The changes are called topological transformations. From a topological point of view, all the shapes in *Figure 3.7* are equivalent.

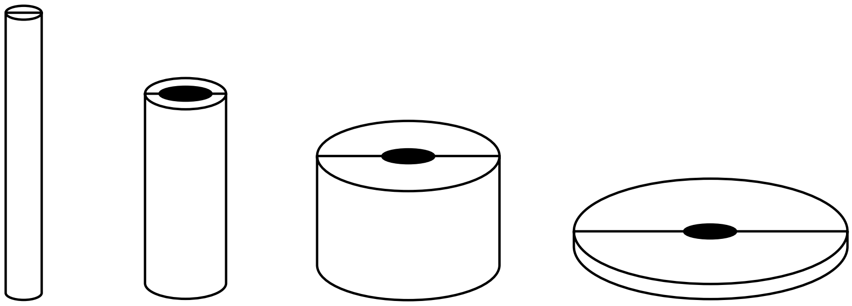


Figure 3.7: Topologically equivalent shapes

Figures 3.8 shows a famous example of topologically equivalent things—a doughnut and a coffee cup.

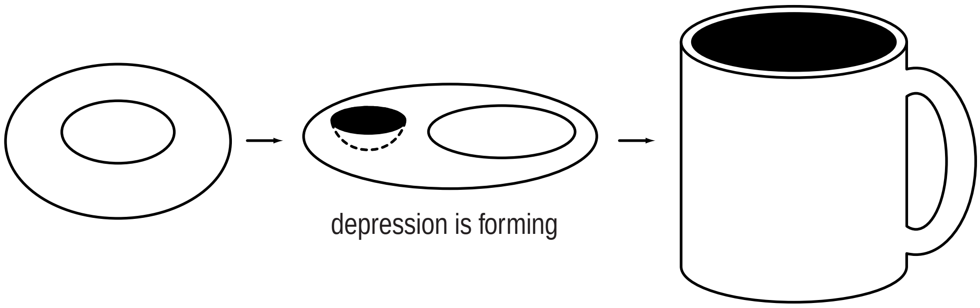


Figure 3.8: Topologically equivalent shapes.

Reflection

Do you think topology does or does not deal with mathematical concepts? Why?

In case you were thinking topology is about crazy ideas, it will be useful to know about Jean Piaget, the great genetic **epistimologist** (a person who studies how knowledge develops). In an article on “How Children Form Mathematical Concepts” published in 1953, Piaget said that for the first three years, a child cannot tell the difference between squares and triangles but can distinguish between open and closed figures (O’Daffer and Clemens, 1992).

Another way to demonstrate the importance of topological thinking is in giving directions for a route. For example, you may give directions to someone as follows: “Get onto the road, turn right, then third on the left and go on until you come to a roundabout, take the second exit…” These instructions do not mention distance, amount of turn (angles) or straightness of sides. Nonetheless, they are vital information in terms of reaching the destination.

When you focus on properties that remain the same when figures undergo change, from a topological point of view, you look at ideas such as **between,**

next, meet, inside, and **outside.** That is, if one junction lies **between** two other junctions in a network of paths, that junction still lies between the two junctions after a transformation—as long as the change does not involve cutting, tearing, cementing, or welding.

Have a look at one more example of the importance of topological thinking. Children will recognise all the letters in Figure 3.9 below as being the same—the letter “b”. But geometrically, in which way are they the same or similar? Topologically, they are the same!

Figure 3.9: A set of topologically equivalent shapes

Practice Activity 3

Take your class on a tour of your school premises with the view to deciding how to give directions for walking from one end of the school to another. Let pupils work in groups of convenient size to give directions in terms of topological properties.

Practice Activity 4

The aim of this activity is to provide further experience with similar figures and topological transformation. You will need to recap these ideas before pupils embark on the activity.

Make enough large charts illustrating the shapes in *Figure 3.10* for each groups of pupils. Within their groups, let them identify the odd shape out and give explanations for their decisions. Let each group make a presentation to the class.

Let the pupils make their own sets of shapes and challenge each other to find, with accompanying reasons, the odd one out.

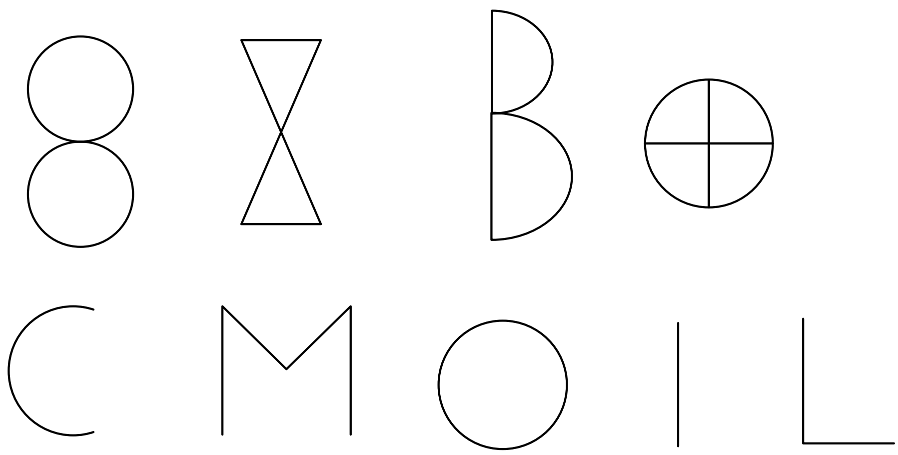


Figure 3.10: Shapes with an odd one out

Unit 4: Constructions – Two-Dimensional Shapes

Introduction

In the previous units on shapes, you explored the properties of two-dimensional and three-dimensional shapes. Now we are going to look at **constructions** that involve two- and three-dimensional shapes. You will extend your knowledge of the properties of two-dimensional shapes, using the investigative approach that you can also use with your pupils. As such, this unit contains more practical work and less text.

Objectives

After working through this unit you should be able to:

- discover polygon relationships
- explore patterns of polygons
- develop a spirit of inquiry and an appreciation of geometry in your pupils

Constructions: Two-dimensional shapes

To **construct** means to draw accurate pictures. At the primary level, we define construction as building, drawing, and making accurate two- and three-dimensional shapes. Note that this definition does not resemble the traditional meaning of “construct” in Euclidean geometry, for which no aids are allowed except the compass and straightedge. In this age of good rulers and protractors, these restrictions are of little value and are being phased out of most geometry curricula.

At the lower primary level, pupils are engaged in activities that mostly involve sorting, identifying, and describing various shapes. Of course they build, make, draw, put together, and take apart shapes—but without using properties and without learning the vocabulary this module uses.

Besides activities involving geometric properties, measurement activities can help pupils discover more relationships among shapes. For example, measuring and comparing area, perimeter, surface area, volume, angles, radii, and circumference of various shapes will reveal interesting relationships.

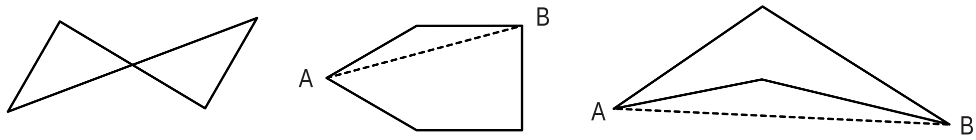
Throughout the units on constructions, you will be involved in the investigations, and you should work through them with an “open mind”, as both a student and a teacher. In the words of O’Daffer and Clemens (1992: xii):

> “Enter the experience of the text with an open mind and with the curiosity of a small child.”

Where feasible, have your students work in groups where they can exchange ideas with others. Your main task is to develop a spirit of inquiry and a deep appreciation for geometry in your pupils. Tiles, geoboards, and dot and grid paper continue to be good construction materials for two-dimensional shapes (see the Appendix of Module 3).

Discovering Polygon Relationships

The figures below illustrate types of polygons.

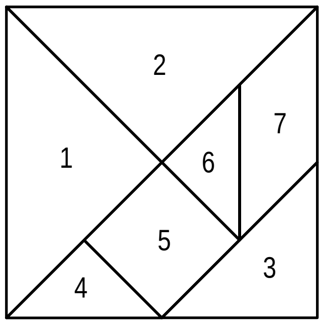


Non-simple polygons have *intersecting sides***.** In a **convex** polygon, any segment AB joining any two points of the polygon lies inside the polygon. In a **concave** polygon, there is at least one segment AB that does not lie inside the polygon.

Geometrical puzzles

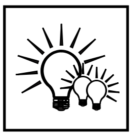
Teaching Geometry with Tangrams

Tangrams are a seven-piece puzzle that originated in China. They help pupils develop geometric concepts and reinforce spatial perceptions.



Thirteen convex polygons can be formed, each using all seven pieces of the tangram. One of them is this square.

- How many of the other twelve polygons can you find?
- Draw outlines of the polygons you made.



Practice Activity 1

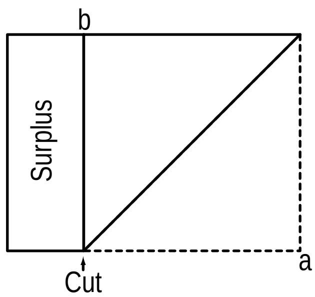
Constructing Tangrams by Paper Folding

Materials: sheet of paper (11 cm by 14 cm), pencil, scissors

- Demonstrate how to make a tangram.
- Have pupils follow the instructions step by step.
- After each step, discuss with pupils the shapes they have made—names, properties, size, relationships to other shapes, and differences.

i) Make a square by folding pieces of paper. Bring point A to B and crease the paper. Cut along the crease (*Figure 4.3*).

Figure 4.3



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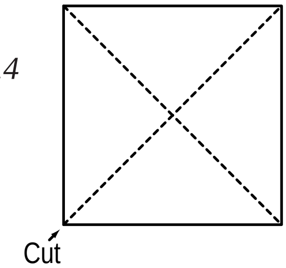
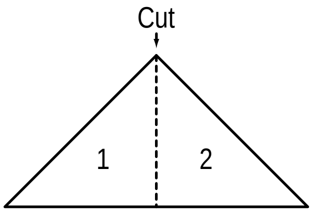
ii) Now fold the paper again, along the other diagonal, and cut along one diagonal to form two large triangles (*Figure 4.4*).

Figure 4.4



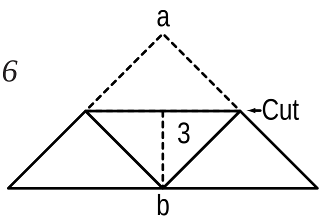
iii) Cut one large triangle along the centre line to form two smaller triangles (*Figure 4.5*). Label them 1 and 2.

Figure 4.5



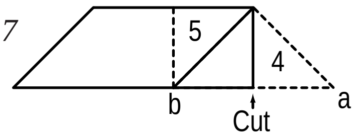
iv) Take the remaining large triangle, bring the vertex of the right angle to the midpoint of the hypotenuse. Crease and cut along the crease (*Figure 4.6*).

Figure 4.6



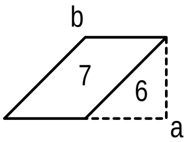
v) Fold corner "a" of a trapezium to the midpoint of baseline "b" (*Figure 4.7*). Crease, cut along crease, and label this triangle 4. Cut off the remaining square and label it 5.

Figure 4.7



vi) Fold the remaining pieces to obtain another right angle triangle and a parallelogram (*Figure 4.8*). Bring point "a" to "b", then cut and label the triangle 6 and the parallelogram 7. Now, each pupil has a set of tangram pieces (*Figure 4.9*).

Figure 4.8



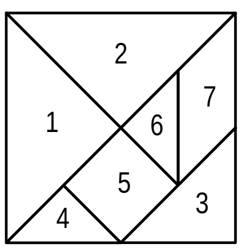


Figure 4.9

Practice Activity 2

Materials: Set of tangrams for each pupil or pair of pupils.

Have pupils measure sides and angles of the different polygons in a tangram.

Practice Activity 3

Materials: set of tangrams for each pupil.

In the introduction, review the names and properties of the polygons. Let pupils explain how the polygons are similar and different. First, have pupils make the stated shapes using a different number of pieces.

Have pupils make different polygons using all the pieces. Check to see they have made all the polygons.

Let them draw outlines of the shapes they have made. These outlines will be used later.

Pentominoes

A pentomino is a two-dimensional shape made by grouping five congruent, square shapes together. Each square can only touch another on a complete side. Two pentominoes are different if they are not congruent. That means they cannot be matched by flipping or rotating the shapes. Activities with pentominoes enhance perception.

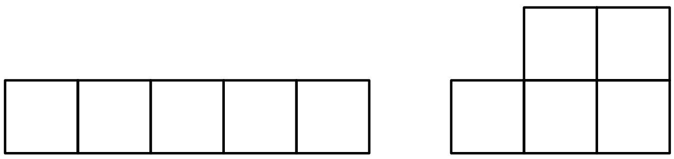


Figure 4.10: Pentominoes

We can use pentominoes to develop the concept of **reflectional symmetry** and **rotational symmetry**. A figure has reflectional symmetry. If a figure can be rotated so it coincides with its tracing, then the figure has a rotational symmetry. The angle the tracing turns through to coincide with the figure is called the angle of rotation. The point that is held fixed is the centre of rotational symmetry. Reflections and rotations are dealt with more in transformations in Unit 6.

Unit Activity 1

Using five congruent squares:

1. Find all the 12 pentominoes and show them on a dot paper.
2. Which pentominoes have 90˚ rotational symmetry about some point? 180˚ rotational symmetry?
3. Which pentominoes have exactly one line of reflectional symmetry?
4. Which pentominoes have more than one line of reflection?

Optional Activity

Use five 45-degree right angles (halves of squares) to make shapes. Sides that touch must be of the same length. How many different shapes have you made?

Practice Activity 4

1. Making Pentominoes

   Materials: graph paper, pencils, rulers, and scissors

   - In cooperative learning groups, have pupils find different pentominoes. Do not tell them how many different shapes there are. Let them cut out the shapes.
   - Ask your pupils if the shapes they have come up with are really different, and if all possible shapes have been found.
   - Have the pupils tell if all the pentominoes have the same perimeter and area, and if they have lines of symmetry.
   - Can they use some of the pentominoes to make a cube without a lid?

2. Pentominoes Puzzle

   Have pupils use all twelve pentominoes to construct a 6 by 10 rectangle. Have them draw the rectangle and show the tessellation of the pentominoes.

3. Explore a Rigid Polygon

   A model of a polygon can be made using homemade cardboard strips. The length of strip is the distance between the holes. Brass fasteners are used for joining two strips. For example, a rectangle can be made by fastening two pairs of strips, but the pairs are not equal in length. Some models of polygons can be “pushed out of shape” (*Figure 4.11*) if pressure is applied to vertex. The polygons whose models cannot be pushed out of shape even when pressure is applied to their vertices are termed as rigid polygons.

Rigid Shapes

Reflection

Why are most roofs triangular?

Unit Activity 2

Rigid polygons retain the same shape even if pressure is applied.

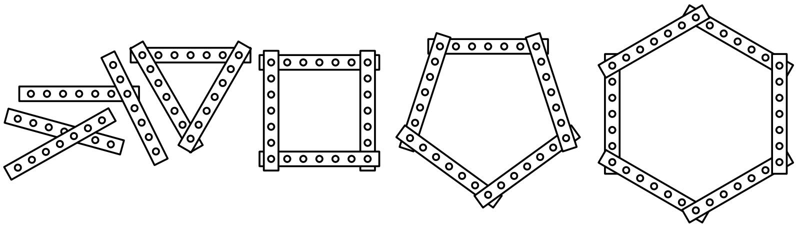


Figure 4.11

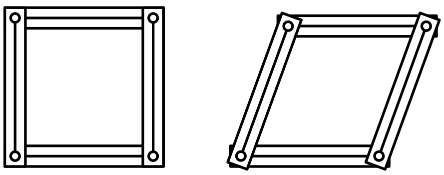
- Use cardboard strips with punched holes and brass fasteners (*Figure 4.11*) to explore the following questions.
- Which of these figures is rigid?
- Did you discover that a triangle is the only rigid polygon?
- For each figure that is not rigid, what is the fewest number of strips required to make it rigid?
- Formulate a general rule for the number required to make an n-gon rigid.

Practice Activity 5

Materials: cardboard strips, brass fasteners.

- Demonstrate how to make a polygon using cardboard strips.
- Have pupils examine what happens when a shape is pushed out of shape. Does the perimeter change?

  Why? When is the area the greatest? How do the sizes of the angles change?



- Ask pupils which shape is rigid (triangle). Discuss with them why most roofs are constructed in the shape of a triangle?

Regular Polygons

A regular polygon is a simple polygon with all sides equal and interior angles equal. A regular polygon with n-sides is called a regular n-gon.

Unit Activity 3

1. Making regular polygons

   Materials: circle geoboard, rubber bands

   - On a circle geoboard (or dot paper) with thirty-six nails (*Figure 4.13*), how many different polygons with sides of equal length can you construct?

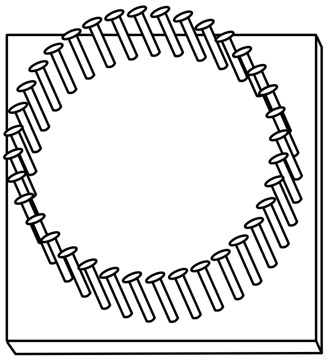
   

   Figure 4.13

   - Can you find any relationship between the number of nails (36) and the number of sides of the polygons you constructed?
   - Find polygons whose sides are equal but interior angles are not equal. Why are these polygons not regular polygons?
   - Find polygons whose interior angles are equal but sides are not equal. Are these regular polygons? Explain.
   - Find the number of degrees in each interior angle of a regular n-gon.

2. Regular Pentagon

   Materials: equal-length cardboard strips, brass fasteners

   - Use five equal cardboard strips to make a pentagon.
   - Make a variety of different pentagons.
   - Measure the interior angles of the pentagon to find out if they are equal.
   - Make different n-gons of the same perimeter.
   - When does an n-gon have the greatest area?

Practice Activity 6

1. Making regular polygons

   Materials: circle geoboard, rubber bands, cardboard strips, brass fasteners.

   - In groups, let pupils find regular polygons.
   - Have pupils find polygons whose sides are equal but are not regular polygons, and those whose interior angles are equal but they are not regular polygons.

Circle Designs

Star polygon

The idea of a star is not a new one. Now and then we see stars in the sky. A star takes the form of a non-simple polygon. For example, a star polygon can be constructed from five equally spaced points on a circle. This star can be made by joining every second point with a segment, for instance, V_1, V_3, V_5, V_2, V_4, V_1. This star is called a regular star polygon because all the segments are equal and the vertex angles are all equal. This regular star is denoted by $\left\{\begin{matrix}5\\2\end{matrix}\right\}$ The number of points is represented by 5 and the pattern of joining is represented by 2. (*Figure 4.14*). A polygon constructed from five points, with consecutive points (V_1, V_2, V_3, V_4, V_5) joined, is not a star polygon.

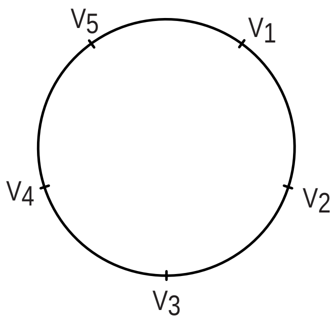


Figure 4.13

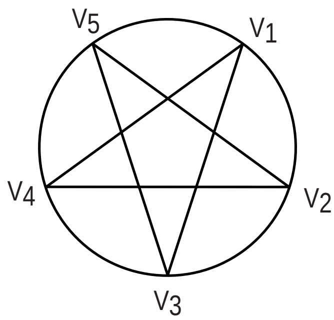


Figure 4.14

Reflection

If the star polygon in *Figure 4.14* has five sides, then what is a side of a star polygon?

Unit Activity 5

Draw a regular star polygon denoted by $\left\{\begin{matrix}5\\3\end{matrix}\right\}$. Is this star different from a $\left\{\begin{matrix}5\\2\end{matrix}\right\}$? How do you know?

Draw $\left\{\begin{matrix}8\\2\end{matrix}\right\}$ and $\left\{\begin{matrix}8\\3\end{matrix}\right\}$. Are these star polygons? How do you know?

How many sides has each polygon?

What is the smallest star polygon?

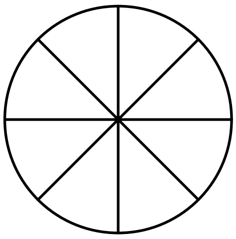
Practice Activity 7

1. Circle Designs

   Materials: sheets of paper, scissors

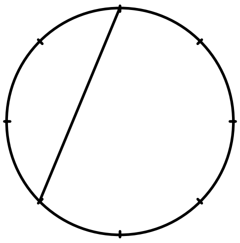
   - Have each pupil trace a circle with a radius of 3 cm or 4 cm. Then have the pupil cut out the circular shape and fold it into eighths, marking a dot where each fold line intersects the circle (*Figure 4.15*).

Figure 4.15



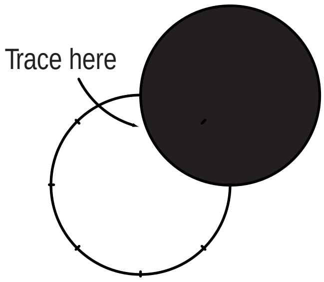
   - Each child traces the original circle several more times and uses this marked figure to help mark dots on each newly traced circle where the fold lines intersect the circle. Now, no cutting out figures.
   - Have pupils keep the traced circles (templates) and folded marked circles for future use.
   - Let pupils use a straight edge to join any two points (*Figure 4.16*). Have pupils examine how many different designs they have come up with.

Figure 4.16



2. Pupils examine how many different designs they have come up with.
   - Have pupils draw different polygons by joining the dots.
   - Discuss with the pupils the properties of the polygons they have made.
   - Have pupils use their imaginations to come up with their own designs.
   - Discuss the shapes they have come up with. Any star polygons?
3. Variation on drawing with a straight edge.
   - Instead of drawing straight lines to join the dots, let your pupils draw curves to join the dots. They can use the edge of their template (*Figure 4.16*) to do this.

Figure 4.17



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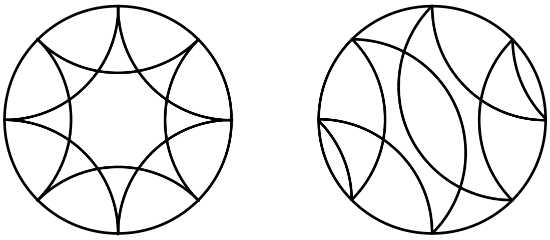
- Let pupils discover different designs made by tracing the curve of the template (*Figure 4.18*).
- Discuss with pupils the differences, similarities, and lines of symmetry of shapes they have made.

Figure 4.18



Circuzzles

Have ever heard of a circuzzle? A circuzzle is a puzzle made from quarters of a circle. Two pieces are placed so that the straight sides are together.

Like this

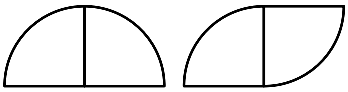


Figure 4.19a

Not this

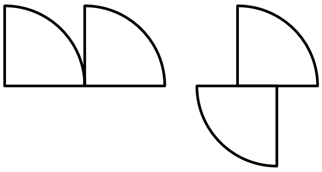
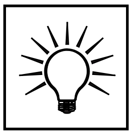


Figure 4.19b



Unit Activity 5

1. How many quarters make the following circuzzles?

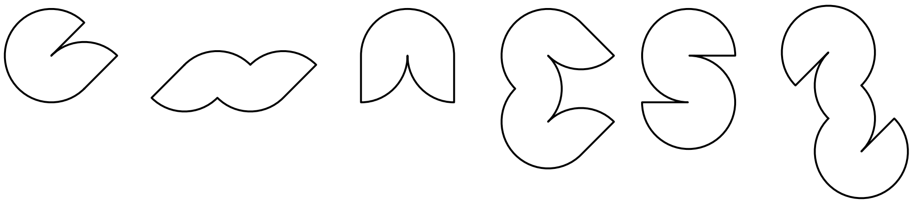


Figure 4.20

2. Can you continue the following pattern? What is the pattern?

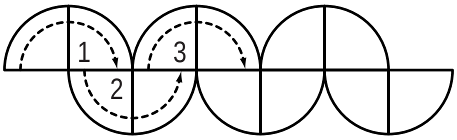


Figure 4.21

Practice Activity 8

Materials: manila paper, scissors

- Have pupils draw equal-sized circles onto manila paper, divide them into quarters, and cut them out.
- Show them a circuzzle and ask if they can use their pieces to build one.
- Now have pupils make different circuzzles from the same number of pieces.
- Check with them if the shapes they make are really different. Let them compare the perimeter and area. How many quarters make each shape?
- Create a pattern and let them continue with it.
- Have them design their own patterns. Ask them to describe the designs they have made.

Tessellation

A **tessellation** is a repeated pattern that completely covers a plane, leaving no “holes” and no “overlaps” (*Figure 4.22*). Tessellating leaves “ragged” edges, showing that the pattern can be continued.

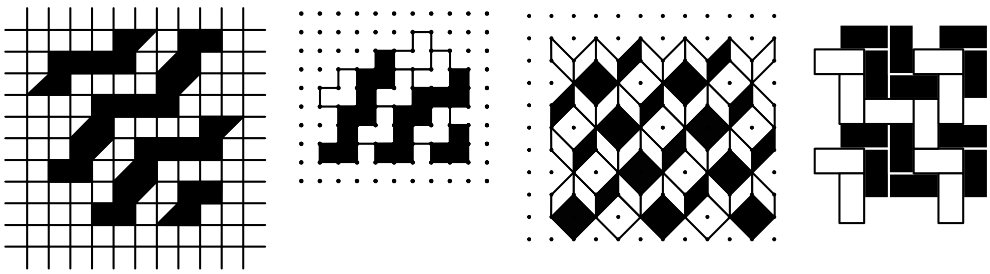


Figure 4.22

Unit Activity 6

Squares tessellate the plane.
Use cardboard or plastic polygons to search for other quadrilaterals, triangles, and regular polygons that tessellate the plane. Copy those polygons (*Figure 4.23*) that tessellate the plane and show the tessellations.

- Can you state which polygons will tessellate a plane?

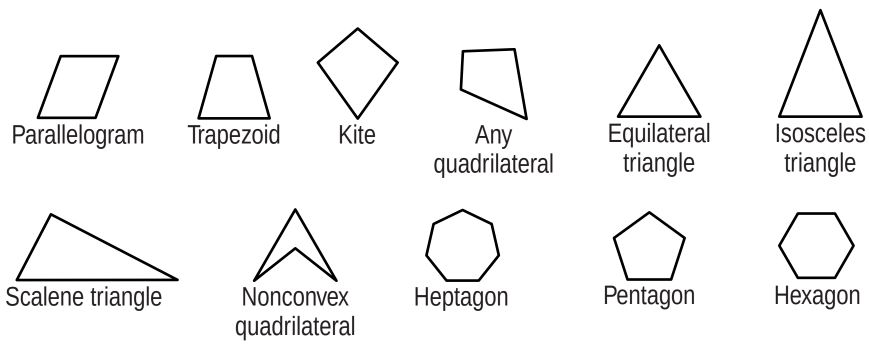


Figure 4.23

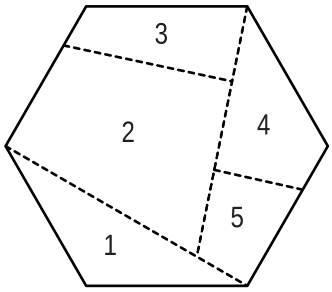
Practice Activity 9

Materials: dot or line grids, pencil, paper.

- Have pupils plan their tessellations with pencil and paper.
- Let them transfer their patterns onto dot or line grids and tessellate.
- Have pupils describe the shapes they used and how many shapes surround each vertex. What angle is formed by all the shapes at each vertex?

Self Assessment

1. Find the number of degrees in the vertex angle of a regular 20-gon.
2. Copy the regular hexagon as shown below. Cut it apart along dotted lines. Make a square using all the pieces.



3. How many shapes can be made with three quarters?
4. (a) Does $\left\{ \begin{matrix} 9 \\ 2 \end{matrix} \right\}$ represent a star polygon?

   (b) How do you know?

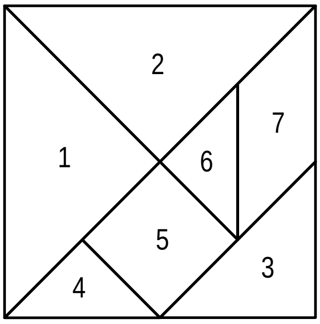
   (c) How many sides does this star polygon have?

Summary

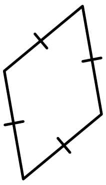
The investigative approach has been used in this unit to enhance the student-centred approach. The models used will enable a learner to visualise shapes. After all, it is more exciting to discover the knowledge than to be told the facts.

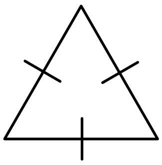
Unit 4 Test: Constructions

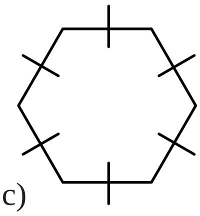
1. A square is a rectangle. True or false?
2. After having pupils make tangram pieces by folding paper, what three questions can you ask them?

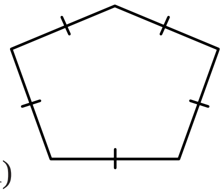


3. If you make quadrilaterals of the same perimeter, what quadrilateral can give you the greatest area? What conclusion can you make?
4. Which of the following shapes are not regular polygons. Explain.


a)

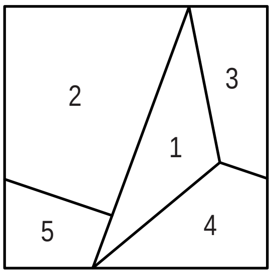

b)


c)

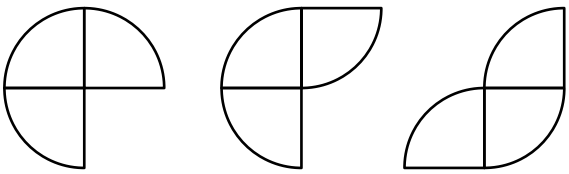

d)

Suggested answers for Self-Assessment

1. $\frac{180^\circ(n-2)}{n} = \frac{180^\circ(20-2)}{20} = 162^\circ$

2. 

3. Three shapes can be made with three quarters.



4. a) Yes.
   b) The polygon is concave.
   c) Nine.

Unit 5: Constructions – Three-Dimensional Shapes

Introduction

In Unit 4, you explored constructions of two-dimensional shapes. In this unit, you will extend the concepts of two-dimensional shapes to constructing three-dimensional shapes. The approach is similar to Unit 4, with lots of activities that you can in turn use with your class. You are encouraged to be creative and enjoy going through the practical work.

Objectives

After working through this unit, you should be able to:

- demonstrate how to make models and nets of three-dimensional shapes
- plan a field trip or geometry walk
- develop your pupils' creative abilities

Polyhedra

A three-dimensional shape with flat, polygonal faces is called a **polyhedron**. In Greek, *poly* and *hedron* mean "many" and "face", respectively. A polyhedron has faces, edges, and vertices (*Figure 5.1*). In the set of polyhedra, we have prisms, pyramids, and regular polyhedra, as well as many mixed forms.

A **prism** is formed by two congruent parallel polygonal faces (bases) and is bounded by parallelograms. Prisms are named after their bases. For example, if the bases are triangles, the prism is called a triangular prism.

A **pyramid** has a polygonal base and an apex. It is also named after its base.

Regular polyhedra have congruent regular polygonal faces. As mentioned in Unit 2, there are only five regular polyhedra.

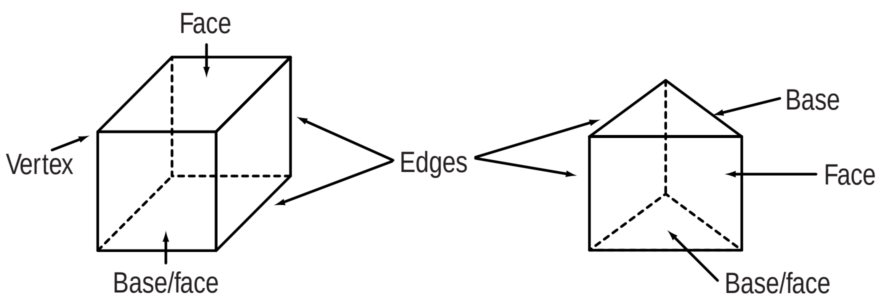


Figure 5.1

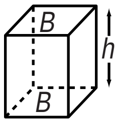
Curved shapes in three dimensions

Cylinders and **cones** are curved shapes that are related to prisms and cones respectively. Spheres will not be covered in this unit.

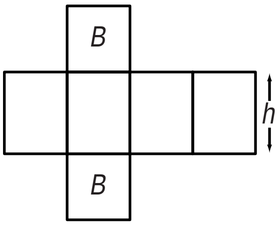
Nets for three-dimensional shapes

A two-dimensional pattern that can be cut out and fastened together to form a three-dimensional polyhedron is called a **net** for the polyhedron. For convenience, in primary classrooms, a net can include tabs around the edges for gluing the polyhedron together. See the square pyramid net below.

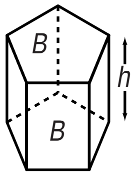
Some nets for prisms



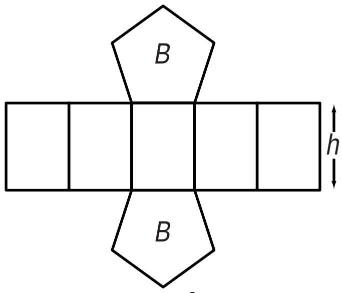
Cuboid



Net of a Cuboid

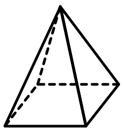


Pentagonal Prism

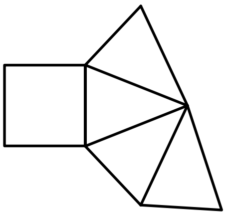


Net of a Pentagonal Prism

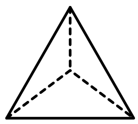
Some nets for pyramids



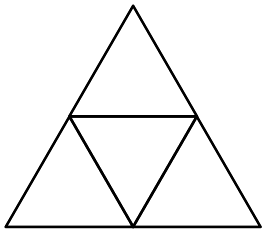
Square Pyramid



Net of a Square Pyramid



Triangular Pyramid



Net of a Triangular Pyramid

Unit Activity 1

1. Fill out the table below

Polyhedron	Number of edges (E)	Number of vertices (V)	Number of faces (F)
Square Pyramid			
Triangular Prism			
Octahedron			

2. How many faces, edges, and vertices does a pyramid with an n-gon base have?
3. How many faces, edges, and vertices does a prism with an n-gon base have?
4. For the polyhedra in exercises 1, 2, and 3, compare F, V, and E. Write an equation that shows the relationship between these variables.
5. Find the number of faces, edges, and vertices of a cone.

The relationship you discovered in question 4, F + V = E + 2 is called **Euler's formula**. Is this formula satisfied for the cones and cylinder? If not, why?

Unit Activity 2

Make string models for exploring cylinders, cones, prisms, and pyramids. See guidelines in *Figure 5.6*. For hollow beads, you can improvise with metal washers.

Materials: soft cotton string or embroidery yarn, metal washer about $^{3}/_{4}$ inch in diameter (about 15 per model), poster board, and a hole punch.

Directions:

- Cut two identical base pieces from poster board. There are no restrictions on the shape. The size should be roughly 9 to 15 cm across.
- Place the two bases together and punch an **even number** of holes around the edges, punching both pieces at the same time so the holes line up and are about 1 cm apart.
- Cut pieces of string about 50 cm long. You will need half as many pieces as holes.
- Run each piece of string through a washer and thread the two ends up through two adjacent holes of both bases. Pull all the ends together directly above the base and tie a knot.

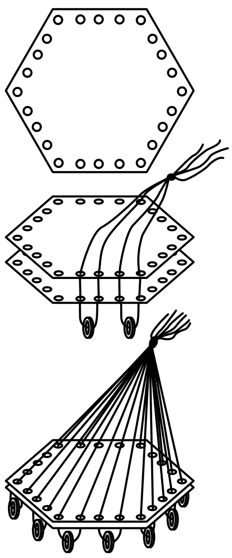


Figure 5.6: A model for cylinder and cones

By varying the position of the vertex you can come up with a family of cones or pyramids with the same base. By manipulating bases you can come up with a variety of models. For instance, moving one base up to the knot and adjusting the other base, can produce a family of cylinders or prisms. Tilting and/or twisting can produce non-cylindrical shapes. For cylinders and prisms, what is the relationship between the string regardless of the position of the bases?

Reflection

Why do most containers for tinned goods take a cylindrical form?

Unit Activity 3

1. Make models of a cylinder and a cuboid whose bases have the same perimeter and height.
   - Do the models have the same volume? If not, what is the relationship?
   - Do the bases of the models have the same area?
   - Do the models use the same amount of materials?
   - Why do most containers have a cylindrical form?
2. Since each face of a cube contains four edges and three edges that meet at each vertex, we can use the notation suggested by Ludwig Schläfli (1814-1895) to describe a polyhedron. The cube is described by (4, 3). Complete the table below to describe other regular polyhedra.

Regular polyhedra	Schläfli
Cube	(4, 3)
Tetrahedron	
Octahedron	
Dodecahedron	
Icosahedron	

   Does Schläfli's notation give us a clue to the number of edges in a regular polyhedron? If we extend this notation to describe prisms, does the notation give us a clue to the number of edges in a prism? How?
3. Make an appropriate sized net and construct a polyhedron with more than six faces. Does Euler's formula work for your polyhedron?

Practice activity 1

1. Materials: construction paper, pictures of polyhedra nets.
   - Show pupils nets of three-dimensional shapes. Let them determine the shapes that can be made from the nets.
   - Have them work in groups to construct nets and make shapes from the nets.
   - Have pupils describe the process they used.
2. Materials: strings, cardboards, washers.
   - Demonstrate how to make a string model.
   - Have pupils make string models.
   - Discuss with the pupils the behaviour of the strings when bases take new positions. Do the strings remain parallel or not? How do they know? Have pupils examine how the angles change relative to each other as the vertex is moved.

Semi-regular Polyhedra

A semi-regular polyhedron is a polyhedron constructed from different regular polygons but with some arrangement of polygons at each vertex.

Several of the semi-regular polyhedra can be produced from the regular polyhedra by "slicing off the vertices." This process is known as **truncating** (*Figure 5.8*). Truncated polyhedra acquire their names from the original shape. For instance a cube whose vertices are sliced so that the square faces are now octagonal is a truncated cube.

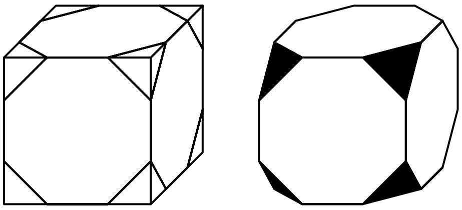
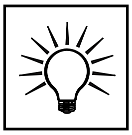


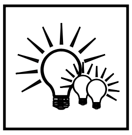
Figure 5.8



Unit Activity 4

Materials: make nets of faces from manila paper.

1 What polyhedra can you make using the following:
   - i) Four equilateral triangles and four regular hexagons?
   - ii) Eight equilateral triangles and six squares?
2. Decide on the names of the polyhedra you have made in 1.
3. Does Euler's formula hold for semiregular polyhedra?



Practice Activity 2

Material: piano wire, oil-based craft clay or ordinary clay (piano wire does a better job of cutting clay than a knife)

- Have pupils make prisms, cones, and cylinders using clay. In this situation, accuracy is not important as long as the essential features are seen.
- Suggest where to make a slice and see if the pupils can predict the shape of the face (*Figure 5.9*).
- Ask pupils to find a way to slice a given shape to produce a slice with a particular shape. For example, how could you slice a cuboid so that the slice is a trapezium?

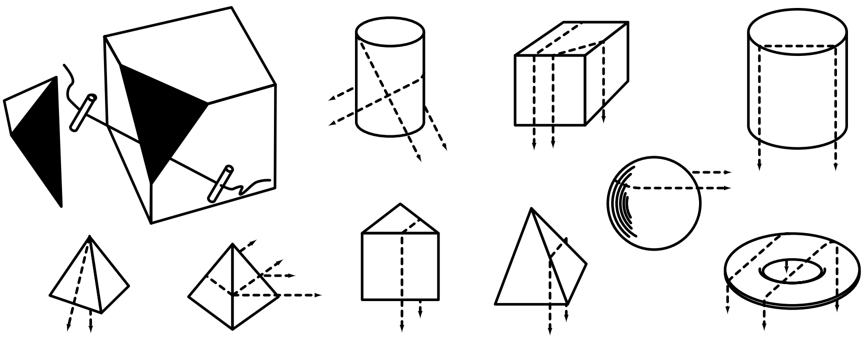


Figure 5.9: Pupils predict the shape of the slice face before you cut a clay model with piano wire

Reflection

When you see a closed dictionary, apart from recognizing it as a book, what shape do you see?

Field Trip: Take a Geometry Walk

Your task is to make mathematics meaningful and enjoyable. Encourage pupils to think mathematically. Geometry is real and we see its existence in the world around us. Creative teachers use models in the classroom to illustrate geometric concepts. For instance, when a pupil sees a book, he/she should see a rectangular prism as well. This approach can be extended by taking a geometry walk outside the classroom.

To promote a greater awareness of geometry in the real world, encourage students to note that:

i) An object's function or use may determine its shape.

ii) The aesthetic or appearance of an object may determine its shape.

iii) The vocabulary of geometry can help describe shapes and spatial relationships. (Hill, 1987: 146)

Before you go out for a field trip, you should have the following in place:

1. Sketch a map for your planned walk. Mark places where you will ask questions.
2. Plan your schedule so there is class time after the walk for a discussion of pupils' observations.
3. Prepare questions that will address functional shape, aesthetic shape, and geometric vocabulary. For example:

   Functional Shape—look at a bicycle. Why is a wheel of a bicycle shaped like circle instead of a square? Pretend you are riding a bicycle with rectangular wheels. Will it be easier or more difficult to ride? Explain.

   Aesthetic shape—look at a pair of shoes. Do you like the shape? How would you redesign its shape to look better.

   Geometric vocabulary—look at a tree. What type of an angle is made by this tree and the ground?

Unit Activity 5

1. Field Trip Preparation.
   - Sketch a map for your planned walk.
   - Plan questions for your geometry walk.
   - Plan the time schedule.

Practice Activity 3

- Take pupils out for a geometry walk.
- Ask appropriate questions.
- Have a discussion with pupils after the walk. Let them tell you their observations.
- Have each student draw the outline of a shape that he or she observed on a walk. See if others can recall seeing that shape.
- Have students sketch functional objects and describe how the shapes are useful in performing their job.

Can you think of other activities to do with your pupils?

Create art

As you may have seen on your geometry walk, the local environment is rich in creative art. We have wood carving, different types of weavings using various materials, and a variety of paintings and tiling. The list is endless.

As such, in your unit activity and practice activities, you are encouraged to come up with any creative work that can allow exploration of properties of three-dimensional shapes.

Unit Activity 6

1. Material: rectangular paper in two different colours, ruler, scissors

   Instructions: Draw two-dimensional shapes with the given measurements (*Figure 5.10*), one shape from each piece of coloured paper. For the rectangular parts, make one 16.4 cm long and the other 15.8 cm long (*Figure 5.10*).

   - Do the shapes have the same area? If not, why?

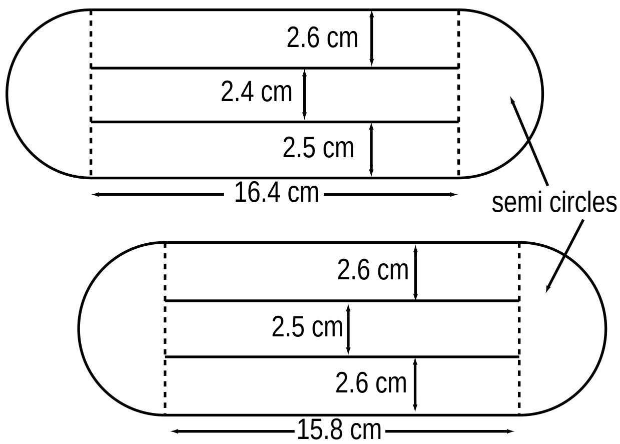


Figure 5.10

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- Divide the rectangular parts into the widths shown, and cut each to the beginning of the semicircles (*Figure 5.10*). You will have three strips attached to the semicircles
- Now you have two curved rectangular pieces ready for weaving. Fold and interweave the two pieces, and at the end you should produce a shape called a Swedish Basket with a pattern as shown in *Figure 5.11*.

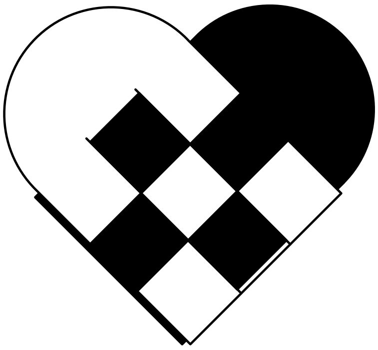
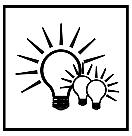


Figure 5.11

Why does the shape have curved ends? How would you redesign this shape to make it better?

- Describe the pattern of the shape.
- What shape have you produced? Is it a two- or three-dimensional shape? Open it! Does it puzzle you that it turns out to be a three-dimensional shape? (a cone)
- Does the shape match with its name? If not what name can you give it and why?
- How long did it take you to construct this shape?

2. Make a three-dimensional shape of your own choice that has attributes of tradition, e.g., incised or relief or decorative design.
   - What geometrical properties are found in your three-dimensional shape?



Practice Activity 4

1. Have your pupils make a Swedish basket. Do not tell them how to do it but show them one.
   - Prepare appropriate questions to ask your pupils.
   - Observe the strategies they use to make the basket.
   - Have them describe the processes they used to produce the basket.
   - How long did it take the first pupil to come up with the shape?
2. Let groups or individual pupils come up with activities on constructing three-dimensional shapes that have attributes of tradition.
3. Have pupils describe the geometrical properties of their three-dimensional shapes.

Summary

In the construction of three-dimensional shapes, we continued to use the investigative approach because it enhances problem solving and induces long-term retention of the principles learned. This approach promotes life skills such as perseverance and creativity. The end result is acquiring self-esteem as well as a working knowledge of solid geometry.

Self Assessment

1. Consider the cube with vertices ABCDEFGH shown in *Figure 5.12*.
   i) Two edges of the cube are called opposite if they are parallel and are not the edges of a single face. List pairs of opposite edges. How many are there?
   ii) Two vertices of a cube are called opposite if they are not joined by an edge and are not vertices of a single face. List opposite vertices. How many are there?
   iii) If you have an ordinary six-sided die, what is the sum of dots on each pair of opposite sides?

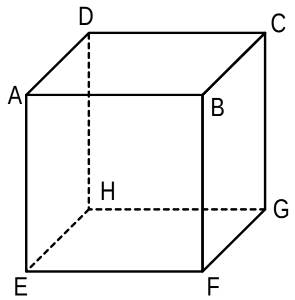


Figure 5.12

2. Which of the following figures in *Figure 5.13* are nets for a cube?

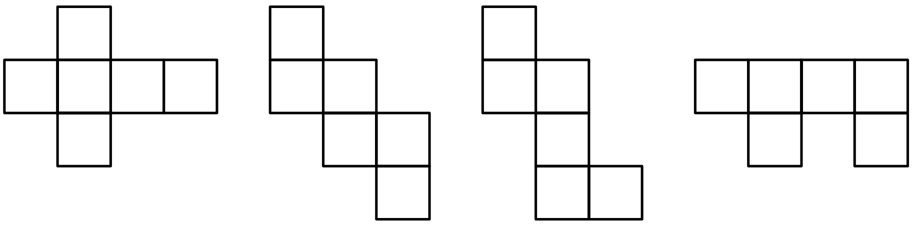


Figure 5.13

3. An antiprism is a polyhedron that is like a prism, except the lateral faces are triangles (*Figure 5.14*). Notice how the bases are rotated. The top and bottom faces are squares and the other faces are equilateral triangles.

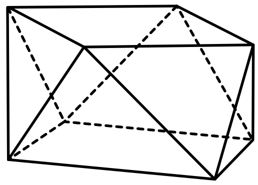


Figure5.14

Explain why the antiprism in *Figure 5.14* is a semiregular polyhedron.

Continued on next page

4. The three-dimensional shapes in *Figure 5.15* are each cut by a plane as indicated. Identify the resulting cross-section.

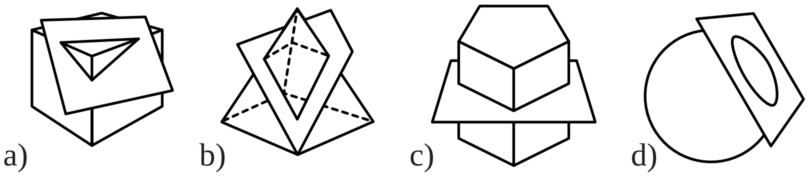


Figure 5.15

5. The picture below is a three-dimensional shape.
   a) Find the number of faces, edges, and vertices.
   b) Is Euler's formula satisfied for this shape?

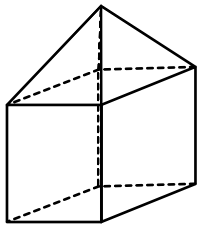
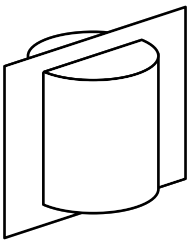


Figure 5.16

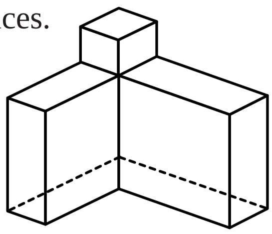
6. Create an investigation card that will involve the pupils you are teaching in the exploration of a basic geometric idea about solid shapes. Try your card with pupils and try to improve its effectiveness.

Unit 5 Test

1. Why are most buildings built with triangular roofs?
2. In a traditional setting, most houses have circular foundations instead of polygonal. Why is this so?
3. Describe four tasks you can give your pupils after making a cube.
4. The three-dimensional shape below is cut by a plane as indicated.
   a) Identify the resulting cross-section.
   b) What errors can a pupil make?



5. The picture below is a three-dimensional shape.
   a) Find the number of faces, edges, and vertices.
   b) Does Euler's formula work here?
   c) What errors can a pupil make?



Suggested Answers to Self Assessment

1. (a) BF and DH; DG and AE; AB and HG; DC and EF; AD and FG; EH and BC

   (b) A and G; C and E; D and F; B and H

2. (a)

3. The antiprism is a semiregular polyhedron because it is bounded by more than one type of regular polygon. In this case, equilateral triangles and squares make the faces of this polyhedron. The arrangement at the vertices is alike.

4. (a) Triangle (b) Kite (c) Pentagon (d) Circle

5. (a) Faces are 9

   Vertices are 9

   Edges are 16

   (b) Yes, Euler's formula is satisfied.

6. Guidelines

   (a) Have a clear objective for the investigation card.

   (b) The card should involve the pupil in an investigation of one of the ideas specified in the objectives.

   (c) The questions are motivating, simple, and clear.

   (d) The card suggests appropriate materials.

   (e) The card leaves room for creativity.

Suggested Answers to Unit test

1. Because triangles are rigid unlike other polygons.

2. A circular house consumes less material but has a greater area than a square or rectangular house.

3. (i) Finding the surface area of a cube

   (ii) Finding the volume of a cube

   (iii) Finding the mass of a cube

   (iv) Finding the vertices, edges, and faces of a cube

4. (a) Rectangle

   (b) Pupils may only focus on the top base and think the cross-section is a semi-circle.

5. (a) Faces are 12, vertices are 17, and edges are 27.

   (b) Yes

   (c) It is likely there will be errors here, but few will be mentioned.

      (i) The bottom face may be counted twice.

      (ii) Forget the major edge behind.

Unit 6: Transformations

Introduction

We live in a world of transformations. Some transformations change the size of an object, some change the shape, while others change neither. In the previous units in this module you dealt with different shapes and you even constructed some. In this unit you will focus on the creation of patterns by doing something to or with simple geometric shapes, and identifying patterns that evolve from the motion of figures/shapes.

Objectives

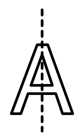
At the end of this unit you should be able to:

- identify and state different types of transformations vis-à-vis translation, reflection, rotation, and enlargement
- introduce transformations to your pupils through a study of flips, slides, and turns, and objects in the real world
- demonstrate ideas of congruency and similarity in transformations

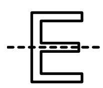
Symmetry

Make a set of 10 cm high letters of the alphabet (capitals)
That is: A B C D E F G H I J K L M
N O P Q R S T U V W X Y Z

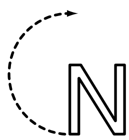
(i) Write down all the letters that appear balanced about a vertical axis. For example,



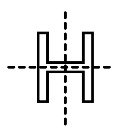
(ii) Write down all the letters that appear balanced about a horizontal axis. For example,



(iii) Write down all the letters that appear the same if you turn it half a complete turn about a point on it or near it. For example,

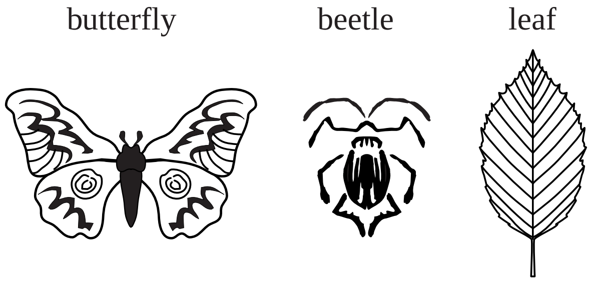


(iv) Write down all the letters that appear balanced about both a vertical and a horizontal axis. For example,



The line (axis) about which a shape/letter balances is referred to as the axis of symmetry. This type of symmetry is called **reflectional symmetry**. The symmetry of a figure is a transformation of the figure for which the image is the original figure.

Symmetry is used in design and is evident in nature, art, and architecture. Designs in nature illustrate a natural tendency to make things balanced, and we look on symmetry as a type of balance.



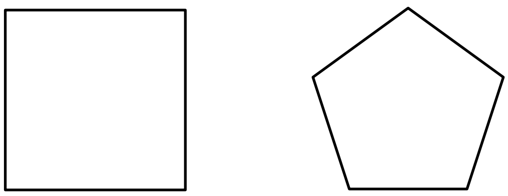
Encourage your pupils to look for examples in natural and manufactured objects, and in pictures, advertisements, etc., which show symmetry, then make a display that illustrates the axis of symmetry of these objects.

If you return to question (iii) on the previous page, you will notice this does not fall in the categories of either vertical or horizontal axis of symmetry. This type of symmetry is called **rotational symmetry**. Rotational symmetry (point symmetry) involves a central point of symmetry, with a rotation of less than 360º about a point, such that the figure moves onto itself.

Unit Activity 1

a) In this activity, trace the above rectangle on another sheet. Determine the central point and call it P. Test P as a point of symmetry by rotating the tracing of the rectangle about the point P.

b) Determine whether the two figures below have rotational symmetry. If so, how much rotation is involved in each?



Practice Activity 1

1. Give each pupil a 18 cm × 12 cm piece of paper. Let them fold it in half and make holes (with any sharp instrument) about 1 cm apart, beginning and ending near the fold. Let them open the paper and join each pair of holes. What do they notice about the mid-point of each pair of holes?

   Let them join points to make a shape that balances on either side of the fold. If you have a mirror, try putting it along the balance line. What do they notice?

2. Give each pupil another piece of paper. Let them repeat activity 1, except now have them fold the paper twice before making the holes. Let them check with a mirror to determine how many lines of symmetry the shape will have.

3. Let pupils cut out the following shapes and then by folding:

   (i) State whether or not the shape has symmetry.

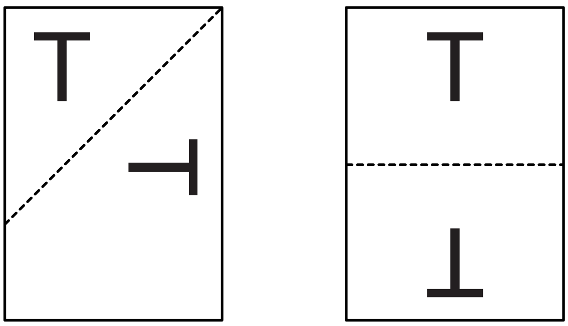
   (ii) Determine how many lines of symmetry each shape has.

CIRCLE	EQUILATERAL TRIANGLE		PARALLELOGRAM
SECTOR	ISOSCELES	”	RHOMBUS
QUADRANT	SCALENE	”	RECTANGLE
	RIGHT-ANGLED	”	SQUARE

   (iii) Let pupils check the rotational symmetry of each of the shapes above.

Reflection

Flips or reflections form the basis for looking at various properties of geometric figures. Consider the diagrams below:



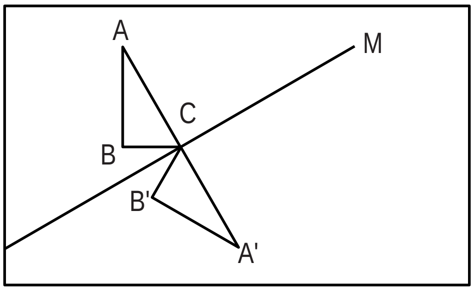
The line of the fold is called the line of reflection—we say reflection about a line or reflection over a line.

Food for thought

In the above section on symmetry, we established what was called an axis of symmetry and sometimes we referred to symmetry as reflectional symmetry. How would you distinguish between the axis of symmetry and a line of reflection? In what ways are they the same or different?

Reflection images or flip images are easily obtained by using paper folding, a mirror, by construction, or just by common sense.

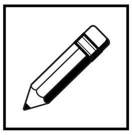
Find the flip image of triangle ABC relative to line M.



The concept of a reflection is a special case of a transformation. A transformation is also called a mapping. In the example above, A is mapped onto A', B onto B', and C is on the line. A' is the image of A.

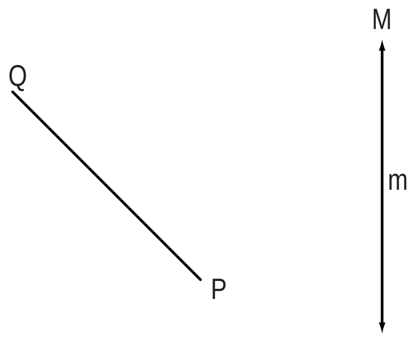
A transformation of a plane is a one-to-one correspondence between two sets of points of a plane. For reflection we can say that it is transformation mapping that maps point A onto A' as follows:
If A is on M (mirror line), then A = A' (the case of C in the example above)
If A is not on M, then M is the perpendicular bisector of AA'.

You could verify this using constructions that you dealt with in the last unit.



Self Assessment 1

Construct a perpendicular to a line from a given point not on the line.
Reflect or flip the line segment $\underrightarrow{PQ}$ about the line M.



Use paper folding to trace the flip image, P'Q'. What do you notice about the distance QM and MQ'? Measure the shortest distance between PM and $\underrightarrow{MP}$'. What is your observation?

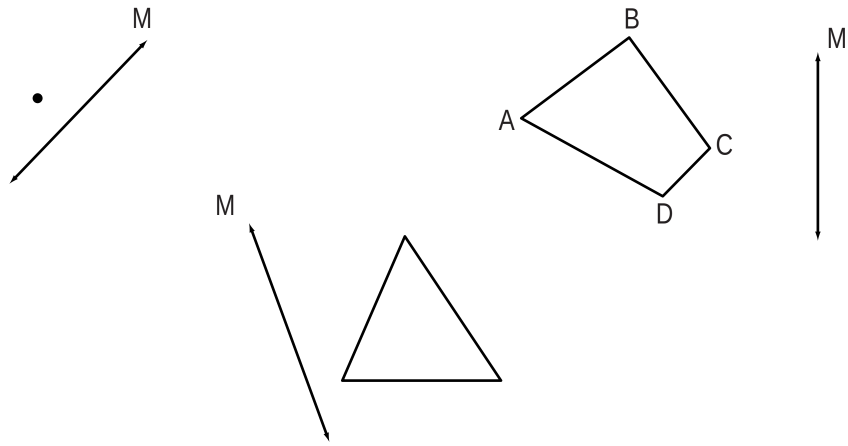
Some properties are *observed* or preserved under a reflectional transformation. You may have noticed that the *distance* between the *image* and the *mirror line* and between the mirror line and the image is the **same**. Further you may notice that the points fall in one line. Are angles preserved under this transformation?

The idea of a reflection is very familiar. Every time you look in a mirror, you see a reflected image of yourself. What do you notice about the orientation of your reflection?

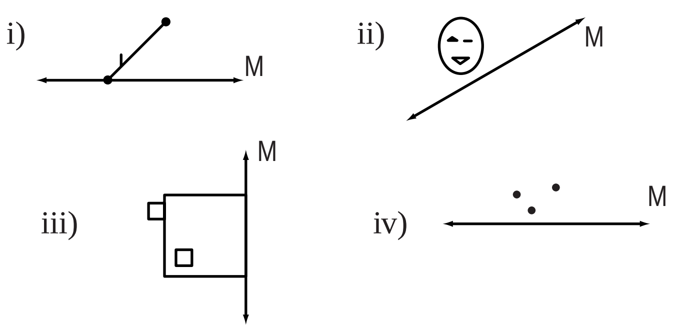
Self Assessment 2

In the activities below, perform the tasks and answer the questions that follow.

1. Trace each drawing onto thin paper. Then fold the paper along the line M and find the reflection image of each given point or object with respect to the given line M.



2. If Q is the reflection image of S over line M, how are M and $\overrightarrow{QS}$ related?
3. Classify as True or False:
   a) A segment and its reflection image have different lengths.
   b) A point and its reflection image are the same distance from the reflecting line.
   c) A reflection is a one-to-one correspondence.
   d) A point and its image coincide if the point is on the reflecting line.
   e) The reflection image of an acute angle is always acute.
   f) When two non-intersecting lines are reflected over the same line, their images may intersect.
4. Flip the following figures about line M and show the flip image on the same diagram.



These transformations can be informally introduced to pupils at the upper primary level, and they should enjoy these activities.

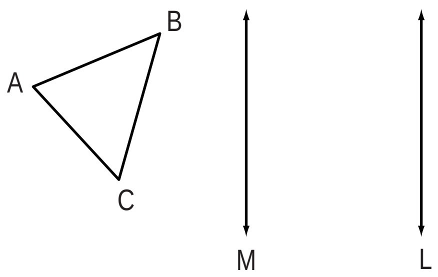
Practice Activity 2

1. Have pupils act as mirror images of each other. Let them discuss the properties observed in a mirror image.
2. Paper folding activities involving points and objects/shapes can be very interesting for pupils. Allow your pupils to establish the properties—distances between pupil or object and their images.
3. Ask pupils to bring manufactured or naturally occurring examples of reflections to class.

Translations (or slides)

Do the activity below and answer the questions that follow:

Enlarge the following drawing, then trace it onto thin paper. Fold the paper and find the reflection image of the drawing with respect to the first line M. Using this image as the object, find its reflection image with respect to the second line L.



When you are finished:

i) Measure the distances between the vertices in the original drawing and the last image.

ii) What do you notice about the distances?

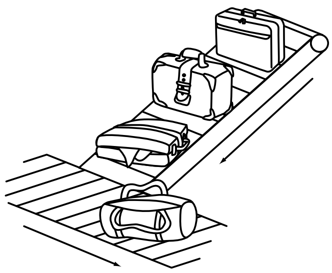
iii) What a do you notice about the size of the drawings?

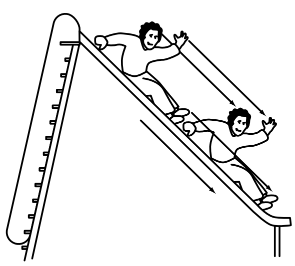
iv) What do you notice about the orientations of the two drawings?

In the activity above, you have a transformation that is a result of two reflections about two parallel lines. The drawing of triangle ABC above has moved a given distance in a given direction. This leads to the concept of a slide or, more formally, a translation.

A translation of an object or figure may be thought of as a slide along a straight line. It has both direction and distance.

There are examples of this transformation in everyday life. Look at the pictures below:





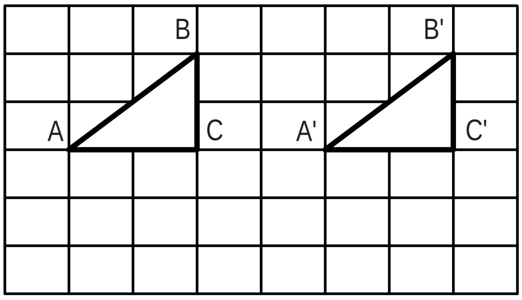
The corners/vertices of the bags on the conveyor belt will move the same distance and since they move along parallel lines, they will also move in the same direction.

Unit Activity 2

Do these activities on grid paper or ruled paper.

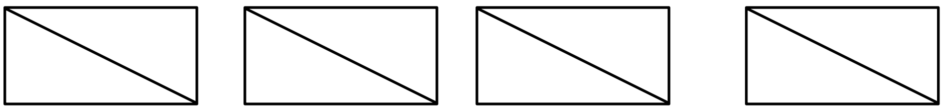
(i) Draw different shapes on the grid, e.g., triangles, rectangles, a straight line, etc.

(ii) Move each of the objects/shapes four grids either to the right, left, up, or down.



The exercises above provide a valuable background for understanding the idea of translations. However, we need to identify some of the mathematical concepts required to discuss the ‘sameness’ and differences involved in the two or more shapes. For pupils, they need to establish these properties of the transformations on their own.

Consider the shapes below:

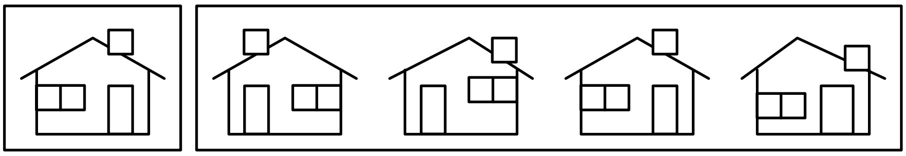


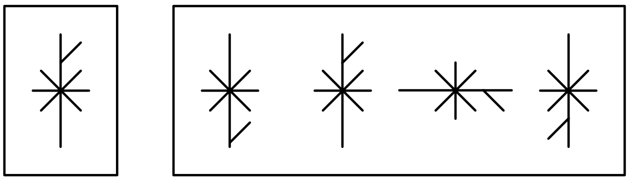
Ask pupils whether the above shapes are the same or not. Let pupils establish that they are all rectangles with a diagonal drawn from the top-left to the bottom right-hand corner, and that they have the same dimensions, the same angles, corresponding sides point in the same direction, etc.

Let pupils observe the difference that each rectangle is drawn in a different position. The transformation that slides, without turning from one position to another, is referred to as a translation. To describe such a transformation, we need concepts of direction, for example, up, down, left, or right.

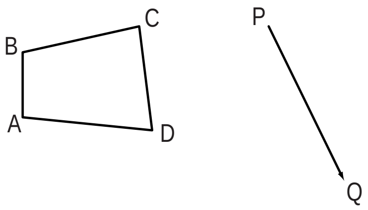
Self Assessment 3

1. In the figures below, which of the figures on the right is a slide image of the figure on the left?

a) 

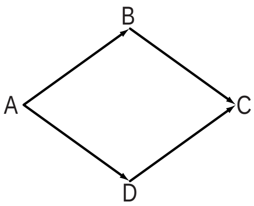
b) 

2. Draw a quadrilateral ABCD and arrow PQ on your paper as follows:



Find the slide image of the quadrilateral after the slide PQ.

3. In the figure below it can be said that the motion, i.e., the slide from AD followed by slide DC is the same as slide AB followed by slide BC. Explain.



Rotation

We experience a turning motion in our daily living, for example, in the door handle, the hands of a watch or clock, a wheel, propeller, etc. The concept of the turning motion is refined and extended from the everyday experiences we have. To illustrate the turning motion, we shall use physical materials.

Consider two sheets of acetate fastened with a paper fastener, illustrated below in *Figure 6.1*.

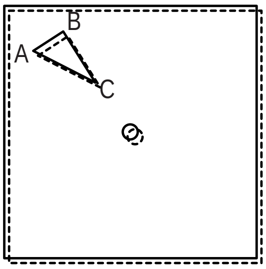


Figure 6.1

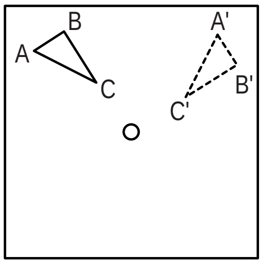


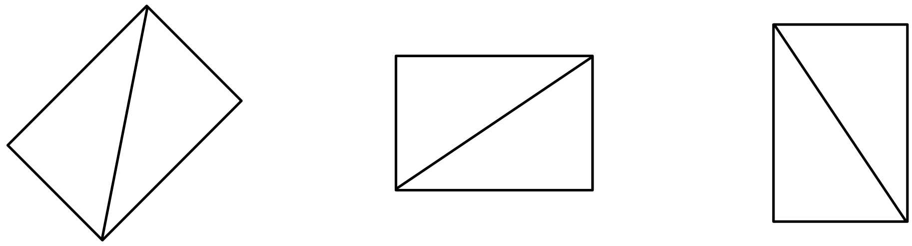
Figure 6.2

Triangle ABC is drawn on the bottom sheet and traced on the top sheet. Then, turn the bottom sheet as shown in *Figure 6.2* and label the figure A'B'C'.

The resulting figure (A'B'C') is the turned image of triangle ABC. The triangle was turned around the point O (called the centre of the turn, or pivot point, around which the amount of turn forms a measurable angle).

This transformation is called a rotation or a turn in a plane. Every rotation has a centre and an amount of turn.

Pupils need to be given an opportunity to identify the properties of this transformation. Consider the following set of shapes:

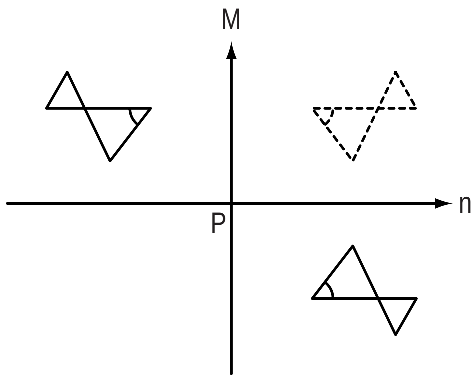


Ask pupils whether the shapes are the same or different. Have them explain their answers. Establish with the pupils that the shapes are the same, but differ in their orientation in space. Corresponding lines in the shapes point in different directions. To get one shape from another, you have to turn it from an identified centre, with a specific amount of turn.

Food for thought

When two or more motions have been performed, it can be difficult to visualise what happened. Would two successive reflections result in a single motion of a rotation?

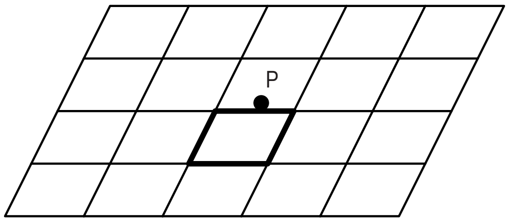
Consider the diagram below**:**



It can be shown that the dotted figure in the diagram is the reflection of the first figure about line M. It can further be shown that the third figure is the reflection of the dotted figure about line *n*.

Now you can rotate the first figure about point P to fit into the third figure, demonstrating that turns or rotations may have the same effect as reflections. Thus, rotations have the same properties as reflections.

Unit Activity 3



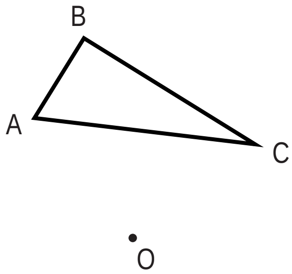
Place a sheet of tracing paper over the tessellation above and trace it. If the tracing is held firmly at point P by the tip of a pencil and rotated 180 degrees, the tracing will fit back on the tessellation.

a) How many other points can you find, on or inside the parallelogram with darker lines, for which this is true? Draw the tessellation and label these points.

b) If rotations other than 180 degrees are allowed, how many points can you find?

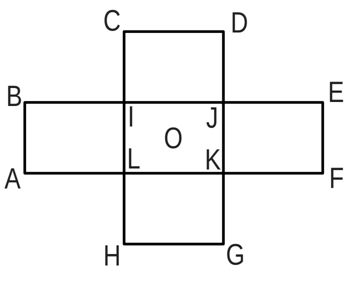
Self Assessment 4

1. Draw a triangle ABC and a point O on your paper. For example:



   Use a protractor and tracing paper to find the 90-degree counter-clockwise turn image of triangle ABC about point O.

2. The pattern of five squares is centred at O in the figure below:



   Find the 90-degree counter-clockwise rotation image about O of:

   (i) A

   (ii) Square ABIL

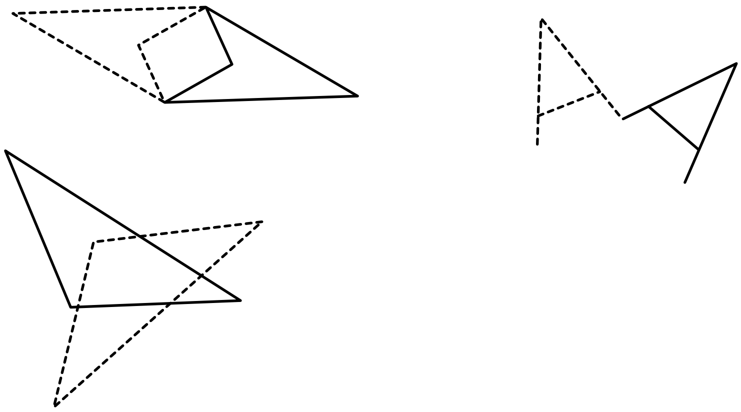
   (iii) Rectangle ABEF

   (iv) Rectangle CDLK

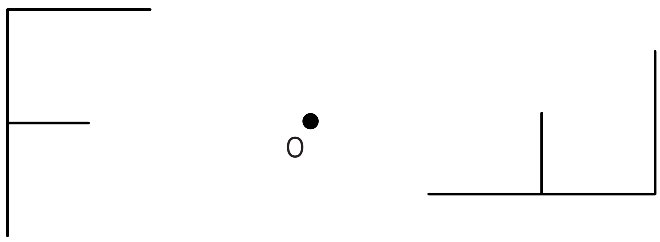
   (v) Rectangle IJKL

   Continues on next page

3. In the figures below, the dotted figure is the image of the black figure under a turn. Label the centre of the turn and indicate the angle measure for each turn. Tracing paper will be helpful.



4. Are the two shapes below turn images of each other? Explain your answer.

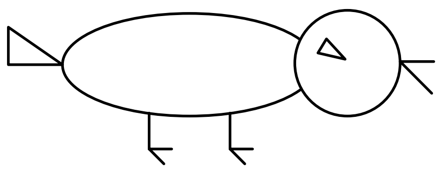


Congruence (identical shapes)

Do the following activity and write down what you observe.

(a) Take a thin sheet of paper and place it on top of the diagram below.

(b) Trace the diagram on the new sheet of paper. Make sure you go over every line.



(c) What do you notice about the diagram and the traced figure in terms of size and shape?

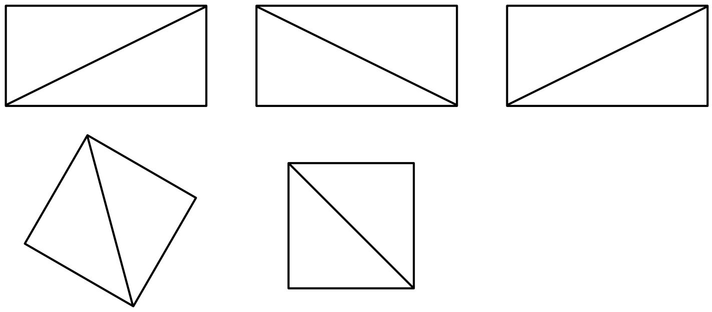
From the activity above, it can be calculated that moving the tracing is, in most cases, translating the figure. We obtain exactly the same size and shape as the original. Every line and point matches. The movement of the tracing can be described by either a translation, a reflection, or a rotation.

When an image is exactly the same in size and shape as the original object, the mathematical term that describes this relationship is **congruence**.

Congruence is at the heart of modern technology, for example, production of parts ‘congruent’ to each other—same make, size, and shape of motor vehicles, etc. When we sort coins from a handful of change, we use congruence, i.e., putting coins with the same denomination in one place.

Practice Activity 3

Give the following set of shapes to your pupils.



Ask pupils to investigate:

(i) Lengths of segments in each of the figures.

(ii) Angles in each of the figures.

(iii) Corresponding sides of the figures.

(iv) Corresponding angles of the figures.

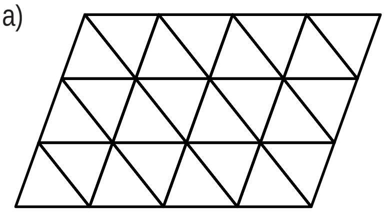
In this activity, pupils should establish that congruent segments have the same length, that congruent angles have the same measure, that corresponding sides of congruent figures are congruent, and that corresponding angles of congruent figures are congruent.

Remember that the motions—reflection, translation, and rotation—preserve the shape and size of the original figure. Thus we can conclude that "two figures A and B are congruent if and only if there is a composite of reflection, translations, or rotations that map Figure A onto Figure B or Figure B onto Figure A."

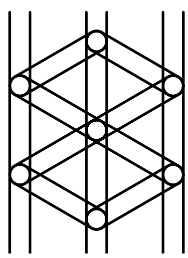
Self Assessment 5

1. Classify each of the following statements either as TRUE or as FALSE:
   (i) If AB is congruent to AD, then B and D name the same point.
   (ii) The union of two different segments may be congruent to one of the segments.
   (iii) Two congruent figure have the same shape.
   (iv) If segments AB and BC are congruent, then they are of equal length.
2. Describe one composite transformation of reflections, translations or rotations that map each of the following designs onto itself.

a)



b)



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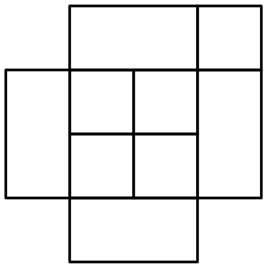
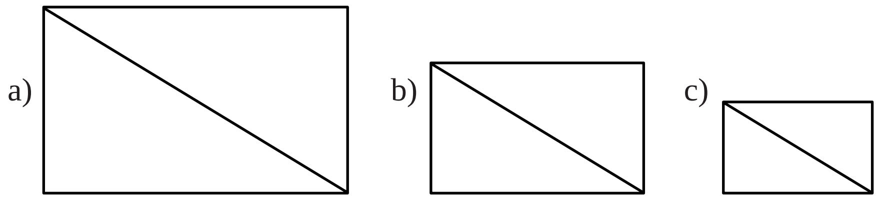
3. How many squares can you find in the figure below? How many are congruent to the one containing four small squares?



Similarity transformation

Magnification (Enlargement and reduction)

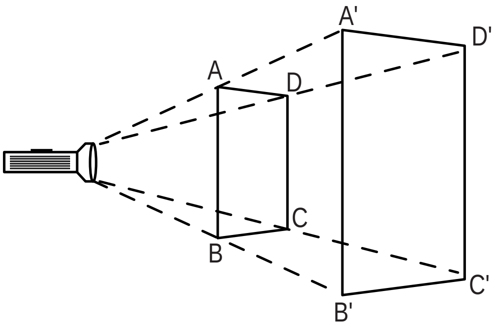
In Unit 3 we introduced the special mathematical term of **similarity**. Similarity refers to shapes that have the same shape, but differ in size. This can also be seen as scaling down. For example, the shapes below demonstrate **scaling**.



The key concept required to specify scaling is a scale factor. In the shapes above, the lengths are scaled by the same factor, hence the ratio of any two lengths in the shape remain the same. Angles in a shape also remain unchanged.

Pupils will have experienced scaling in model cars, play houses, photographs, photocopying, etc. Now try the following activity.

Take a pen flashlight or a torch and hold it in front of a rectangular piece of cardboard with the light rays perpendicular to the centre of the cardboard as shown in the diagram below.



(i) What do you notice about the shape of the shadow on the cardboard?

(ii) Measure the distances between the torch and the object (cardboard) and also between the torch and the shadow.

(iii) Divide the distance between the torch and the shadow by the distance between the torch and the cardboard.

(iv) Measure the lengths of the cardboard and their corresponding lengths on the shadow. Divide the corresponding lengths of the shadow by the lengths on the object.

(v) Are the results in (iii) and (v) the same? Or different?

(vi) Measure the angles for both shapes.

In the activity above you should have found that the ratios of the distances between the corresponding lengths and the torch and the object and the image are the same. This is the scale factor of the motion. You should have established further that the angles remained unchanged. A motion that maintains the shape but not necessarily the size is referred to as an enlargement or similarity transformation.

You should establish with your pupils the following properties for a similarity transformation. Given an enlargement with a scale factor K:

(a) Each segment is mapped onto another segment K times long.

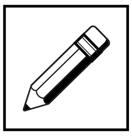
(b) Each angle is mapped onto a congruent angle.

(c) Lines are preserved.

(d) Polygons are preserved; since the image of a segment is a segment, it follows that the image of a polygon is a polygon.

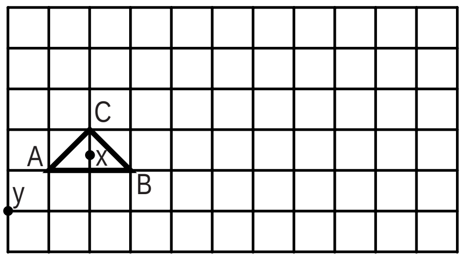
(e) The ratio of the distances is preserved, that is the ratio AB/BC is equal to the ratio A'B'/B'C'.

(f) ‘Between-ness’ is preserved, that is if B is between A and C, then B' is between A' and C'.



Self Assessment 6

1. Use dot paper or graph paper for these exercises.



(a) Draw the image of triangle ABC in the diagram for the magnification with centre X and scale factor 2.

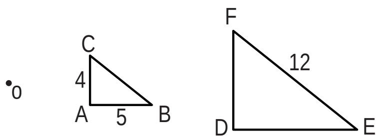
(b) Draw the image of triangle ABC for the magnification with centre Y and scale factor $^1/_2$.

2. Refer to the shapes below. Triangle DEF is the image of triangle ABC for a magnification with a scale factor of 2. Complete the following:

(a) DE =

(b) DF =

(c) BC =



Practice Activity 4

Have your pupils draw many different shapes with similarity transformation properties. Ask them to establish the properties mentioned above.

Unit Activity 4

Materials: protractor, straight edge, tracing paper

1. For each pair of similarly shaped figures below:

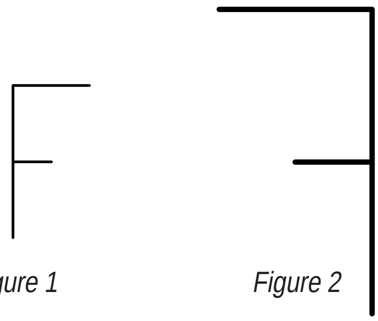


Figure 1 *Figure 2*

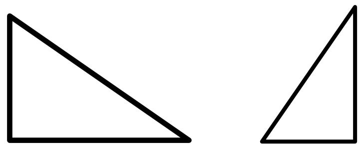


Figure 1 *Figure 2*

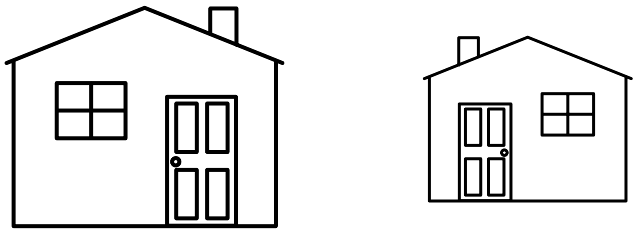


Figure 1 *Figure 2*

   (a) Find a centre of turn or a line of reflection and centre O for a magnification such that the motion followed by the magnification maps Figure 1 onto Figure 2. In how many ways can this be done?

   (b) Repeat this process showing that Figure 1 can be mapped onto Figure 2 by magnification followed by a motion.

Summary

Geometrical thinking is mainly about classifying shapes. This involves recognising both equivalence and transformations—ways in which shapes can be regarded as being the same and ways in which shapes differ or change. In this unit we looked at symmetry, translation, reflection, rotation, and similarity transformation. For pupils to grasp these ideas they need to do a lot of handling of shapes, tracing figures, turning them around, etc. Appropriate geometric language should be introduced gradually. By the end of these activities, your pupils should be able to:

- explore transformations of geometric figures
- represent and solve problems using geometric models
- understand and apply geometric properties and relationships
- develop an appreciation for geometry as a means of describing the physical world

Unit 7: Scale Drawing and Direction

Introduction

In the last unit, you touched on the basic concepts used in scale drawing. In this unit you will learn more about scale drawing and ideas for giving directions. You will use the knowledge of lengths and angles that you acquired in Module 3, and the knowledge of ratios that you dealt with in Module 2, Unit 5. In this final unit of Module 5, we link many concepts that you worked with in the earlier units.

Objectives

After you complete this unit, you should be able to:

- draw lines according to a given scale
- read maps and give directions
- interpret measurement scales
- develop a sense of scale
- effectively teach scale drawing and directions to your pupils
- give bearings and do calculations involving bearings

Introducing scale drawing

Consider the following statements from Haylock and Cockburn (1997):

1. "One of my boys measured the playground to make a plan, then said he couldn't do it. He said the paper was not big enough, because it was only about metre long, but the playground was thirty metres!"
2. "We were reading a book about a monster and there was this picture of a huge foot next to a small man. None of my children got it. They did not seem to appreciate the significance of scale at all."

The two statements above reveal problems that pupils have in developing a sense of scale, or seeing a scale drawing as an attribute of scaling. Though pupils are taught to measure objects and do calculations involving measurements, they are not allowed to 'stand back' and reflect on how they can draw pictures of objects large enough to fill a room.

A sense of scale comes intuitively to pupils. Consider those circumstances when you drew a large shape on the board and, saying nothing about the measurements, requested your pupils to draw the shape in their exercise books. Usually they all draw something fitting in their exercise books—pure common sense—and no questions are asked. A sense of scale comes in, and they scale the shape on the board to something that will fit in their books.

Complete the activity below, and write down the process you follow:

Unit Activity 1

a) Measure the length of the longer side of a block of classrooms. Record your results in metres.

b) Draw a representation of this length on a sheet of paper (a straight line)

c) What decisions did you make?

The above activity calls for scaling the measurement so that a representation of it will fit on a sheet of paper. Suppose the measurement above was 95 m.

We could let 10 m be represented by 1 cm, meaning 20 m will be represented by 2 cm. Therefore, 9.5 cm would represent 95 m, which would be the line we would draw on A4 paper—a 9.5 cm line.

In this process we have used a scale of 1 cm to represent 10 m (or 1000 cm) on the ground. Ratio notation is usually used to indicate such a scale on a map—in this case 1:1000. That means 1 cm on the map equals 1000 cm, or 10 m, on the ground. A fraction notation (1/1000) can also be used. For every 1 cm on the book, there are 10 m on the ground. The sense or skill of finding an appropriate ratio lies in estimating skills in measurements—what references or benchmarks do you have for standards units?

Scale drawing is common in everyday life. Many pictures and diagrams are drawn to scale—plans for buildings, models for the manufacturing industry, maps, etc. The challenge, as mentioned above, is in finding a suitable scale factor. Pupils may make several false starts, but they need such activities to develop a sense of scale.

Self Assessment 1

1. If the scale of a map is 1:100 000, what does 1 cm represent?
2. What would the distance of 12.5 km be on a map with a scale of 1:100 000?
3. What scale would you use to make each of the following scale drawings in your exercise book?
   (a) A drawing of yourself.
   (b) Map of Zambia.
   (c) A drawing of this Module you are using.
4. A ______________________________ B

   C ______ D

   Line CD is a scale drawing of line AB. It is drawn to a scale of 2 mm representing 1 cm. Write down the scale as a ratio of the form 1:a or 1/a.
5. Draw lines to represent the following lengths (use a scale of 1:50).
   a) 1.5 m   b) 75 cm

You should have noticed that most scale drawings start with 1 to some other amount. For example, 1 cm represents 10 m. In ratio form the units are the same if they are not stated. For example, 1:10 000 implies the 1 and the 10 000 are in the same measuring unit. In maps and other scale drawings, the centimetre unit is used. For example, a map with a scale of 1:50 000 means 1 cm represents 50 000 cm.

Unit Activity 2

Using a map of Central Africa, answer the questions below:

a) Measure the shortest distance between Lusaka and Harare in cm. Write down your answer.

b) Use the scale on the map to find the actual distance on the ground.

c) With the use of a string, follow the road network to find the actual travelling distance on the ground between Gaborone and Johannesburg (shortest route possible).

d) If Zimbabwe was to cover the whole sheet of A4 paper, what scale would you propose?

Practice Activity 1

To introduce scale drawing to your pupils, try drawing a large square or any other shape on the board. Ask the pupils to draw the shape in their exercise books.

Discuss with the pupils what is meant by scale drawing, discuss the scale used on various maps and scale drawings, and translate these scales into actual lengths and distances. Increase and decrease the sizes of the scales.

Explain the three ways to express scales:

- a fractional scale, for example 1/5
- a ratio, for example 1:5
- a statement, for example 1 cm represents 5 cm

The discussions are meaningful if done in context. The class activities below should provide teaching and practice situations for your pupils.

Class Activity 1

Let pupils measure the length and breadth of their classroom.
For each metre that they measure, let them put down a small convenient measure, for example a drawing pin.
Let them lay pins side by side, forming the rectangular shape of the room.

Class Activity 2

Ask pupils to make a scale drawing of their class showing the perimeter, the teacher’s table, and their tables.

Class Activity 3

Let pupils make a model of their village/compound or school showing their house, major shopping centre, and a church. Watch for appropriate use of scale.

Class Activity 4

Let pupils make a scale model of a box (provide a box) so that the net of the box just fits onto a sheet of A4 card/paper.

Class Activity 5

Give pupils a picture. Let them enlarge it five times. Let them compare their final product with the original figure.

Class Activity 6

Allow pupils to make a model of their table or desk. Let them decide on the scale to use. They should measure all the important lengths.

Reflection

Why is it important to use the same scale in one diagram?

Direction

This section extends the work you did with angles in Unit 1 of this module. We will look at this aspect in the sense of providing a useful and accurate way to give and locate places.

Understanding of direction is a necessary skill for map reading and construction. We often give each other directions, make journeys to unknown places with the aid of maps, etc.

What information do you need to give someone in order for that person to reach a particular destination?

A compass is one of the tools that will indicate direction. The distance one has to cover is another aspect to be considered. You may also need a reference point. But can the position of an object/town be located using distance only? Or can the position be fixed only by using directions?

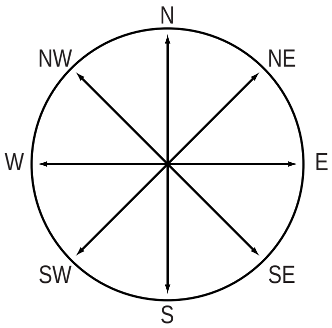
Unit Activity 3

Ask pupils to give each other directions to get to the nearest church building from the school. What ideas do they have about giving adequate and accurate directions?

Directions using a compass

There are different types of compasses: a compass card, magnet compasses, sun compasses, etc. For our purposes, we will refer to the magnetic card. This is circular and is marked in 360 equal units or degrees, beginning with zero and proceeding in a clockwise direction.

A typical magnetic card has North marked at 0, East at 90, South at 180, and West at 270. These are the main reference points, called **cardinal points**. Falling midway between the cardinal points are the **intercardinal points**: northeast (45), southeast (135), southwest (225), and northwest (315). There are other points that fall midway between cardinal and intercardinal points. Further points can still be located on the compass, but the basic card is shown below.



Pupils should be familiar with the card and its use. The magnetic compass will always point North.

Practice Activity 2

For this activity you will need maps. The goal is to teach pupils to give appropriate directions and with adequate information.

(i) Indicate the direction North in the classroom and let them give the other directions.

(ii) Take your pupils to a point on the school grounds, indicate east, and let them give the other directions.

(iii) Look at several maps and indicate the four directions.

Practice Activity 3

Let pupils give commands to each other as follows:

(i) Stand facing north, turn $^1/_2$ a right angle to the right. What direction are you now facing?

(ii) Stand facing west, turn $2^1/_2$ right angles to the right. What direction are you now facing?

(iii) Stand facing south, turn $1^1/_2$ right angles to the left. What direction are you now facing?

Locating specific points also requires specific distances. Compass directions alone may not be enough. The use of a grid can demonstrate this point.

N

6					
5					School
4					
3					
2		Post office			
1					
	A	B	C	D	E

The location of the post office is in the rectangle B2. What is the location of the school?

Assuming we are at the school (E5) how would you give a compass direction for the location of the post office in relation to the school?

From the school, the post office is in southwest direction. We can move three rectangles to the west and three steps to the south. Or we can move three steps south and three steps to the west to arrive at the post office in rectangle B2.

The activities below, which you should do with your pupils, may require reading measurements on the maps.

Practice Activity 4

1. a) Draw a grid on the ground, mark an X in one square, and give the pupils directions to move on the grid. For example, move six spaces east from X, make a $^{3}/_{4}$ turn and move two spaces.

   b) Locate or put 'treasure' on some spot on the grid. Let pupils locate the treasure.

2.

N

	A		B		C		D	
		E		F		G		H
	I		J		K		L	
		M		*X*		N		O
	P		Q		R		S	
		T		U		V		W
	Y		Z		a		b	
		c		d		e		f

Continues on next page

Start at the point marked X on the grid. Move three spaces south, three spaces west, and two spaces north. Note the letter you stop on.

Move three spaces southeast and one space east, then one space north. Note the letter again.

Make a $^{3}/_{4}$ turn and move three spaces. Then move one space south and note the letter.

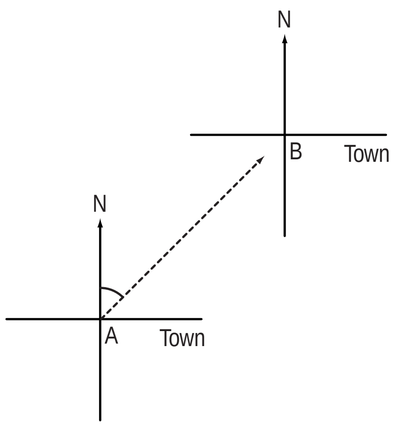
Move four spaces east and note the last letter.

Unit Activity 4

Give pupils different maps. Show them how direction is indicated on the map. Set tasks for the pupils to find the positions of towns or places of interest.

Three Figure Bearings

The directions can also be given using angles. You dealt with angles in the first unit of this module. A conventional three-figure notation is adopted in this case. North on the compass card is 000/360. East is 090, south is 180, and west is 270. The use of angles in degrees is referred to as **bearings**. All directions involving bearings are measured **clockwise**, and maintain a three-figure notation. For example, given the two towns below:

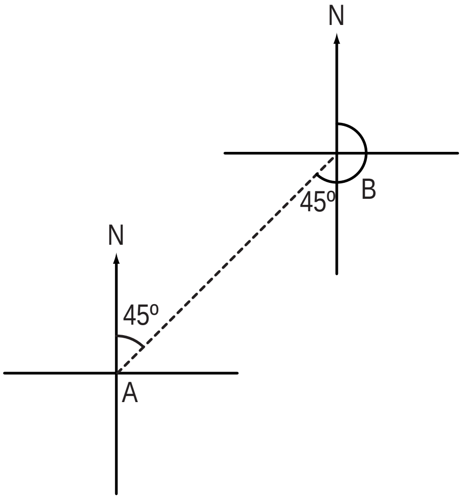


If the town B is 45° northeast of A, then the bearing of town B from town A is 045.

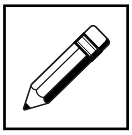
Reflection

Does it follow that the bearing of town A from town B is also 045? Explain your answer.

Clearly the bearing of A from B cannot be 045. Remember bearing is always measured from the North in a clockwise direction only. Thus, the bearing of town A from town B is the angle shown below.

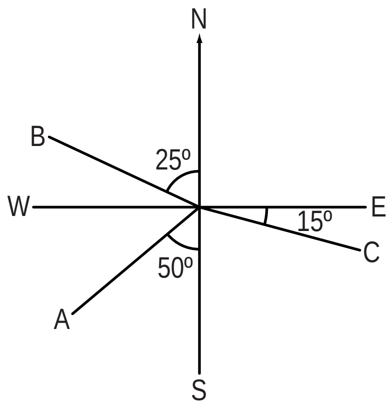


Therefore, 45° + 180° = **225°**

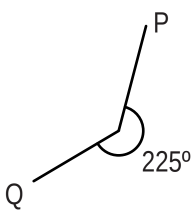


Self Assessment 1

1. Write three-figure bearings for the directions of the points indicated below, i.e., A B, and C.



2. The diagram shows the bearings of P from Q. Find the bearings of Q from P.



3. The distance between two villages, Kasoka and Panguli, is 5 km and the distance between the villages Panguli and Vupa is 8 km. The bearing of Panguli from Kasoka village is 070 and the bearing of Vupa from Panguli is 270. Find the distance between the villages Kasoka and Vupa.

Summary

This unit has dealt with scale drawing with the aim of helping pupils develop a sense of scale. It further explored ways to give directions and locate towns/places on a map. The teaching approach in this unit emphasised the practical use of everyday situations. Keep a record of your activities and observations in your portfolio or teaching journal.

Unit Test

1. The scale factor of the map is 1:250 000. Find the actual distance of two towns that are 2.5 cm apart on the map.
2. Two towns A and B are 32 km apart. A is due North of B. Town C is 20 km from B, with a compass bearing of 056 from B. Find:
   (a) The distance between the towns A and C.
   (b) The bearing of C from A (use a scale of 1 cm:5 cm).
3. Write a lesson plan for a forty-minute lesson on scale drawings, indicating activities that you would perform with your class.
4. P is 40 km east of Q, and R is 30 km north of P. What is:
   (a) the bearing of R from Q?
   (b) the distance between Q and R?

Selected Answers to Unit 5 Test

1. 625 000 cm, or 6.25 km apart

4. (a) R lies northeast of Q. Bearing of R is therefore less than 090. By applying a protractor to triangle PQR, we find that the measure of angle PQR is about 37°. Therefore, measuring from north, the bearing of R from Q is (090 – 037) = 053. (One can also measure the bearing from north directly. Using sines, the procise value of the bearing is 53.13°)

   (b) Triangle POR is a classic Pyghagorean 3-4-5 triangle. Therefore the distance from Q to R is 50 km.

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