Module 1

Upper Primary Mathematics

Number and Numeration

The Commonwealth of Learning

Science, Technology and Mathematics Modules

for Upper Primary and Junior Secondary School Teachers of Science, Technology and Mathematics by Distance in the Southern African Development Community (SADC)

Developed by
The Southern African Development Community (SADC)

Ministries of Education in:

- **Botswana**
- **Malawi**
- **Mozambique**
- **Namibia**
- **South Africa**
- **Tanzania**
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SCIENCE, TECHNOLOGY, AND MATHEMATICS MODULES

This module is one of a series prepared under the auspices of the participating Southern African Development Community (SADC) and The Commonwealth of Learning as part of the Training of Upper Primary and Junior Secondary Science, Technology, and Mathematics Teachers in Africa by Distance. These modules enable teachers to enhance their professional skills through distance and open learning. Many individuals and groups have been involved in writing and producing these modules. We trust that they will benefit not only the teachers who use them, but also, ultimately, their students and the communities and nations in which they live.

The twenty-eight Science, Technology, and Mathematics modules are as follows:

Upper Primary Science
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Module 3: *My Health*
Module 4: *My Natural Environment*

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Module 2: *Making Things Move*
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Module 4: *Materials*
Module 5: *Processing*

Junior Secondary Technology
Module 1: *Introduction to Teaching Technology*
Module 2: *Systems and Controls*
Module 3: *Tools and Materials*
Module 4: *Structures*

Upper Primary Mathematics
Module 1: *Number and Numeration*
Module 2: *Fractions*
Module 3: *Measures*
Module 4: *Social Arithmetic*
Module 5: *Geometry*

Junior Secondary Mathematics
Module 1: *Number Systems*
Module 2: *Number Operations*
Module 3: *Shapes and Sizes*
Module 4: *Algebraic Processes*
Module 5: *Solving Equations*
Module 6: *Data Handling*

A MESSAGE FROM THE COMMONWEALTH OF LEARNING

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Dato' Professor Gajaraj Dhanarajan
President and Chief Executive Officer

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UPPER PRIMARY MATHEMATICS PROGRAMME

Introduction

Welcome to the programme in Teaching Upper Primary Mathematics. This series of five modules is designed to help you strengthen your knowledge of mathematics topics and acquire more instructional strategies for teaching mathematics in the classroom.

Each of the five modules in the mathematics series provides an opportunity to apply theory to practice. Learning about mathematics entails the development of practical skills as well as theoretical knowledge. Each topic includes examples of how mathematics is used in practice and suggestions for classroom activities that allow students to explore the maths for themselves.

Each module also explores several instructional strategies that can be used in the mathematics classroom and provides you with an opportunity to apply these strategies in practical classroom activities. Each module examines the reasons for using a particular strategy in the classroom and provides a guide for the best use of each strategy, given the topic, context, and goals.

The guiding principles of these modules are to help make the connection between theory and practice, to apply instructional theory to practice in the classroom situation, and to support you, as you, in turn, help your students to apply mathematics to practical classroom work.

Programme Goals

This programme is designed to help you:

- strengthen your understanding of mathematics topics
- expand the range of instructional strategies that you can use in the mathematics classroom

Programme Objectives

By the time you have completed this programme, you should be able to:

- guide students as they work in teams on practical projects in mathematics, and help them to work effectively as members of a group
- use questioning and explanation strategies to help students learn new concepts and to support students in their problem solving activities
- prepare your own portfolio of teaching activities

The relationship between this programme and the mathematics curriculum

The content presented in these modules includes some of the topics most commonly covered in the mathematics curricula in southern African countries. However, it is not intended to comprehensively cover all topics in any one country's mathematics curriculum. For this, you need to consult your national or regional curriculum guide. The curriculum content presented in these modules is intended to:

- provide an overview of the content in order to support the development of appropriate teaching strategies
- use selected parts of the curriculum as examples of the application of specific teaching strategies
- explain those elements of the curriculum that provide essential background knowledge, or that address particularly complex or specialised concepts
- provide directions to additional resources on the curriculum content

How to work on this programme

As is indicated in the goals and objectives, this programme requires you to participate actively in each module by applying instructional strategies when exploring mathematics with your students and by reflecting on that experience. There are several ways to do this.

Working on your own

You may be the only teacher of mathematics in your school, or you may choose to work on your own so you can accommodate this programme within your schedule. If this is the case, these are the recommended strategies for using this module:

1. Establish a schedule for working on the module. Choose a date by which you plan to complete the first module, taking into account that each unit will require between six and eight hours of study time and about two hours of classroom time to implement your lesson plan. For example, if you have two hours a week available for study, then each unit will take between three and four weeks to complete. If you have four hours a week for study, then each unit will take about two weeks to complete.
2. Choose a study space where you can work quietly without interruption, such as a space in your school where you can work after hours.
3. If possible, identify someone who is interested in mathematics or whose interests are relevant to it (for example, a science teacher in your school) with whom you can discuss the module and some of your ideas about teaching mathematics. Even the most independent learner benefits from good dialogue with others. It helps us to formulate our ideas—or as one learner commented, "How do I know what I'm thinking until I hear what I have to say?"

Working with colleagues

If there are other teachers of mathematics in your school or in your immediate area, then it may be possible for you to work together on this module. You may choose to do this informally, perhaps having a discussion group once a week or once every two weeks about a particular topic in one of the units. Or, you may choose to organise more formally, establishing a schedule so that everyone is working on the same units at the same time, and you can work in small groups or pairs on particular projects.

Your group may also have the opportunity to consult with a mentor, or with other groups, by teleconference, audioconference, letter mail, or e-mail. Check with the local coordinator of your programme about these possibilities so you can arrange a group schedule that is compatible with these provisions.

Colleagues as feedback/resource persons

Even if your colleagues are not participating directly in this programme, they may be interested in hearing about it and about some of your ideas as a result of taking part. Your head teacher or the local area specialist in mathematics may also be willing to take part in discussions with you about the programme.

Working with a mentor

As mentioned above, you may have the opportunity to work with a mentor, someone with expertise in maths education who can provide feedback about your work. If you are working on your own, communication with your mentor may be by letter mail, telephone, or e-mail. If you are working as a group, you may have occasional group meetings, teleconferences, or audioconferences with your mentor.

Resources available to you

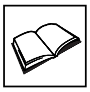
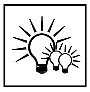
Although these modules can be completed without referring to additional resource materials, your experience and that of your students can be enriched if you use other resources as well. A list of resource materials is provided at the end of each module. You might also find locally available resource material that will enhance the teaching/learning experience. These include:

- manipulatives (other than the abacus that this module describes), such as algebra tiles, geometry tiles, and fraction tiles
- magazines with articles about maths
- books and other resources about maths that are in your school or community library

ICONS

Throughout each module, you will find some or all of the following icons or symbols that alert you to a change in activity within the module.

Read the following explanations to discover what each icon prompts you to do.

	**Introduction**	Rationale or overview for this part of the course.
	**Learning Objectives**	What you should be able to do after completing this module or unit.
	**Text or Reading Material**	Course content for you to study.
	**Important—Take Note!**	Something to study carefully.
	**Self-Marking Exercise**	An exercise to demonstrate your own grasp of the content.
	**Individual Activity**	An exercise or project for you to try by yourself and demonstrate your own grasp of the content.
	**Classroom Activity**	An exercise or project for you to do with or assign to your students.
	**Reflection**	A question or project for yourself—for deeper understanding of this concept, or of your use of it when teaching.
	**Summary**	
	**Unit or Module Assignment**	Exercise to assess your understanding of all the unit or module topics.
	**Suggested Answers to Activities**	

CONTENTS

Module 1: Number and Numeration

Module 1
Number and Numeration

Introduction to the Module

Numbers form the basis for all Mathematics. In the same way we learn letters of the alphabet before learning the vocabulary, we learn the ten basic numerals 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 before learning the operations of addition, subtraction, multiplication, and division. This module is about the whole number system.

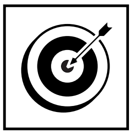
Aim of the Module

The aim of this series of modules is to guide teachers to make teaching of Mathematics simple, enjoyable, and understandable. In particular, this module explores ways in which the foundation of Mathematics can be laid and consolidated in the early stages of learning the subject. It looks at the evolution of numbers. It also explores interesting and practical activities for teaching whole numbers and their operations.

Structure of the Module

There are six units in this module. Unit 1 is basically a historical review of the number systems. In Unit 2, the concept of place value is emphasised. Unit 3 discusses the two operations of addition and subtraction. In Unit 4 we look at multiplication and division operations. In Unit 5, an attempt is made to try to link the knowledge of number operations with problem solving. Unit 6 looks at the enrichment topic of number bases.

The module offers a series of unit and practice activities, which you are encouraged to do in order to consolidate the content and the teaching strategies proposed. Some self assessment exercises are given for you to evaluate yourself. Do them also.



Objectives of the Module

The specific objectives of the module are to:

- help teachers consolidate the basic concepts of whole numbers and number bases
- explore interesting and practical ways of teaching whole numbers
- introduce teachers to problem solving skills

Unit 1: Number Systems

Introduction

This unit introduces the number systems used by various civilisations in the past. By understanding the need for number systems and how these needs have changed over time, it is hoped that your ability to teach number systems will be strengthened.

Objectives

At the end of this unit you should be able to:

- use number systems from a variety of civilisations
- teach the concept of number from a utilitarian point of view
- use projects to enable pupils to develop their knowledge of numeration

Practice Activity 1

Organise your pupils to play a game. Ask six pupils to stand at the back of the classroom and pretend they are visitors from a country with a different number system. Explain to the rest of the class that these visitors do not know your number system, and you do not know theirs.

Without using any of the number systems they already know, pupils must find a way to communicate the following information to the visitors:

a) the number of pupils who sit at one desk
b) the number of pupils in the class
c) the number of pupils in the whole school

Let the pupils work in groups of convenient sizes to allow all of them to participate actively. Let pupils work on the activity for about an hour, but they can go beyond that time depending on the progress they make. Have the groups present their plans to each other. The explanations should include how numbers will be represented and how the numeration system will work. Guide the discussion to identify advantages and disadvantages of each group's proposed system and jot down the ideas for use after you have worked through this unit.

Points arising from the discussion might include the fact that the proposed numeration system is not clear or that it is cumbersome. The essence should be to provoke critical thinking on number and numeration. Pupils should hold on to their work for further development.

Numbers and Numerals

Your experience with the above activity should have provided you with ideas on the importance of having numeration systems, which have an agreed way of writing and clear rules on combining the numbers. These ideas will be useful in discussing the various numeration systems that are presented later in this unit.

Reflection

What is the difference between a number and a numeral?

Note
Various groups of pupils in your class probably came with very different ways of expressing the number of pupils sitting on one desk. Take for example, that you have two pupils per desk. With each number system, the 'two' was written in different *numerals*, but it is the same *number*—'two'.

Keeping Records

One of the problems you most likely faced with the activity you did with your class is that the numbers were represented in a clumsy way. Early numeration systems had the same problems. The earliest forms of counting used two ideas only—one thing and many things. For, example, if a person had four children, the children were considered to be many. If the children were seven, they were still said to be many.

Are there people in your community who still count in terms of one thing and many things?

Reflection

Literacy and numeracy are still big problems in sub-Saharan Africa. You will, therefore, most likely have people count in terms of one thing and many things. But such people, as has been the case for many generations, have their own ways of keeping records. *Figure 1.1* illustrates a simple way to keep records of cows.

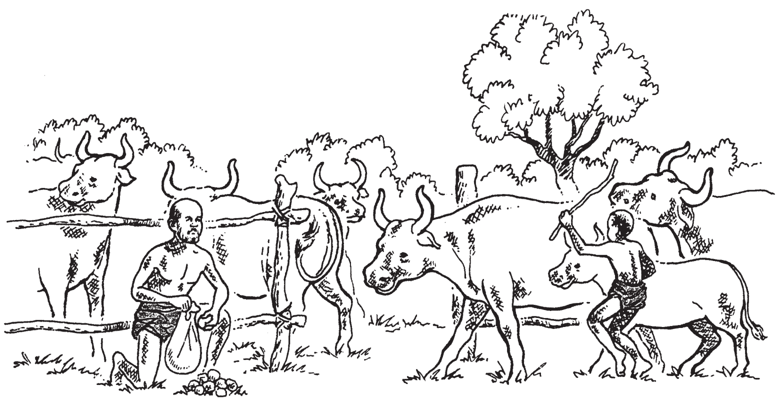


Figure 1.1: An ancient way of keeping records of cattle.

The record keeping system in *Figure 1.1* involves picking up one stone for each cow. By keeping the stones in one place, the owner of the cows has a record of the number of cows in his herd. The next time the cows are counted, one stone is simply removed from the pile for each cow.

Reflection

What do you think the following statements mean?

1. The number of stones is greater than the number of cows.
2. The number of cows is greater than the number of stones.
3. The number of cows and the number of stones are the same.

If you assumed that the ‘counting’ was accurate, the number of stones being greater than the number of cows means that some cows are missing. If the number of cows is greater than the number of stones, it means some cows have strayed into the herd. Should the number of cows and the number of stones match, it means each cow has been accounted for.

Of course, as our ancestors found out, stones are not very practical if you have many things to count. Stones are heavy to carry and they are easy to lose. People invented the tally stick to solve this problem.

A tally stick is piece of wood in which cuts have been made to represent the number of things recorded. In *Figure 1.2(a)*, the six cuts on the tally stick represent, for example, six cows.

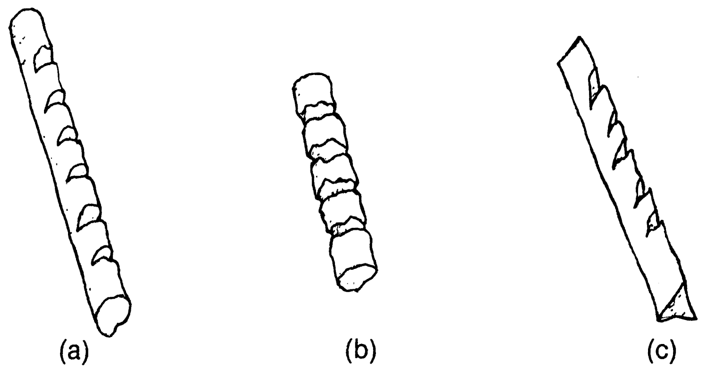


Figure 1.2: Tally stick for keeping records

You see that *Figure 1.2(b)* represents four cows and *Figure 1.2(c)* represents five cows. Your pupils will see that this is still a cumbersome way of maintaining records.

The tally stick system can be extended so one type of notch represents ten while another type represents one. Yet tally sticks are a very cumbersome way of keeping records. They are time consuming and difficult to change as the number of items being recorded changes.

Practice Activity 2

Explore the methods people in your locality used to keep their records. Have your pupils gather the information. Pupils should relate this to what they did in the practice activity, keep revising their ideas, and develop a project under your supervision.

Early Number Systems

From the earliest ways of keeping records, people eventually invented signs or symbols to represent numbers of the things they wanted to count. These symbols, as we said earlier, are called *numerals*.

The Egyptian System (about 3400 BC)

The Egyptian system was a step forward in representing numbers. The only principles used were those of repetition and addition. The symbol for one, called a *staff* and similar to the present day numeral 1, was repeated an appropriate number of times to indicate a number less than 10. For ease of recognition, no more than four of the same objects were grouped together as illustrated below:

|	1
| |	2
| | |	3
| | | |	4
| | | | |	5
| | | | | |	6
| | | | | | |	7
| | | | | | | |	8
| | | | | | | | |	9

To present the number 10, the Egyptians selected the symbol ∩, called the *heelbone*. Any number less than 100 was represented as combination of heelbones and staffs as follows:

∩	10
∩ | |	12
∩∩ | | | |	24
∩∩∩ | | | |	34

To represent 100 or ten heelbones, another symbol, 𓍢, called the *scroll* was invented. Now the number 134 could be represented as 𓍢∩∩∩ | | | |. The

Egyptians thus used two principles in writing numerals, *repetition* and *addition*. The principle of repetition was used in representing a number, such as 3, by writing the same symbol three times. The principal of addition was understood in that the numeral 𓎆𓏺𓏺 represented 10 + 2 or 12. The system did not have place value and, therefore, the order in which the numerals were written did not matter. For example, 𓎆𓎆𓏺𓏺𓏺𓎆𓏺 = 10 + 10 + 3 + 10 + 1 = 34.

The Egyptian system in which numbers from 1 to 9 999 999 could be represented involved the seven different symbols shown below:

Modern Day	**Symbols**	**Name**
1	𓏺	Staff
10	𓎆	Heelbone
100	𓍢	Scroll
1 000	𓆼	Lotus flower
10 000	𓂭	Pointed finger (bent line)
100 000	𓆐	Burbot (fish)
1 000 000	𓁨	Astonished person

Reflection

In your view, what are the merits and demerits of the Egyptian numeration system?

The merits you have identified in the Egyptian system may include the fact that it was economical, e.g., the fact that it had a single symbol for 100, 101, 102, … 106. Other advantages include the facts that the symbols were taken from their local environment and they had an additive property. On the demerits side, the system did not have place value and, therefore, was not very flexible for operations like addition, subtraction, multiplication, and division.

Self Assessment 1

What do the following Egyptian numerals represent in the modern day numeration system?

𓎆𓏺𓏺𓏺𓏺

𓍢𓎆𓎆𓎆𓏺𓏺𓏺

𓎆𓎆𓎆𓎆𓏺𓏺𓏺

𓎆𓏺𓎆𓏺𓎆𓏺𓎆

𓂭𓆼𓆼𓍢𓍢𓍢𓎆𓏺𓏺𓏺𓏺𓏺𓏺

𓆼𓍢𓍢𓍢𓍢𓍢𓍢𓎆𓎆𓏺𓏺

Write your own answers, then check them against the given answers. Answers to most self assessments are at the back of each unit.

The Babylonian System (about 3000 BC)

In the Babylonian civilisation, in the Middle East, people wrote numerals on clay or wood. It was a simple system with two symbols only—a vertical wedge ∇ to represent 1 and the symbol ≺ to represent 10. To write numbers 1 to 59, they made additive use of these symbols. One example is ≺≺≺∇∇ which meant 32.

One way in which the Babylonian system was an advancement on previous systems is that it had “place value”. The concept of place value is developed further in Unit 2. The different *places* in the set of numerals represented multiples of 60. The various places were indicated by wider spaces between the characters or by changing the order of ∇ and ≺. Should ∇ appear to the left of ≺, it meant that ∇ was in a new *place*. Study the examples in the table below:

Symbols	**Value**
≺≺≺≺∇∇∇	43
∇ ≺≺∇∇∇	$1 \times 60 + 23 = 83$
≺∇∇ ≺∇	$12 \times 60 + 11 = 720 + 11 = 731$
∇∇ ≺≺ ≺≺∇∇	$2 \times 60 \times 60 + 20 \times 60 + 22 = 8422$
∇∇ ≺≺≺	$2 \times 60 + 30 = 150$

Reflection

What do you consider to be the advantages and disadvantages of the Babylonian numeration system?

One aspect you could consider to be positive for the Babylonian is the simplicity of the symbols and the rules. More significantly, the place value system makes it more versatile. The demerits include the lack of 0. This means for a system which has place value, ∇∇ could mean 2, it could also mean $2 \times 60 + 0$, $2 \times 60 \times 60 + 0 + 0$ and so on.

Self Assessment 2

What do the following numerals represent in the modern day system?

≺≺≺∇∇∇∇∇

≺   ≺≺∇∇∇

≺≺∇   ≺∇∇   ∇

∇   ≺∇   ≺∇   ∇∇∇∇

Write each of the numbers above in Egyptian system.

The Roman System (AD 100)

The Roman system of numeration was an improvement over the Egyptian system in certain respects. Fewer symbols were necessary to represent many numbers. For example, the Roman symbol V replaced the Egyptian $|\,|\,|\,|\,|$ for representing five. Along with the principles of repetition and addition used by the Egyptians, the Romans used the principle of subtraction. Subtraction was used to represent numbers involving fours and nines by placing a smaller number before a larger number, as in IX for 9, XL for 40 and XC for 90. The Romans developed what is called a quinary binary base system, that is five 1's are represented by V, two V's by X, five X's by L and two L's by C.

The Roman system is still in use today, especially on some clocks, in outlining written work, in numbering pages, and in the dates on the cornerstones of buildings and films (movies).

The convention adopted for representing subtraction needs to be explained. The number 95, for example, was not expressed as VC because the numeral V was never used to indicate subtraction. Rather such numbers are expressed in terms of tens and hundreds (powers of 10).

In the number 95, there are nine tens, so the notation would be XC (100 – 10) for 90, to which is added V and would be written XCV. Similarly, 995 is represented as 900 + 90 + 5 or CMXCV rather than as 1000 – 5 or VM. Here is an outline of some Roman numerals:

Present	Roman	Present	Roman	Present	Roman
1	I	7	VII	21	XXI
2	II	8	VIII	50	L
3	III	9	IX	100	C
4	IV	10	X	500	D
5	V	11	XI	1000	M
6	VI	20	XX		

Self Assessment 3

Write the following numerals in the modern day system:

XLIV
LXXVI
MCMXC
DCXLVII
MDXXI

Write the following numbers in Roman numerals:

18
94
256
1999
2112

The Hindu-Arabic Numeration System (about 800 AD)

Our present day numeration system has its origins from the Hindus in India in about 800 AD. The Arabs adopted the system and improved on it up to the stage we know it today—hence the name Hindu-Arabic system. The Hindu-Arabic system is also called the decimal system. Why do you think it is called the decimal system?

You are already aware that the modern day numeration system (Hindu-Arabic) has ten basic symbols called digits. These are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. The system has the following notable characteristics:

- each symbol represents a *cardinal* number (cardinal numbers are basic numbers from which all other numbers are constructed, that is, the numbers 0 to 9)
- the position of the symbol in the numeral has a place value with the place values based on repeated groupings of ten
- there is a numeral for zero. This is a major advance in numeral systems. As far as we know, none of the previous civilisations realised that "none" is a number. The Hindu civilisation grasped this concept, and so added a numeral to its system to represent the number "none". We call that numeral "zero" or "0". After thousands of years of changing *numeral* systems, this is the first major change in a *number* system.

Just as a reminder, you can represent the place value with a number such as 6025 as in *Figure 1.3* below (the idea of place value is developed further in Unit 2).

Thousand (Th.)	Hundreds (H)	Tens (T)	Units (U)
6	0	2	5

Figure 1.3

Note that in expanded notation, 6025 can be written as:

6×1000	=	6000
0×100	=	0
2×10	=	20
5×1	=	5
		6025

Unit Activity 1

Now that you have had some practice using various numeration systems and trying related activities with your pupils, compare and contrast the effectiveness of the systems described in this unit—including the system that was, or still is, used in your area.

Engage your pupils in a discussion about numeration systems and write a comprehensive profile of their ideas—before, during, and after the lessons in this unit. Based on your pupils' comments, will you make any changes to the way you teach future lessons on this topic? Keep your profile in a folder as evidence of your work in this unit.

Summary

In this unit you have learned that:

- The need to keep records and communicate quantity gives rise to numbers and numerals.
- Numerals are symbols that represent numbers, e.g., III, ǀǀǀ, 3, ∇∇∇ are different numerals representing the same number, three.
- Early numeration systems had varying deficiencies, mostly due to the lack of place value and a numeral for the number zero.

You also learned the difference between a number system and a numeral or numeration system:

- A number system is a set of mental constructs for describing or measuring quantities. For example, “three” and “twelve” are members of the counting number system. Three cows in a corral are described by the number (the quantity or concept) “three”, regardless of the words, symbols, sticks, stones, numerals, or set of fingers that any humans might use to count the cows.
- A numeral system or numeration system is a way of recording and manipulating numbers for the purpose of record-keeping or communicating to others. Whereas “three” is a quantity or number, “III” and “3” are numerals that one can use to record that quantity or number on paper—depending on the numeration system that is appropriate to use in your society.

Perhaps you are thinking of teaching this number-numeral distinction to your upper primary Maths class. If so, think again. To distinguish between number systems and numeral systems requires a degree of abstract thinking beyond most young people. Some students will understand the difference between numbers and numerals, but many will be unable to grasp the distinction, no matter how carefully you explain it. The best they could do is memorize the definitions and repeat them on a test.

Since the ability to differentiate between numbers and numerals is not required for numeracy in a technical society, you may prefer to sidestep the distinction in your class. For example, the following test (which you should complete for practice) exercises a student’s ability to use different numeral systems, without asking what a numeral system is. This is the approach we recommend for assessing your students in Upper Primary Maths.

Unit 1 Test

1. Which numerals were represented by the following:
   a) Babylonian numeral ≺≺
   b) Roman numeral IX
   c) Egyptian numeral ∩|||

2. How would an Egyptian have written ≺?

3. How would a Babylonian have written ∩||?

4. What numbers do the following Arabic numerals represent in each of the following systems: Babylonian, Egyptian and Roman?
   a) 8
   b) 303
   c) 769
   d) 174

5. For each of the following, tell which numeral represents the greater number and why?
   a) CDXXIV and CDXXVI
   b) 4,632 and 46,032
   c) 𓍢𓍢𓍢∩∩|| and 𓆼∩|

6. Lorata had a dream in which she was selling diamonds on an international market. She started out with 120 diamonds. The first customer was an Egyptian who bought ∩|||||| diamonds. Next was a Babylonian who bought ≺≺≺∇∇∇∇∇∇∇ of them. A Roman bought XXIV, and a Hindu-Arabic person bought the remaining diamonds. How many diamonds did the Hindu-Arabic person buy?

Unit 1: Answers to Self Assessments

Self Assessment 1

∩|||| - 14

𓍢∩∩∩||| - 133

∩∩∩∩||| - 43

∩|∩|∩|∩ - 43

𓂭𓆼𓆼𓍢𓍢𓍢∩|||||| - 12 316

𓆼𓍢𓍢𓍢𓍢𓍢𓍢∩∩|| - 1 622

Self Assessment 2

≺≺≺∇∇∇∇∇ - 35

≺   ≺≺∇∇∇ - 10 × 60 + 23 = 623

≺≺∇   ≺∇∇   ∇ - 21 60 60 + 12 60 + 1 = 76 321

∇   ≺∇   ≺∇   ∇∇∇∇ -

1 × 60 × 60 × 60 + 11 × 60 × 60 + 11 × 60 + 4 = 256 264

In Egyptian system:

∩∩∩|||||

𓍢𓍢𓍢𓍢𓍢𓍢∩∩|||

𓂭𓂭𓂭𓂭𓂭𓂭𓂭𓆼𓆼𓆼𓆼𓆼𓆼𓍢𓍢𓍢∩∩|

𓆐𓆐𓂭𓂭𓂭𓂭𓂭𓆼𓆼𓆼𓆼𓆼𓆼𓍢𓍢∩∩∩∩∩∩||||

Self Assessment 3

In modern day system:

44
76
1990
647
1521

In Roman numerals:

XVIII
XCIIII
CCLVI
MCMXCVIIII
MMCXII

Answers to Unit 1 Test

1. a) 20, XX, ∩∩.
   b) 9, | | | | | | | | |, ∇∇∇∇∇∇∇∇∇.
   c) 13, XIII, ≺∇∇∇.

2. ∩
3. ≺∇∇

4. What numbers do the following Arabic numerals represent in each of the following systems Babylonian, Egyptian and Roman?
   a) ∇∇∇∇∇∇∇∇, | | | | | | | |, VIII.
   b) ∇∇∇∇∇ ∇∇∇, 𓍢𓍢𓍢| | |, CCCIII
   c) ≺∇∇ ≺≺≺≺∇∇∇∇∇∇∇∇∇,
   𓍢𓍢𓍢𓍢𓍢𓍢𓍢∩∩∩∩∩∩| | | | | | | | |,
   DCCLXIX
   d) ∇∇ ≺≺≺≺≺∇∇∇∇, 𓍢∩∩∩∩∩∩∩| | | |, CLXXIIII

5. a) CDXXVI represents 426, the greater number, because VI adds one to five rather than subtract one from five as in IV, the first numeral.
   b) 46 032 represents the greater number because it has an extra 0, thus multiplying the numeral by a value of 10.
   c) 𓆼∩| represents the greater Egyptian number because the 𓆼 symbol's value is 1000 whereas the value of the 𓍢 symbol is only 100. Thus the first numeral represents only 322, not 1101, the second numeral.

6. 45 diamonds.

Unit 2: Place Value and Order of Numbers

Introduction

In Unit 1 you learned about different numeration systems. Of the many numeration systems developed throughout the history of mathematics, the Hindu-Arabic system proved superior over the other systems because of the use of the **place value**, the **decimal system**, and the use of zero as a **place holder**. The place value system made it possible for numbers of any size to be written using only the ten numerals 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. The place value system also made it easier to add, subtract, multiply, and divide numbers.

In this unit, you will learn about the place value and the order of numbers in the Hindu-Arabic system and the strategies to teach place value.

Objectives

By the end of this unit, you should be able to:

- make a simple abacus and use it to represent numbers from one up to 9 999 999
- make and use place value cards to represent numbers
- read and write numbers in numerals and words from ten to millions
- plan and teach lessons on place value
- order numbers using less than (<) and greater than (>) symbols

Place Value

The Hindu-Arabic numeral system uses the place system to write numbers using the numerals 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. The decimal system that we use in counting puts numbers in groups of ones, tens, hundreds, thousands, ten thousands, hundred thousands, millions, etc. For example, the number 7 246 528 represents:

7 Millions
2 Hundred Thousands
4 Ten Thousands
6 Thousands
5 Hundreds
2 Tens
8 Ones

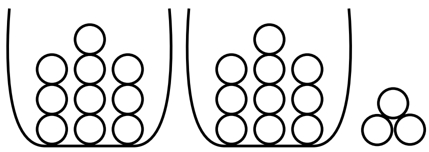
i.e.	M	H Th	T Th	Th	H	T	O
	7	2	4	6	5	2	8

The number 7 246 528 is read as:

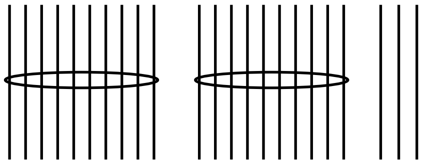
"Seven million two hundred and forty-six thousand five hundred and twenty-eight." The idea of place value is introduced to pupils in the early grades, the time when their number system is extended from 10 to 20. By the end of the second grade or so, pupils should have learned numbers up to 99. Thereafter, one place value after another is introduced systematically. Two **models** are used to develop the categories—**proportional** and **non-proportional** models.

Proportional Models

A proportional model is one in which a Ten model is physically 10 times larger than the model for One, and a Hundred model is 10 times larger than the Ten model. The proportional models for base ten are further divided into two categories, **groupable** models as illustrated in *Figures 2.1(a)* and *2.1(b)* and **pregrouped** models as illustrated in *Figures 2.2(a)* and *2.2(b)*.



Counters and cups
Figure 2.1 (a)



Bundles and sticks
Figure 2.1 (b)

Each model in *Figure 2.1* represents the number 23, i.e., 2 Tens and 3 Ones

Pregrouped models

Figures 2.2(a) and *2.2(b)* show pregrouped models.

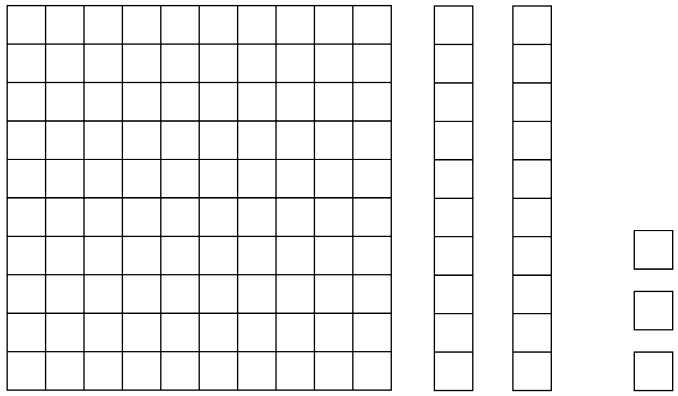


Figure 2.2(a): Place value cards: representing the number 123

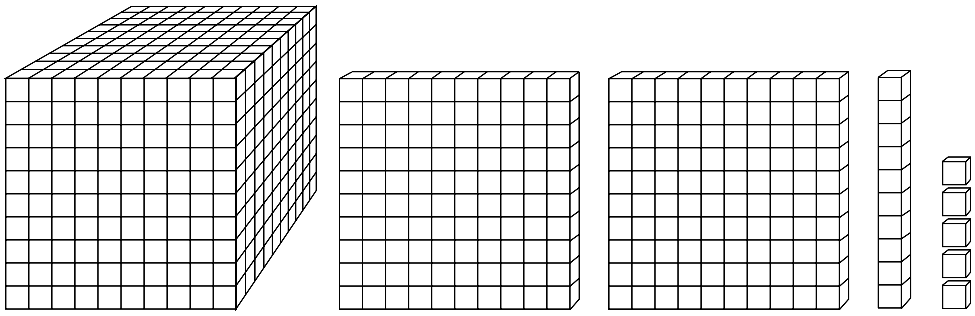


Figure 2.2(b): Place value blocks: representing the number 1215.

Place value blocks are sometimes called **Dienes Blocks,** named after the famous educational psychologist Zoltan Dienes.

Non-Proportional Models

Figures 2.3(a) and *2.3(b)* show non-proportional models.

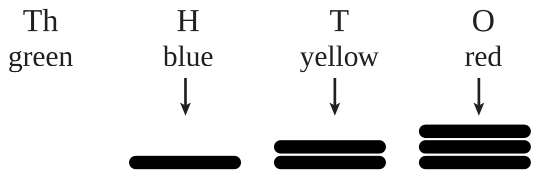


Figure 2.3(a): Chip-trading material showing 123

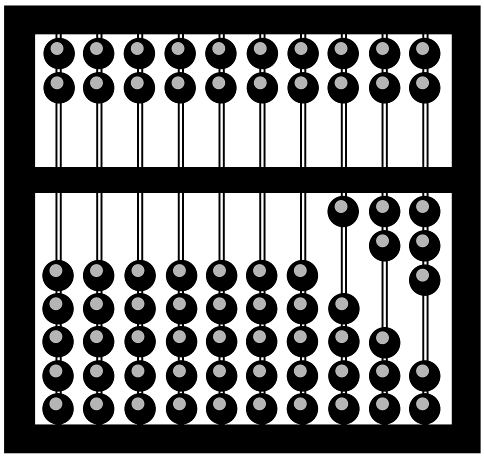


Figure 2.3(b): Chinese abacus representing 123

In non-proportional models for base 10, the models for Ones, Tens, Hundreds, etc., are not proportional. For example, in the chip-trading materials in *Figure 2.3(a)*, 10 red chips representing Ones are traded for 1 yellow representing a Tens, 10 yellow chips are traded for 1 blue chip representing Hundreds and 10 blue chips are traded for 1 green chip representing Thousands. Otherwise the chips are of the same size.

The Chinese abacus in Figure 2.3(b) is still in use in the Orient, but being quinary-binary based (like Roman numerals) it does not work too well for teaching ordinary base-ten arithmetic. We will use a simpler model which we simply call an “abacus”.

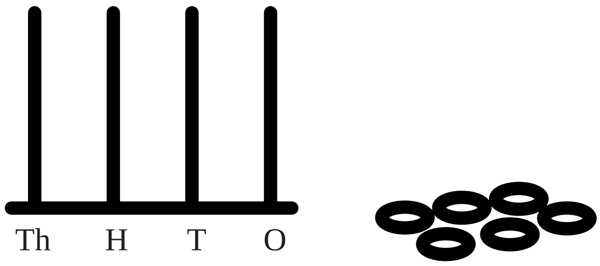
In the abacus, all counters are the same size. Ten counters on the peg for Ones will be traded for 1 counter on the peg for Tens; ten counters on the peg for Tens will be traded for 1 counter on the peg for Hundreds, etc.

Self Assessment 1

1. There are two categories of models for representing numbers in base ten, proportional and non-proportional models. Explain the difficulty pupils may experience in learning the concept of place value using the non-proportional models as compared to the proportional models.
2. The two proportional models for representing numbers in base ten are groupable and pregrouped models.
   (a) Explain the difference between groupable and pregrouped models.
   (b) Which do you think are easier to use in explaining the concept of place value, the groupable or the pregrouped models?
   (c) Which do you think are more convenient to use in a teaching situation, the groupable or pregrouped models?

The Abacus

The abacus in *Figure 2.4* can be used to represent numbers from 1 up to 9999.



Wooden base with vertical wooden or wire rods

Counters (These could be bottle tops, washers, wood cuttings, or beads)

Figure 2.4

Unit Activity 1: Making an abacus

Make an abacus for representing numbers up to 9 999 999. The picture of it is shown in *Figure 2.5*.

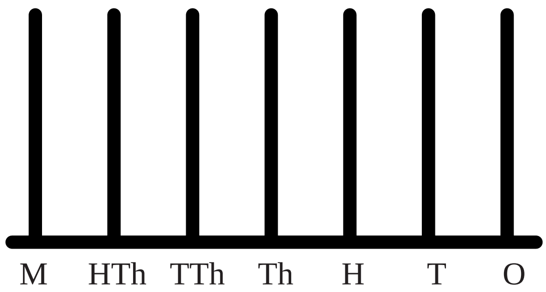


Figure 2.5

Continues on next page.

What you need

- A strip of wooden board about 1 cm high, 4 cm wide and 30 cm long.
- Seven wooden or wire rods, of uniform size of about 15 cm long.
- 63 counters (bottle tops, soft wood cuttings, washers, eyelets, or beads).

What you do

Leaving a space of 2 cm on either side of the wooden strip, fix seven rods in a straight line approximately 4 cm apart. Ideally, each rod should hold nine counters but not ten.

Mark the columns O(Ones), T(Tens), H(Hundreds), Th(Thousands), TTh(Ten thousands), HTh(Hundred thousands), M(Millions). Make six of these abacuses for your class.

Introducing the Abacus

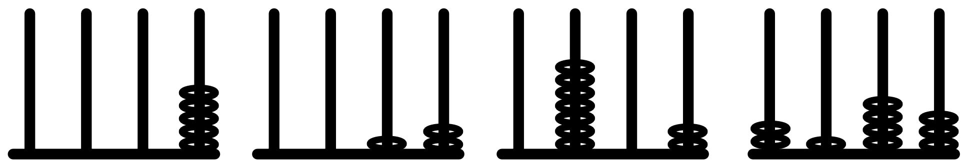
For a class of fifty pupils you require six abacuses, each with thirty counters. Put your pupils in five groups of ten and give each group an abacus. Keep one abacus for demonstration. Lead the groups in counting up to one hundred.

Start counting from one, each time placing a counter on the peg for Ones. When you get ten counters you trade (exchange) them for one counter on the peg for Tens. Continue with the process of counting and trading until you have ten counters on the peg for Tens, which you should trade for one counter on the peg for Hundreds. Pupils require a lot of practice on this.

Once the pupils are familiar with the abacus through the process of counting and trading, introduce them to direct presentation of numbers on the abacus.

Example 1

Represent each of the following numbers on an abacus: 5; 12; 702; and 2143.



(a) 5 (b) 12 (c) 702 (d) 2143

In the number 2143: 2 represents Thousands
1 represents Hundreds
4 represents Tens
3 represents Ones

In words, we write the number 2143 as: Two thousand one hundred and forty-three.

Expanded form:

The numbers 5, 12, 702 and 2143 can be written in expanded form as follows:

$5 = 5 \times 1$
$12 = (1 \times 10) + (2 \times 1) = 10 + 2$
$702 = (7 \times 100) + (0 \times 10) + 2 \times 1) = 700 + 00 + 2$
$2143 = (2 \times 1000) + (1 \times 100) + (4 \times 10) + (3 \times 1) = 2000 + 100 + 40 + 3$

Common Errors

Children usually make mistakes when writing the number "Three hundred and four" as 3004 instead of 304. The use of an abacus to represent numbers will help remedy this problem.

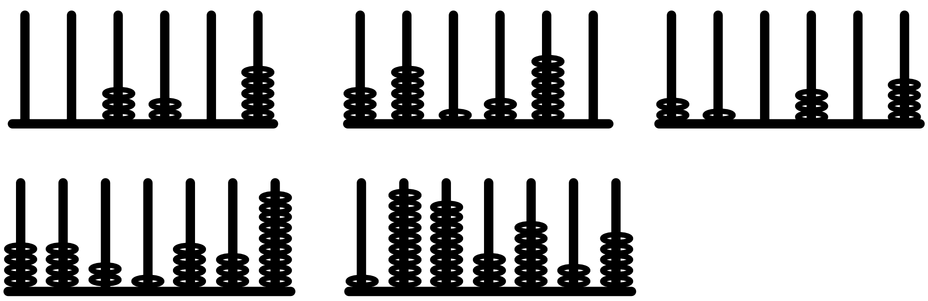
Give pupils numbers like 105, 201, 405, and ask them to represent the numbers on the abacus. Let them write the numbers in words as they represent them on the abacus.

After several practice exercises of this type, give them numbers written in words, and ask them to represent the numbers on the abacus. Let them write the numbers in figures as they represent them on the abacus.

Now give pupils examples of the same kind represented on the abacus, and ask them to write the number down in words and in figures.

Self Assessment 2

1. Show the following numbers on an abacus in a drawing
   (a) 1009 (b) 10 450 (c) 27 314 (d) 324 532
2. Write the numbers in question 1 in expanded form.
3. Write the numbers in question 1 in words.
4. Write the numbers represented on the abacuses below in
   (a) Words (b) Numerals



Unit Activity 2: Making place value cards

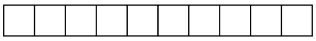
What you need:

- Manilla or any hard paper. You may use ordinary writing plain paper but it is not durable.
- Ruler and pencil
- Scissors

What to do:

Draw ten squares of 1 cm by 1 cm and cut them out.

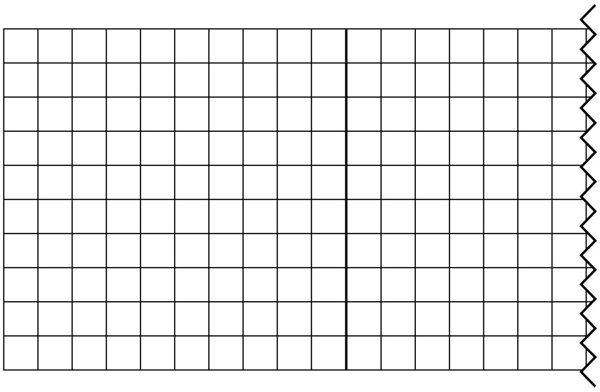
Draw ten strips, each with ten squares of 1 cm by 1 cm, and cut them out.



A strip of ten squares

Draw ten large squares, each made up of ten strips, and cut them out.

Make ten long strips, each made up of ten large squares.

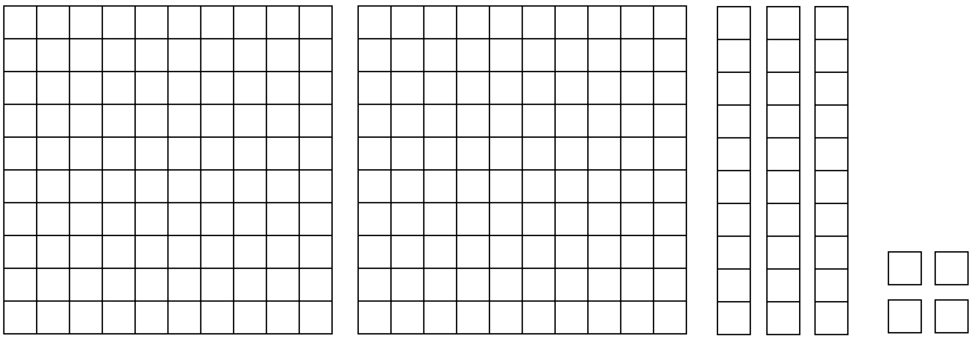


Part of a long strip with ten large squares.

You can use these cards to represent numbers up to 9999.

Example 2

Represent the number 234 using place value cards, and write the number in expanded form and in words.



The number 234 can be written in two other forms as follows:

Expanded form:
$234 = (2 \times 100) + (3 \times 10) + (4 \times 1) = 200 + 30 + 4$

In words:
Two hundred and thirty-four.

Self Assessment 3

1. Represent the following numbers using place value cards:
   a) 109 b) 450 c) 1273 d) 322 e) 739
2. Write the numbers in question 1 in words and expanded form.

Self Assessment 4

1. Draw pictures of place value blocks to represent the following numbers:
   a) 1720 b) 672 c) 3254
2. Write the numbers in question 1 in:
   a) Expanded form b) Words
3. Write the following numbers in numerals:
   (a) Five hundred and seventy-two
   (b) Two thousand seven hundred
   (c) Twenty three thousand four hundred and four
   (d) Eight thousand and twenty-two
   (e) Sixteen thousand one hundred

Practice Activity 1

1. Design practical activities for your pupils to make the following models appropriate to the grade levels you teach, Grade 5, Grade 6, or Grade 7.
   a) Abacuses b) Place value cards c) Place value blocks
2. Do the following activities with your pupils in class.
   - Arrange your class in five groups and let each group sit in a round-table formation.
   - Give each group an abacus.
   - A team leader for each group should be recording the group's work.
   - Write the following exercise on the board and let each group do it. First give an example to show how a number, e.g., 1275 can be represented on an abacus.

Continues on next page.

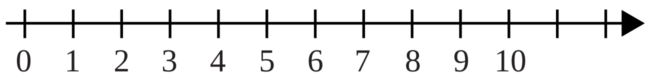
Exercise for pupils

1. Show the following numbers on abacuses:
   a) 7 b) 48 c) 252 d) 2034 e) 42 503
2. Write the following numbers in:
   (a) Expanded form
   (b) Word form
       (i) 315 (ii) 2582 (iii) 27 364 (iv) 14 000 (v) 3008
3. Write the following numbers in numerals:
   (a) Six hundred and eight.
   (b) Four thousand three hundred and twenty three.
   (c) Twenty-three thousand four hundred and eight.
   (d) Five hundred and twenty-one thousand seven hundred and four.
   (e) Three thousand eight hundred and two.
4. Draw diagrams to represent the number 1432 using:
   (a) Place value blocks (b) Place value cards.

Note
Record your pupils' experiences in this practice activity.

Order of Numbers

If we were to represent numbers on a line, we would have the following:



The line on which numbers are represented is called a **number line**. The numbers on the number line are arranged in order of size starting with the smallest on the left and continuing to the right.

Of the numbers 5, 12, 702 and 2143, the number 5 is the smallest and the number 2143 in the largest.

The number 702 is smaller (or less) than the number 2143 but it is larger (or greatest) than 12.

We use the symbol ‘<’ to mean ‘less than’;

And the symbol ‘>’ to mean ‘greater than’.

Therefore, the sentence “702 is greater than 12” can be written as “702 > 12”; and the sentence “702 is less than 2143” can be written as “702 < 2143”.

Examples

Complete the blank spaces in each of the following pairs of numbers by using either < or > symbol:

(a) 72…27 (b) 132…432 (c) 1001…1 000 101 (d) 7542…899

Solutions

(a) 72 > 27 (b) 132 < 432 (c) 1001 < 1 000 101 (d) 7542 > 899

Self Assessment 5

1. Put either < or > in place of the blank spaces:

   (a) 15…25 (b) 631....482 (c) 111…1111 (d) 7564…17 564
   (e) 801…900 809…100

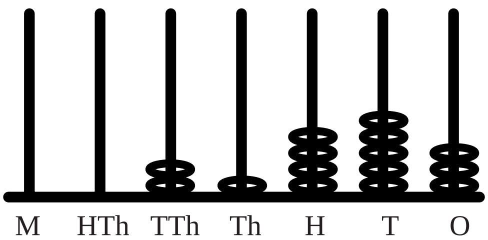
1. Use either < or > to fill in the blank spaces:

   (a) 440…44 (b) 251…521 (c) 222…2222 (d) 15 000…15 001
   (e) 781…871 (f) 417 019…420 (g) 102 000…1 000 001

Summary

In this unit you have learned about the place value system and order of numbers. You have written numbers in expanded form and in words. You have learned how to represent numbers using the abacus, place value cards, and place value blocks.

- The number 21 453 can be represented on an abacus as follows:



- The number 21 453 can be written in expanded form as follows:

  $$\begin{aligned} 21\,453 &= (2 \times 10\,000) + (1 \times 1000) + (4 \times 100) + (5 \times 10) + (3 \times 1) \\ &= 20\,000 + 1\,000 + 400 + 50 + 3 \end{aligned}$$

- The number 21 453 can be written in words as:
  Twenty-one thousand four hundred and fifty-three.
- The sentence “27 is less than 127” can be written as “27 < 127”.
- The sentence “1 000 000 is greater than 100 000” can be written as “1 000 000 > 100 000”

Reflection

In this unit you have been presented with three methods of representing numbers: the abacus, place value cards, and the place value blocks.

1. Which of the three methods do you find most convenient to represent numbers?
2. What are the advantages of using the abacus?
3. What are the disadvantages of using the place value cards?

Unit 2 Test

1.

HTh	TTh	Th	H	T	O
6	7	5	4	3	2

Write this number in:

(a) words

(b) expanded form

(c) numerals

2. (a) What is the value of the digit 7 in each of the following numbers?

(i) 7245 (ii) 627 (iii) 5743 (iv) 3471 (v) 570

(b) What is the value of the digit 7 in each of the numbers given in (a) if each number is multiplied by 1000?

3. Represent the number 2145 using:

(a) an abacus (b) place value cards.

4. What is the value of 3 in each of the following numbers if all the zeros are removed?

(a) 530 534 (b) 370 002 (c) 560 003 (d) 3 000 000

5. Fill in the blank spaces using either < or >:

(a) 25…75 (b) 101…1001 (c) 99…90 (d) 10 090…10 009

(e) 1 000 001…1 001 000

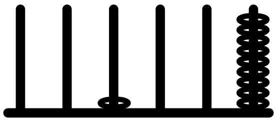
Unit 2: Answers to Self Assessments

Self Assessment 1

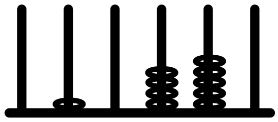
1. (a) Groupable models are models that can be rearranged to form new numbers. For example, with 15 sticks you can represent any number from 1 to 15. Pregrouped models cannot be rearranged to form new numbers. For, example, a place value strip representing 10 cannot be rearranged to form other numbers, unless you cut the strip into smaller squares of one.

Self Assessment 2

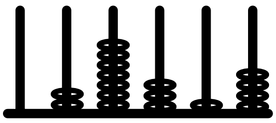
1. (a)



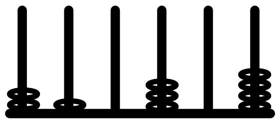
(b)



(c)

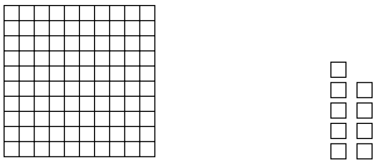


(d)

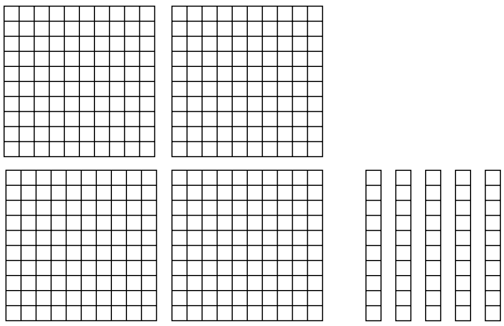


Self Assessment 3

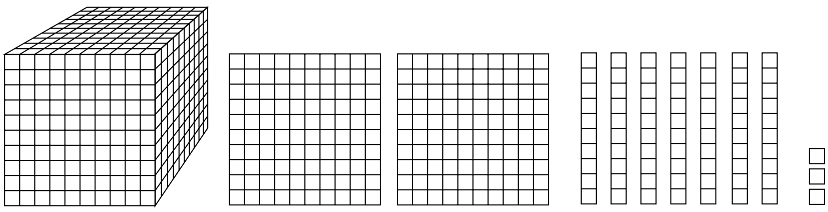
1. (a)



(b)



(c)



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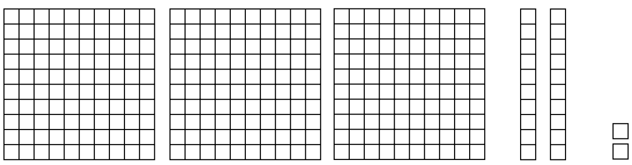
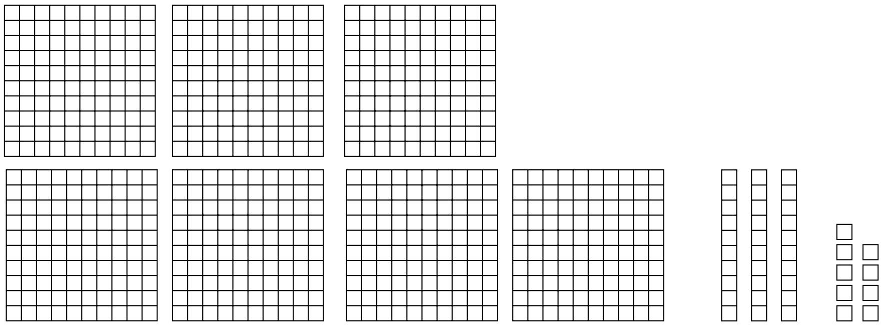
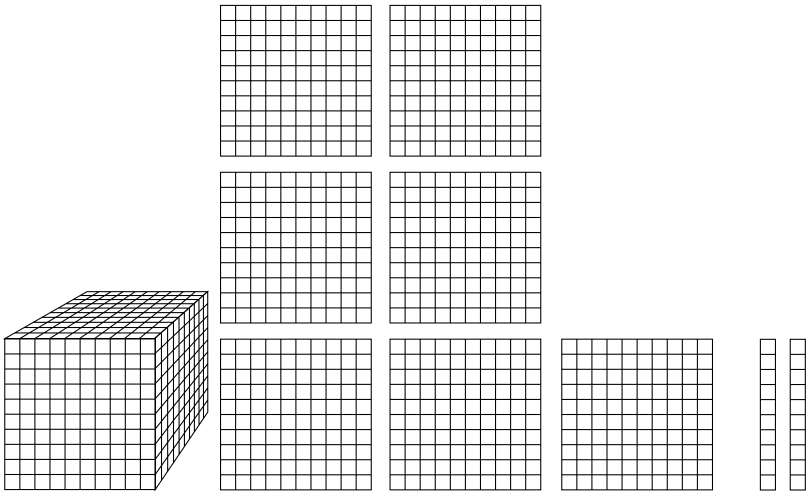
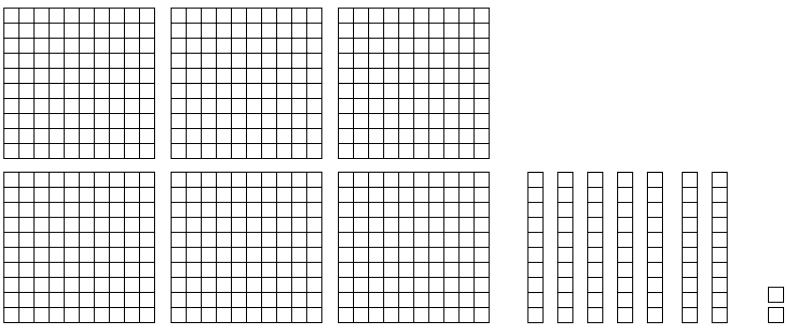


(e) 

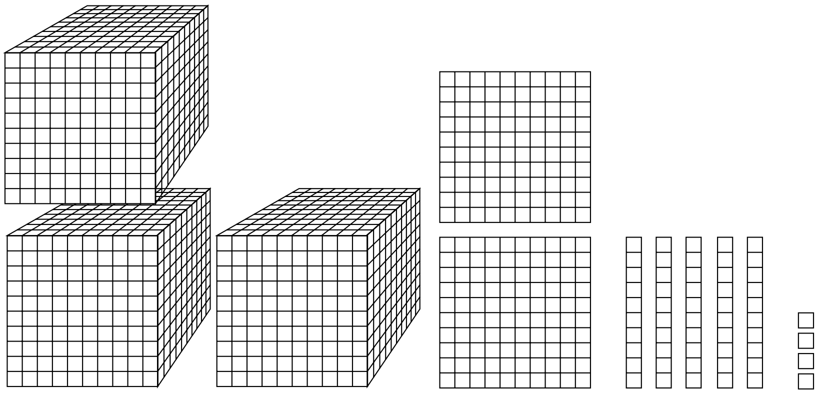
2. (a) 109 = 100 + 0 + 9: One hundred and nine.
   (b) 450 = 400 + 50 + 0: Four hundred and fifty.
   (c) 1273 = 1000 + 200 + 70 +3:
   One thousand two hundred and seventy three.
   (d) 322 = 300 + 30 + 9: Three hundred and twenty two.
   (e) 739 = 700 + 30 + 9: Seven hundred and thirty nine.
3. (a) 572 (b) 2,7

Self Assessment 4

1 (a) 

(b) 

(c)



2. (a) 1720 = 1000 + 700 + 20: One thousand seven hundred and twenty.
   (b) 670 = 600 + 70 + 0: Six hundred and seventy.
   (c) 3254 = 3000 + 200 + 50 + 4: Three thousand two hundred.
3. (a) 572 (b) 2700 (c) 23 404 (d) 8022 (e) 16 100

Self Assessment 5

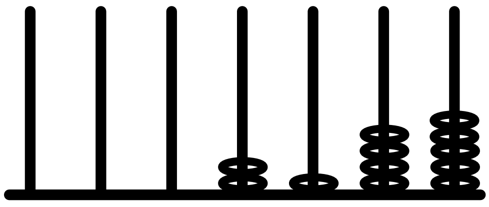
1. (a) 15 < 25 (b) 631 > 482 (c) 111 < 1111 (d) 7564 < 17 564
   (e) 801 900 < 809 100
2. (a) 440 > 44 (b) 251 < 521 (c) 222 < 2222 (d) 15 000 < 15 001
   (e) 781 < 871 (f) 417 019 > 420 (g) 102 000 < 1 000 001

Answers to Unit 2 Test

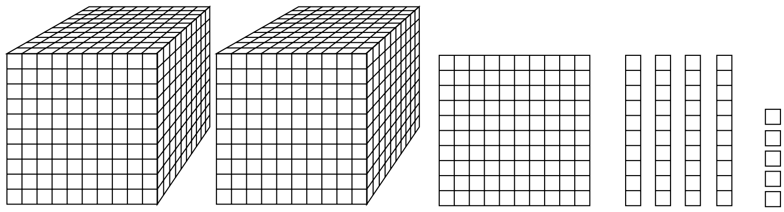
1. (a) Six hundred and seventy-five thousand four hundred and thirty-two
   (b) (6 × 100 000) + (7 × 10 000) + (5 × 1000) + (4 × 100) + (3 × 10) + (2 × 1)
   (c) 675 432

2. (a) (i) 7000 (ii) 7 (iii) 700 (iv) 70 (v) 70
   (b) (i) 7 000 000 (ii) 7000 (iii) 700 000 (iv) 70 000 (v) 70 000

3. (a) 2145



(b)



4. Write down the value of 3 in each of the following numbers if all the zeros are removed.
   (a) 3000; 30 (b) 300 (c) 3 (d) 3

5. Fill in the blank spaces using either < or >:
   (a) 25 < 75 (b) 101 < 1001 (c) 99 > 90 (d) 10 090 > 10 009
   (e) 1 000 001 < 1 001 000

Unit 3: Addition and Subtraction of Whole Numbers

Introduction

You have just learned about the place value system in Unit 2. The place value system makes it possible for numbers to be written using only the ten digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. It also makes it possible to add, subtract, multiply and divide numbers. In this unit you are going to learn about some addition and subtraction facts.

Objectives

You should, by the end of this unit, be able to demonstrate mastery of:

- addition and subtraction **basic facts**
- commutative and associative properties
- ways of adding large numbers
- mental arithmetic strategies
- strategies for teaching addition and subtraction to upper primary (Grade 5–7) pupils

Addition

Developing the concept of addition at the lower primary level involves using concrete objects that children can manipulate. Children need to **master** basic facts of addition to enable them to add large numbers. According to Van de Walle (1990), mastery of basic facts requires a three-stage approach:

1. Provide a strong foundation in number relationships and operations
2. Introduce **mental skills** to help children master the facts
3. Allow adequate time for children to understand the operations and acquire the necessary skills to carry them out

In addition to understanding number relationships, children need to construct meanings for operations. For all four operations—addition, subtraction, multiplication, and division—models and word problems are the teacher's basic tools. Again, pupils must be given adequate time to master the concepts.

Mental skills for learning basic facts

A mental skill is the use of specific number and/or operation relationships to help memorise a particular set of basic facts. For example, to do the addition 9 + 6, use the relationships with 10: add 1 (from the 6) to 9 to make 10, and 5 more is 15. These skills can be taught using a number of teaching strategies such as **teacher-directed** discussions and **individualised** and **small-group** exercises.

Teacher-directed discussions

When you start a new skill, begin with teacher-directed discussions. Models or drawings can be used to explain how a mental skill works. Children also can use models at their desks as they begin to figure out a thought process.

Individualised and small-group exercises

Use as many different activities as possible for each mental skill or group of facts. Flash cards are a useful approach to fact skill practice. Cards can be prepared with reminders, **cues**, or conceptual references that remind children to use the skill being practised.

Other activities involve the use of dice made from wooden cubes, teacher-made spinners, matching activities, and simple games. Drawings, models, and labels are frequently used to remind the children of the thought process they should practice. Some of these activities have been suggested for you below. Adapt a method used in one activity to teach another skill. Be resourceful and try to provide variety for your pupils.

Addition Facts

One-more-than, Two-more-than facts

+	0	1	2	3	4	5	6	7	8	9
0		2	2							
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3		4	5							
4		5	6							
5		6	7							
6		7	8							
7		8	9							
8		9	10							
9		10	11							

Table 3.1

Table 3.1 shows 36 facts with addends of 1 or 2. The facts are the result of the One-more-than or the Two-more-than relationships.

Unit Activity 1

1. In this activity, you should make the following teaching aids, which you will later use to teach your pupils to practice the One-more-than and the Two-more-than facts. Make several sets of flash cards (7 cm × 10 cm) with “One-more than” and “Two-more-than” facts. In the sample cards below, the “arithmetic” side of the card represents the more-than fact in three different ways.

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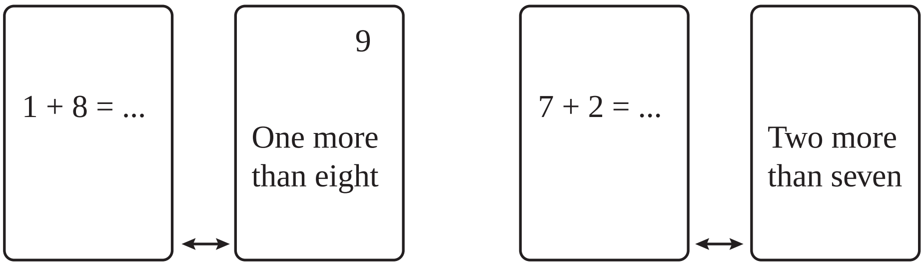
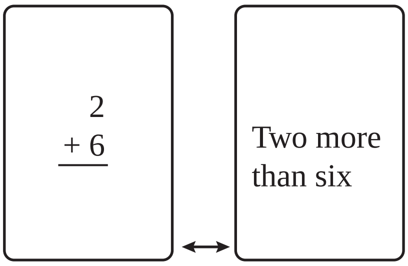


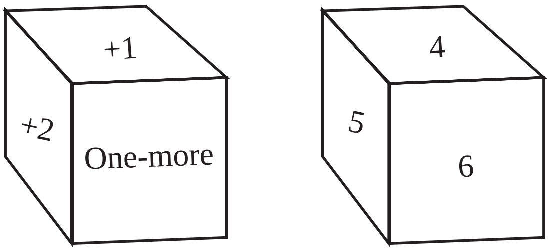
Card 1

Card 2



Card 3

2. Make two dice, one labelled +1, +2, +1, +2, One-more, Two-more, and another labelled 4, 5, 6, 7, 8, and 9.



3. Make five large cards (22 cm × 30 cm) for a matching activity.

	Two more	+2 fact
7.	.8	.3 + 2
3.	.10	.5 + 2
4.	.9	.2 + 8
5.	.5	.7 + 2
6.	.6	.4 + 2
8.	.7	.2 + 6

Using flash cards, dice and matching cards

Flash cards

In a teacher-directed class discussion, ask pupils “What is one-more-than-five?” As soon as you get a response, ask “What is one plus five?” or hold up the 1 + 5 flash card. Make sure you relate the one-more-than-five relationship to both 1 + 5 and 5 + 1. Discuss several similiar examples.

Dice

Roll the two dice together. Let the pupils read the complete fact. For example, if the first die shows + 1 and the second die shows 6, this would read 6 + 1 is 7, or 6 and 1 is 7. Let pupils work in groups, with each group using a pair of dice. This requires you to prepare enough pairs of dice for the groups in your class.

Matching cards

In a matching activity, pupils begin with a number in the left column, match it with the one in the centre column which is two-more, and then connect that with the corresponding basic fact in the right column.

Practice Activity 1

Try the activities described above in your class for consolidation of the One-more-than and Two-more-than facts.

Record your experiences and keep them in your evidence portfolio.

Facts with zero

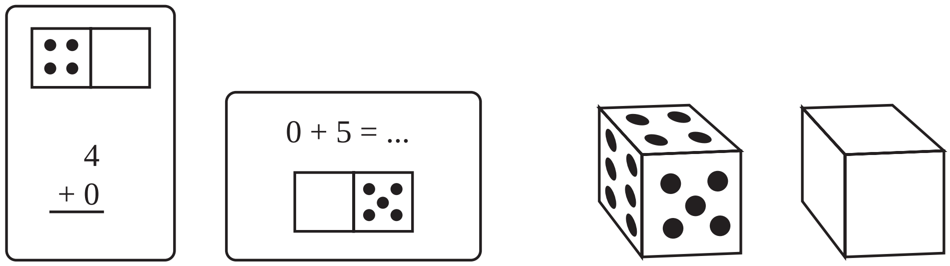
+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1										
2										
3										
4										
5										
6										
7										
8										
9										

Table 3.2

Table 3.2 shows 9 zero facts for addition.

Unit Activity 2

Make flash cards and dice for consolidating the zero-addition facts.



Flash cards showing the fact and model. Pupils should provide the answer when the card is flashed

Two dice, one numbered from 4 to 9, and a zero die. Roll the dice and say the fact.

Practice Activity 2

1. Prepare an activity for consolidation of the zero addition facts, and try them in your class.
2. Record your experiences with your pupils.

Doubles facts

+	0	1	2	3	4	5	6	7	8	9
0	0									
1		2								
2			4							
3				6						
4					8					
5						10				
6							12			
7								14		
8									16	
9										18

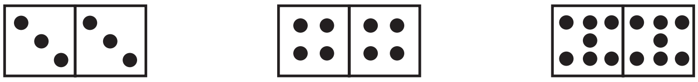
The table shows doubles facts.

Practice Activity 3

Make picture cards and flash cards for the doubles facts. See examples below.

Back of card: 3 + 3 = 4 + 4 = 7 + 7 =

Front of card:



Domino doubles. Say and write the fact.

This activity involves matching the flash cards with pictures of the doubles.

Doubles-plus-1 facts

The doubles-plus-1 facts are also called near-doubles. The mental skill involved in these facts is to double the smaller of the two addends and add 1. The table below shows the doubles-plus-1 facts.

+	0	1	2	3	4	5	6	7	8	9
0		1								
1	1		3							
2		3		5						
3			5		7					
4				7		9				
5					9		11			
6						11		13		
7							13		15	
8								15		17
9									17	

Unit Activity 4

Make several cards for the doubles plus 1 facts and a matching sheet or a matching game board, as shown below:

2 +2	8 +8	4 +4	7 +7	1 +1
3 +3	5 +5	6 +6	0 +0	9 +9

7
+6   5
+4   5
+6   7
+8   etc.

Practice Activity 4

After discussing the strategy with your class, write ten doubles-plus-1 facts on the board. Use vertical and horizontal formats of addition. Vary the addends, beginning with small ones. Go through the facts. Ask pupils to identify which number to double, then ask them to name the double that will be used. In the third stage, ask pupils to say the double, then the double-plus-1 fact, and the answer.

Prepare the doubles and the doubles-plus-1 matching activity for your class. Use the activity to consolidate the doubles-plus-1 facts with your pupils. Record your experiences.

Five and Ten facts

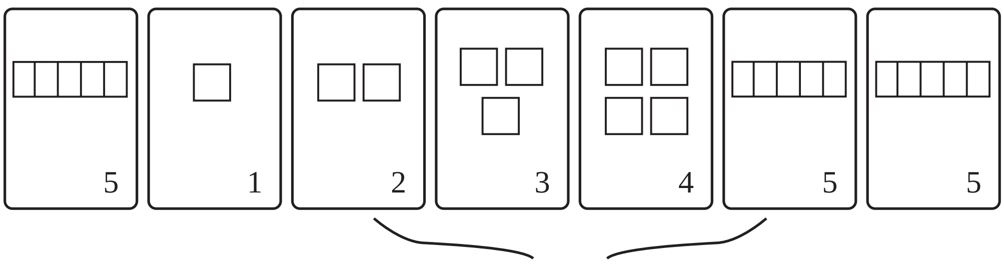
These are two facts in one, those with five as one addend and those with a sum of 10.

The table below shows the Five and Ten facts

+	0	1	2	3	4	5	6	7	8	9
0						5				
1						6				10
2						7			10	
3						8		10		
4						9	10			
5	5	6	7	8	9	10	11	12	13	14
6					10	11				
7				10		12				
8			10			13				
9		10				14				

Practice Activity 5

Try this in your class and record your experiences.
On the board, draw seven cards as shown below.



2 plus 5 is 7

Select a number from 5 to 9 and discuss what two cards can be used to make the given number. In each case, at least one of the two cards should be a

Continues on next page.

five-bar card. For each number, ask the class to say an addition fact and write it on the board.

Extend the activity to target numbers up to 14. For numbers 10 to 14, the task is to find two numbers that make the target, using the cards in the set to model these numbers. For the target 13, for example, 5 and 8 or 6 and 7 can be used and each can be made from the cards.

Make Ten facts

+	0	1	2	3	4	5	6	7	8	9
0										
1									9	10
2									10	11
3									11	12
4									12	13
5									13	14
6									14	15
7									15	16
8			10	11	12	13	14	15	16	17
9		10	11	12	13	14	15	16	17	18

The table shows the make-ten facts. These facts all have at least one addend of 8 or 9. The skill involves building on the 8 or 9 up to 10 and then adding on the rest. For 6 + 8, start with 8, then 2 more makes 10 and that leaves 4 more or 14. For this skill to be effective, pupils should have learned to think of the numbers 11 to 18 as 10 and some more.

Unit Activity 5

Make an appropriate number of mats (see *Figure 3.3*) with two ten-frames, each mat with twenty counters. Also make several flash cards with make-ten facts.

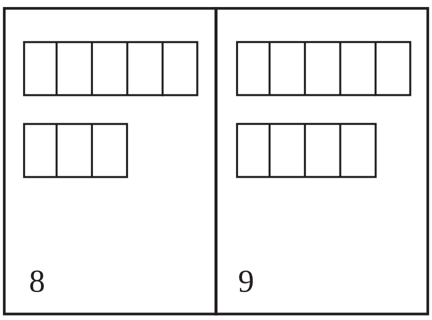


Figure 3.3: A mat of two Ten-Frames. Student counters should be of the same shape and size as the individual rectangles in the frame.

Practice Activity 5

Put your pupils in groups and give each group a mat with two ten-frames and twenty counters.

Show a flash card with a make-ten fact or call out the fact orally. Ask the pupils to model each number on the make-ten fact, one on each of the two ten-frames. Then ask them to decide on the easiest way to show (without counting) what the total is. The easiest choice is to move counters into the frame showing either 8 or 9. Ask pupils to explain what they did. Ask them to explain that 9 + 7 is the same as 10 + 6, for example. Give them more time on the last part of this task. Record pupil's experiences with this task.

Self Assessment 1

List the addition facts you learned in the previous sections.

Explain why it is essential for pupils to master these facts.

Write a short paragraph explaining the teaching strategies you would use to teach these facts.

Addition is Commutative

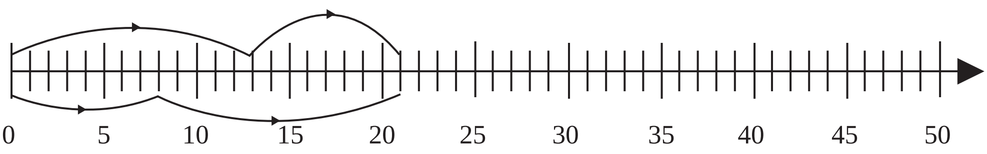
What do we mean when we say addition is commutative?

If you walk 13 steps forward starting at 0, then walk 8 more steps forward, you end at 21. Your walk represents the number sentence 13 + 8 = 21, as shown on the number line below:

Similarly if you walk 8 steps from 0, and then walk 13 more steps forward, you end at 21. Your walk this time represents the number sentence 8 + 13 = 21.

The two walks show you that 13 + 8 = 8 + 13 = 21. This is shown on the number line below.

13 + 8 = 21



Try several similar examples: 4 + 5 and 5 + 4; 6 + 7 and 7 + 6; 7 + 9 and 9 + 7. What do you notice and conclude from these examples?

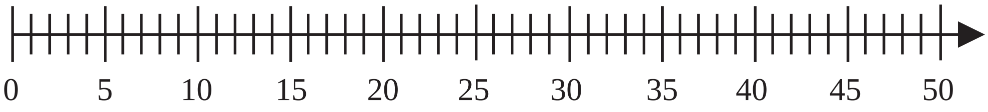
You should see that for any pair of numbers you select, say *x and y, then* $x + y = y + x$.

We say that addition is commutative because for any two numbers x and y, $x + y = y + x$.

Addition is Associative

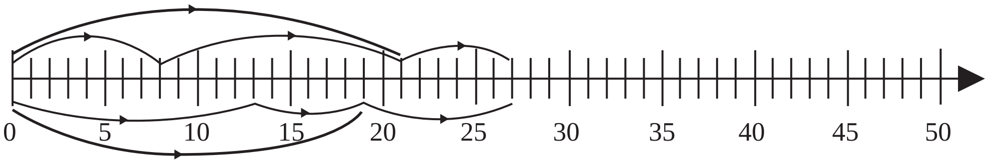
What do we mean we say that addition is associative?

Draw a number line like this one below:



Does the order in which you carry out the following addition matter, (8 + 13) + 6 or 8 + (13 + 6)? Look at these additions represented on the number line below:

(8 + 13) + 6 = 27



8 + (13 + 6) = 27

Regardless of the order, we still find that: (8 + 13) + 6 = 8 + (13 +6).

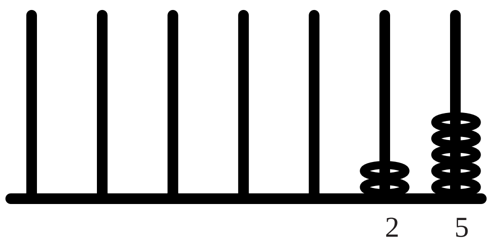
Try several examples of your own.

You will see that whatever three numbers *a*, *b*, and *c* you choose, $(a + b) + c = a + (b + c)$. In this case we say that *addition* is *associative*.

Addition using an abacus

You can also add on an abacus.

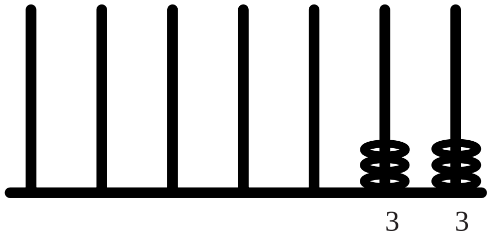
Example
Add 25 + 38
First represent the number 25 on the abacus as shown below.



You require 11 counters to represent 38:
3 counters for 3 tens and 8 counters for 8 ones.

Step 1

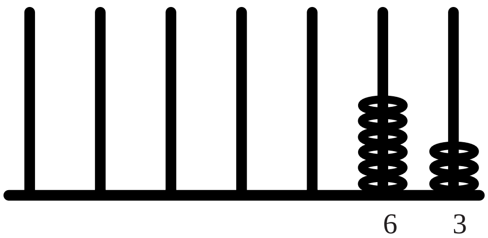
- Take the 8 counters for ones and add to the 5 already on the abacus. You get 13.
- Remove 10 and take one of them and add it on the 2 which are on the tens column.
- Put away the other 9 counters.
- You now have 3 on the tens column and 3 on the ones column, i.e., 33, as shown below.



Step 2

Add the 3 counters for tens on to the tens column.
The answer is 63, as shown below.

$$\begin{array}{r} 25 \\ \underline{+\ 38} \\ 63 \end{array}$$



Addition Algorithms

An algorithm is a multi-step process to calculate or solve a mathematical problem. Large numbers can be added in different methods. Here are three ways of adding 4761 and 2129:

Method 1

4761 + 2129
First you may write each number as follows:

4761 = 4000 + 700 + 60 + 1
2129 = 2000 + 100 + 20 + 9

Now, add as follows:

$$\begin{aligned} 4761 + 2129 &= (4000 + 2000) + (700 + 100) + (60 + 20) + (1 + 9) \\ &= 6000 + 800 + 80 + 10 \\ &= 6000 + 800 + 90 \\ &= 6000 + 890 \\ &= 6890 \end{aligned}$$

Method 2

4761 + 2129
= 6761 + 129 (Thousands have been added)
= 6861 + 29 (Hundreds have been added)
= 6881 + 9 (Tens have been added)
= 6890 (One have been added).

Method 3

$$\begin{array}{r} 4761 \\ \underline{+\ 2129} \\ \underline{6890} \end{array}$$

1 *carried over from the ones column and added to the tens column*

This is the method which is commonly used.

Some common addition errors made by pupils

1. Writes the sum of each column.

   For example: $\begin{array}{r} 57 \\ \underline{+\ 24} \\ 711 \end{array}$

2. Adds all the digits.

   For example: $\begin{array}{r} 48 \\ \underline{+\ 9} \\ 21 \end{array}$

3. Carries all the time.

   For example: $\begin{array}{r} 165 \\ \underline{+\ 24} \\ 99 \end{array}$

Remedy to errors

- Identify the errors by looking for cases where they occur.
- Ask pupils to explain procedures they used to arrive at the correct and incorrect answers.
- Determine where the specific difficulties lie.
- Correct the cause of the difficulty.

Mental Calculations and Estimations

Mental calculations play a vital role in developing and enhancing numeracy skills of children. Numeracy is measured in terms of the following attributes:

- ‘Being at home’ with numbers and the ability to make use of mathematical skills which enables an individual to cope with the practical mathematical demands of his/her everyday life.

- An appreciation and understanding of information which is presented in mathematical terms, for instance, in graphs, charts, or tables or by reference to percentage increase or decrease.

On the basis of the above definition of numeracy, we can outline the reasons for the importance of mental calculations as follows:

- Mental calculations develop the thinking processes of children.
- Ability to carry out simple mental calculations is part of being ‘numerate’ and ‘educated’.
- Mental calculations are the methods which many adults use in their real lives, for example, in social contexts like shopping, games such as darts, golf, etc., and in TV news and other programmes.
- Mental calculation is a facility and gives a feeling of ‘comfort’ with numbers.

What to do in the classroom

Pay serious attention to improving mental ability of your pupils in the teaching/learning process. Various approaches should be used in order to motivate pupils in this respect.

Many pupils feel stressed when doing mental work. It is important to design mental work ‘lessons’ that appeal to pupils and ensure a high rate of success.

We all know that even written calculations involve some form of mental work. But oral mental work in particular should be encouraged if the mental agility of pupils is to develop.

Approaches to mental calculations

Mental calculations can be approached in a number of ways:

1. As a revision exercise to the previous lesson.

   This revision could be in the form of a quick, short ten-question oral exercise given at the beginning of a new lesson.
2. As consolidation of the work covered in the course of a lesson.

   Again, a quick oral exercise given the end of the lesson.
3. As mental work lessons.

   Mental work lessons should be planned to take place once in a while. In these lessons mental skills and common mental methods are taught to pupils (see the teachers’ notes in the next section).

Mathematical games, puzzles, and problem-solving tasks are ideal ways to improve mental calculations.

Mnemonics are a tool to aid memory and an essential element to mental work. For example, one mnemonic is BODMAS (Brackets Of Division, Multiplication, Addition and Subtraction), which gives the order in which the four operations should be followed, and thus is used to simplify expressions. In higher grades, pupils should be encouraged to design their own mnemonics for certain learning situations.

Extending the mental skills

Earlier in this unit, we discussed mental skills that help consolidate the basic addition facts. Here are some additional mental skills, to help with the addition of big numbers.

Extending basic facts

Patterns of numbers are important. For example:

3 + 6 = 9
13 + 6 = 19
23 + 6 = 29 etc.

The 100 squares table

The 100 squares table below generates patterns like the one above.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Serial counting in number bases

Counting in tens, the 100 squares table illustrates sequences like 4, 14, 24, 34, 44. Similar tables can be constructed to illustrate examples of counting in other number bases, as in 2s, 3s, 4s, etc.

44 + 9 = 44 + 10 – 1
= 54 – 1 = 53, etc…

Mental methods for addition

To carry out written calculations with speed and accuracy requires mastery of the basic addition facts and the mental methods for addition of big numbers. There are several methods used in mental calculations. Here are some of them:

1. Adding by serial counting
   For example 47 + 38: 47 57 67 77 85
2. Adding tens first
   For example: 47 + 38: 47 and 30 is 77 and 8 is 85
3. Adding tens and units separately
   For example, 47 + 38 is 40 + 30 and 7 + 8 which is 70 + 15 making 85
4. Adding a single digit by 'bridging' ten
   For example, 36 + 7…36 and 4 is 40 and 3 is 43

5. Adding units first

   For example, 47 + 38: 47 and 8 is 55 and 30 is 85

6. Adding by ‘extending’ the use of basic fact

   For example, 36 + 7…6 and 7 is 13 so 43

7. Special method – Adding on

   (a) Add on 10 and subtract 1, for example 27 + 9: 27 + 10 is 37, 37 subtract 1 gives 36.

   (b) General version: for example, 47 + 38: Add 40 and take away 2, 47 + 40 is 87, 87 – 2 is 85.

8. Adding in a different order (Associative Law)

   For example, 7 + 8 + 3: 7 and 3 is 10 and 8 is 18.

   Also applies to the mental part of written calculation.

$$\begin{array}{r} 47 \\ 58 \\ \underline{+\ 53} \end{array}$$

9. Addition by adjusting both numbers

   For example, 47 + 38 is the same as 45 + 40 (add 2 to 38, subtract 2 from 47)

Practice Activity 7

Plan lessons to teach the mental methods of addition mentioned above and try them out in your class. Keep your lesson plans and comments of the experiences with your class. Give pupils exercises for practice on each method.

Subtraction

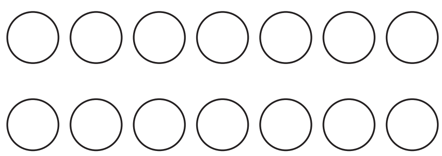
Remember what it means to subtract? Subtraction simply means take away. If 15 of your 45 pupils are called to report to the Headteacher’s office, then 30 pupils remain. This sentence can be written as, 45 – 15 = 30. In the equation 45 – 15 = 30, 45 is called a **minuend**, 15 is the **subtrahend**, and 30 is the **difference**. Note that we do not use these terms when teaching primary pupils. Basic subtraction facts are related to addition, but in reverse. It is important that pupils have a mastery of subtraction facts by the time they reach grade 5. The number line is a common model for teaching subtraction facts.

Subtraction Facts

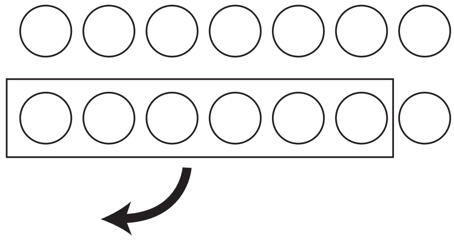
Subtraction facts, although related to addition, are more difficult for pupils. Pupils need more time to practice and master them. Generally, the mental skills for mastering subtraction facts are modelled on the “**think addition**” basis as illustrated in the steps below:

(i) Take-away: 14 – 6

Step 1: Count out 14 counters



Step 2: Count and remove 6

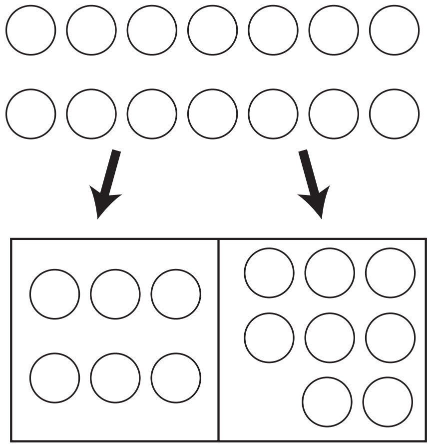


Step 3: Count what is left

14 – 6 = 8

(ii) Missing part: 14 – 6

Step 1: Count 14 counters

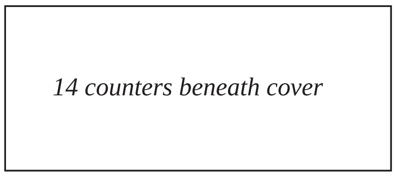


Step 2: Count 6 and put in one side

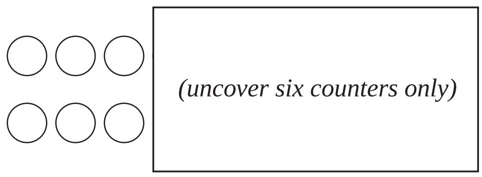
Put the rest in the other side

(iii)Take-away

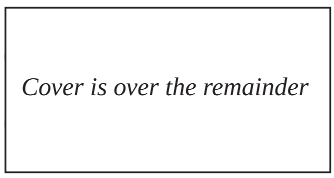
Step 1: Count 14 counters and cover



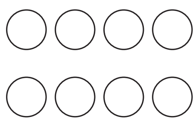
Step 2: Count and take out 6 and put them aside



Step 3: Think – “Six and what makes 14?”
8 left. So 14 – 6 = 8



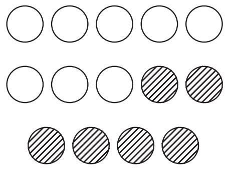
Step 4: Uncover
“8 and 6 is 14”



Make–Ten Facts

Where the subtracted number is either 8 or 9, for example, 14 – 9 and 17 – 8:

$$\begin{array}{r} 14 \\ -8 \end{array}$$



Start with 8. How much to 10? (2)
How much more to 14? (4)
So 14 – 8 is? (6)

2 to get 10 and 4 more

Practice Activity 8

1. Draw a ten frame with nine dots on the board. Discuss with the class how you can build numbers between 11 and 18, starting with 9 in the ten frame. Emphasize the idea of one more to get to 10 “and then the rest of the number”. Repeat for a ten frame showing 8.
2. Next, with either the 8 or 9 ten frame in view, call out numbers from 11 to 18 and ask pupils to say the difference between that number and the one on the ten frame. Show the fact cards, e.g., 16 – 9 to connect this idea with the symbolic subtraction fact.

Back off to 10 strategy

“Back off to ten” is a take-away strategy. It is useful for subtraction facts that have a difference of 8 or 9, for example 14 – 5, 13 – 5. For 14 – 5, start with the total 14 and **back off**, and that gets you to 10. Then take off 1 more to get 9. Thus, 14 – 5 is 9.

Give pupils a lot of practice to master these facts.

Using an abacus to subtract

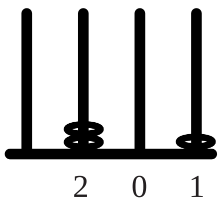
You can also use an abacus to subtract.

Example:

Use the abacus to carry out the subtraction 201 – 19.

Solution

First represent the number 201 on the abacus.



You need 21 counters for this activity. Why do you need 21 counters? If you can’t answer this question now, try to answer it after you have gone through the example.

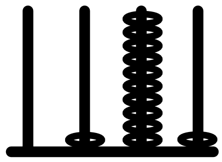
Step 1

Start subtracting the numbers in the units column, i.e., 1 – 9.

There is only one counter on the ones column and we cannot remove 9 counters from 1 counter.

Go to the tens column and borrow 1 counter. Unfortunately, there is no counter in the tens column.

Go to the hundreds column and borrow one counter. When you bring this counter to the tens column, it changes to 10 counters. Why? Because one hundred equals ten tens.



H T O

Step 2

Now borrow one counter from the tens column and leave 9.

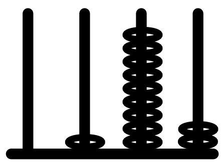
When you bring this counter to the ones column, it changes to 10 counters.

Why? Because one ten equals 10 ones.

Now you have 11 counters on the ones column. Take away 9 counters.

You remain with 2 counters on the ones column.

Now you have 192 on your abacus, as shown below.



1 9 2

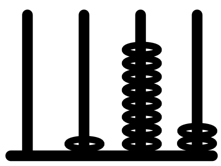
Step 3

Subtract the numbers in the tens column.
There are 9 counters.

Take away 1 counter, and you are left with 8.

The final answer is 182.

This is shown in the diagram below.



1 8 2

Therefore, 201 – 19 = 182

Lay out your work as follows:

$$\begin{array}{r} {}^{1}{}^{1} \\ 201 \\ \underline{-\ 19} \\ 182 \end{array}$$

Some common mental methods for subtraction

1. Counting on ("Shopkeeper" subtraction)

   Example 1: 82 – 38 (units first); 38 and 2 is 40 and 40 is 80 and 2 is 82 – 38 is 44.

   Example 2: 82 – 38 (tens first); 38 and 40 is 78 and 4 is 82, so 82 – 38 is 44.

   Example 3: How long is it from 03.15 to 04.05? 03.15 to 04.00 to 04.05 is 45 minutes + 5 minutes = 50 minutes

2. Subtraction by 'bridging ten'

   Example: 23 – 4 as 23 – 3 is 20 and 20 – 1 is 19

3. Subtracting tens, then ones

   Example, 82 – 38 as 82 – 30 is 52 – 8 is 44.

4. Subtraction by adjusting one of the two numbers

   Change to an easier calculation, then adjust the answer
   Examples: 69 – 7 = 62 relates to 9 – 7 = 2

   82 – 38: Add 2 to both numbers gives the easier calculation 84 – 40 = 44

5. Subtraction by 'extending' the use of a basic fact

   Example: 69 – 7 = 62 related to 9 – 7 = 2

6. Subtraction by counting back

   Example: 23 – 4 as 22, 21, 20, 19.

7. Subtracting units then tens

   Example: 82 – 38 as 82 – 8 is 74; 74 – 30 is 44.

Common errors in subtraction

$$\begin{array}{r} 63 \\ \underline{-\ 47} \\ 24 \end{array}$$

Subtracts smaller from larger

$$\begin{array}{r} 482 \\ \underline{-\ 165} \\ 327 \end{array}$$

Fails to reduce tens column in the process of "borrowing"

$$\begin{array}{r} {}^{4\ 1\ 2} \\ 520 \\ \underline{-\ 286} \\ 240 \end{array}$$

0 – n = 0 instead of trading

$$\begin{array}{r} {}^{8\ 1\ 7} \\ 497 \\ \underline{-\ 135} \\ 3\ 512 \end{array}$$

"Borrowed" item when not needed

$$\begin{array}{r} 68 \\ \underline{-\ 5} \\ 13 \end{array}$$

Subtracts single digit from both columns.

Self Assessment 2

1. Construct an addition table for numbers from 10 to 20.
   (a) How are the sums for 10 + 10, 11 + 11… up to 20 + 20 arranged?
   (b) How are the sums of the form $x + y = y + x$ arranged?
   (c) Find the following sums: (11 + 12) + 15 and 11 + (12 + 15). What do you know about the answer? What addition property is illustrated in these results?
2. The enrolment figures for girls and boys in the Copperbelt Province of Zambia in Grades 1 and 7 in 1996 are shown below:

Enrolment	**Girls**	**Boys**
Grade 1	19 789	19 280
Grade 7	20 826	22 352

   (a) Represent these enrolment figures on abacuses.
   (b) What was the total enrolment for both girls and boys in 1996 in:
   (i) Grade1 (ii) Grade 7?
   (c) How many more girls were there in Grade 1 than boys?
   (d) How many more boys were there in Grade 7 than girls?
   (e) Which Grade had a higher enrolment and by how many?
3. For each of the following pairs of numbers:
   (a) Find the number that must be added to the first number to give the second number.
   (b) Find the sum of each pair of numbers.
   (i) 569; 594 (ii) 4395; 4859 (iii) 54 674; 87 202
   (iv) 436 542; 500 201
4. Apply the following mental methods to carry out the addition 68 + 26, by filling in the blank spaces:
   (a) Adding by serial counting in tens
   68 + 26: … … … …
   (b) Adding tens first
   68 + 26: … and … is … and … is …
   (c) Adding tens and units separately
   68 + 26 is … + … and … + … which is … + … is …
   (d) Adding a single digit by ‘bridging’ ten
   68 + 26 is … + … is … + … is …
   (e) Adding ones first
   68 + 26: … and … is … and … is …
   (f) Adding by ‘extending’ the use of a basic fact
   68 + 26: … and … is 14 and … is … and … is …
   (g) Special method – Adding on
   68 + 26: Add … and take away … gives 68 + … is …;
   then …-… is …
   (h) Addition by adjusting both numbers
   68 + 26 is the same as … + …

Practice Activity 9

These activities should be done with your class. Have your pupils do their work in their exercise books. Keep a record of your experiences with pupils.

1. Let each pupil construct an addition table for numbers from 10 to 20 in their exercise books.

   (a) Using their tables, let them find the answers to the following pairs of additions:

   (i) 12 + 15 and 15 + 12 (ii) 17 + 11 and 11 + 17

   (iii) 16 + 19 and 19 + 16

   (b) Are their answers to each pair of additions the same? If not, then their tables are wrong. To confirm that their tables are correct, the numbers in the diagonals should be the same.

2. (a) Ask pupils to show each of the following pairs of additions on a number line.

   (i) 12 + 15 and 15 + 12 (ii) 17 + 11 and 11 + 17

   (iii) 16 + 19 and 19 + 16

   (b) Let pupils show, by the use of a number line, that each of the following additions are true:

   (i) (8 + 11) + 15 = 8 + (11 + 15) (ii) (12 + 7) + 9 = 12 + (7 +9)

3. Ask pupils to carry out the following addition: 3254 + 8179, using:

   (a) An abacus

   (b) Each of the three methods discussed in the unit content.

   **This activity should be prepared for each Grade levels 5, 6, 7 whenever you teach addition and extended on the place value up to millions.*

4. Design appropriate exercises for your pupils to practice subtraction on an abacus.

5. The methods outlined in the unit content are vital for teaching mental arithmetic. Mental arithmetic is an essential tool in learning of mathematics and should be encouraged at all levels and in all mathematics lessons. Design appropriate lessons to teach your classes (Grades 5 to 7) the mental methods for addition and subtraction.

Summary

You have learned in this unit that:

- Numbers can be added in any order and the result is the same. Examples are:

  $8 + 9 = 9 + 8 = 17$. This is the *commutative law of addition*.

  $(8 + 9) + 18 = 8 + (9 + 18) = 35$. This is the *associative law of addition*.

- We can add and subtract using an abacus.
- Mental calculations should be used to consolidate pupils' numeracy skills.
- People use a variety of mental methods to carry out calculations in addition and subtraction. Practicing these methods will help pupils consolidate addition and subtraction facts.

Reflection

Examples of mental methods of addition and subtraction were restricted to two-digit numbers. How can you extend these to three-digit numbers?

Take one example and try to explain the procedure for each mental method for addition.

Take one example and try to explain the procedure for each mental method for subtraction.

Unit 3 Test

Do all questions in this test.

1. Carry out the following addition and subtraction using an abacus, explaining each step with illustrations:

   (a) 64 723 + 27 878   (b) 64 723 – 27 878

2. The table shows the populations of eight southern African Commonwealth member countries in 1996:

Country	**Urban**	**Rural**
Botswana	392 950	1 057 050
Malawi	1 287 924	8 469 075
Mozambique	5 303 104	10 864 896
Namibia	560 835	889 165
Tanzania	6 996 456	22 649 544
South Africa	20 894 328	20 562 672
Zambia	3 842 584	5 135 416
Zimbabwe	3 457 454	7 553 546
	…………..	…………...

   (a) Find the following and write your answers in figures and words:

      (i) The total population of Zambia

      (ii) The total population of Malawi

      (iii) The overall population of the eight southern Africa Commonwealth member countries.

   (b) Which country has the largest urban population and which has the lowest urban population?

   (c) Which country has the highest population and which has the lowest?

   (d) Compare the highest and lowest populations. What is the difference?

3. Describe and illustrate, with examples, any two mental methods used in addition calculations.

4. Describe and illustrate, with examples, any two mental methods used in subtraction calculations.

Unit 3: Answers to Self Assessments

Self Assessment 2

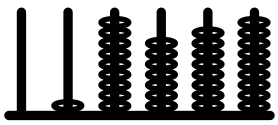
1. Addition table for numbers from 10 to 20.

+	10	11	12	13	14	15	16	17	18	19	20
10	20	21	22	23	24	25	26	27	28	29	30
11	21	22	23	24	25	26	27	28	29	30	31
12	22	23	24	25	26	27	28	29	30	31	32
13	23	24	25	26	27	28	29	30	31	32	33
14	24	25	26	27	28	29	30	31	32	33	34
15	25	26	27	28	29	30	31	32	33	34	35
16	26	27	28	29	30	31	32	33	34	35	36
17	27	28	29	30	31	32	33	34	35	36	37
18	28	29	30	31	32	33	34	35	36	37	38
19	29	30	31	32	33	34	35	36	37	38	39
20	30	31	32	33	34	35	36	37	38	39	40

(a) Diagonally from upper left to lower right.
(b) $y + x$ is the reflection of $x + y$, across the diagonal in (a).
(c) 38 for both; addition is associative

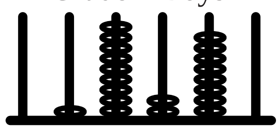
2. (a)

Grade 1 Girls



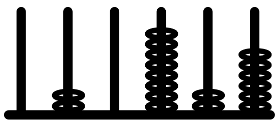
1 9 7 8 9

Grade 1 Boys



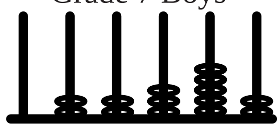
1 9 2 8 0

Grade 7 Girls



2 0 8 2 6

Grade 7 Boys



2 2 3 5 2

(b) (i) 39 069 (ii) 43 178
(c) 509
(d) 1 526
(e) Grade 7; 4109

4. (a) 68 + 26: 78 88 94
(b) 68 + 26: 68 and 20 is 88 and 6 is 94
(c) 68 + 26 is 60 + 20 and 8 + 6 which is 80 + 14 is 94
(d) 68 + 26 is 68 + 2 is 70 + 24 is 94
(e) 68 + 26: 68 and 6 is 74 and 20 is 94
(f) 68 + 26: 8 and 6 is 14 and 60 is 74 and 20 is 94

(g) 68 + 26: Add 30 and take away 4 gives 68 + 30 is 98; then 98 – 4 is 94

(h) 68 + 26 is the same as 70 + 24

Answers to Unit 3 Test

1. (a) 92 601 (b) 36 845

2. (a) (i) The total population of Zambia was 8 978 000 in 1996.
   (ii) The total population of Malawi was 9 756 999 in 1996.
   (iii) The overall population of the eight Southern African Commonwealth member countries was 119 916 999 in 1996.

   (b) South Africa; Namibia

   (c) South Africa; Botswana and Namibia (tie)

   (d) 40 007 000

Unit 4: Multiplication and Division of Whole Numbers

Introduction

In the previous unit, you learned about addition and subtraction operations. This is the common way these operations should be taught. Before multiplication and division operations are taught, pupils should have mastered addition and subtraction facts and skills well.

Objectives

By the end of this unit you should be able to:

- Multiply numbers with products up to millions and explain the algorithms.
- Divide numbers up to millions and explain the algorithms.
- Plan and teach lessons on multiplication and division for primary classes.

Multiplication

Multiplication is introduced to children at the lower primary level. They will continue to develop and build on their multiplication facts and skills as they move from one grade to another. In Grade 5, they are extending multiplication of numbers up to products of 10 000s, in Grade 6 up to products of 100 000s, and in Grade 7 up to products of millions.

Reflection

Recall the meaning of multiplication. How do you define multiplication? Consider the example 2×3.

1×3 means add 3 one time.

2×3 means add 3 two times.

That is $2 \times 3 = 3 + 3$

$= 6$

Similarly, 3×2 means add 2 three times.

That is, $3 \times 2 = 2 + 2 + 2$

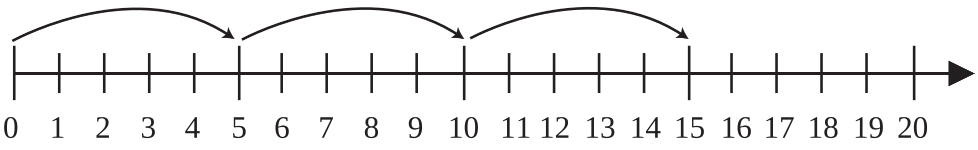
$= 6$

In the multiplication of $3 \times 2 = 6$, 3 and 2 are called **factors** and 6 is a **product**. The first number (3) in the equation 3×2 is called a **multiplicand** and the second number (2) is called a **multiplier**.

Unit Activity 1

Multiplication is sometimes explained as repeated addition using the number line. An explanation is given below.

$5 \times 3 = 5 + 5 + 5 = 15$



1. Draw separate number lines on a big chart to show the following multiplications:

   (a) 7×5 (b) 5×7 (c) 10×4 (d) $(3 + 4) \times 2$ (e) $(3 \times 2) + (4 \times 2)$

2. Draw on the same number line, one above and the other below, the following pair of sentences:

   $(4 + 6) \times 3$ and $(4 \times 3) + (6 \times 3)$.

Multiplication facts

Children need to master the basic multiplication facts before they carry out complicated multiplication algorithms. You should systematically build these through the multiplication of numbers from 0 to 9. The most useful mental skill or strategy is the use of a helping fact.

Consider the 25 facts shown in *Table 4.1* below:

×	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3				9	12		18	21	24	
4				12	16		24	28	32	
5										
6				18	24		36	42	48	
7				21	28		42	49	56	
8				24	32		48	56	64	
9										

Table 4.1

The 25 facts in *Table 4.1* can be learned by relating each to an already known fact or helping fact. For example, 3×8 is connected to 2×8 (double 8 and 8 more).

The 6×7 fact is related to either (5 sevens and 7 more) or to 3×7 (double 3×7). This requires the ability to know the helping fact and to do the mental addition.

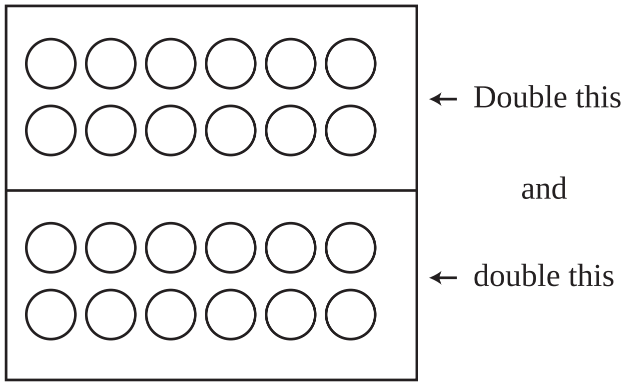
Here are models of how to find a helping fact:

Facts with a 4

This is a strategy to double and double again:

$$\begin{array}{r} 4 \\ \times\, 6 \\ \hline \end{array} \qquad \begin{array}{r} 6 \\ \times\, 4 \\ \hline \end{array}$$

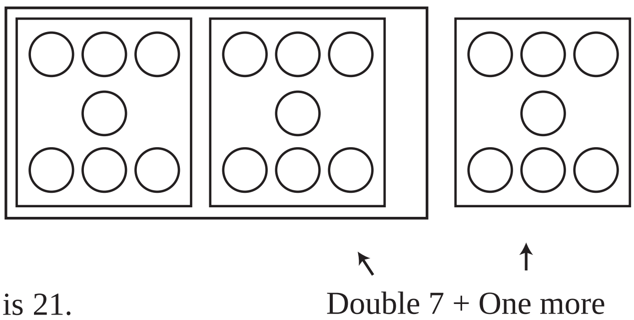
Double 6 is 12
Double again is 24



Facts with a 3

This is a double and one more set:

$$\begin{array}{r} 3 \\ \times\, 7 \\ \hline \end{array} \qquad \begin{array}{r} 7 \\ \times\, 3 \\ \hline \end{array}$$

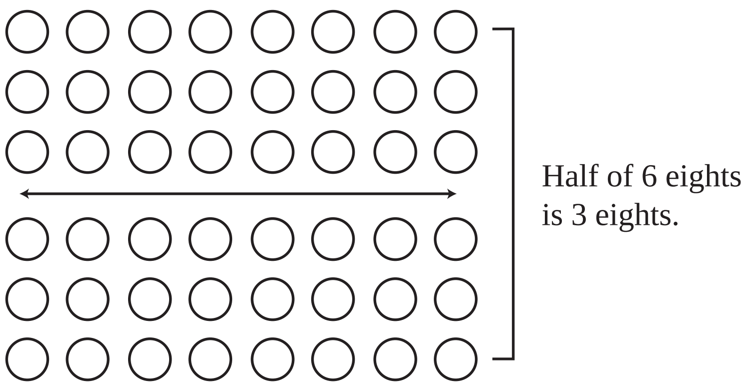


Double 7 is 14. One more 7 is 21.

Facts with an even factor

$$\begin{array}{r} 6 \\ \times\, 8 \\ \hline \end{array} \qquad \begin{array}{r} 8 \\ \times\, 6 \\ \hline \end{array}$$

The strategy is to half the double

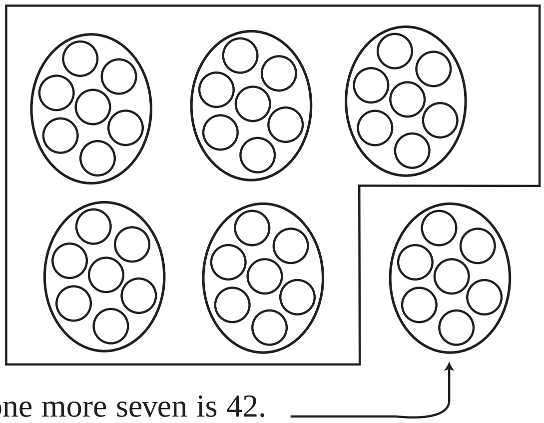


3 times 8 is 24
Double 24 is 48

Any fact

The strategy is to add one more set:

$$\begin{array}{r} 6 \\ \underline{\times 7} \end{array} \qquad \begin{array}{r} 7 \\ \underline{\times 6} \end{array}$$



5 sevens is 35. Five sevens + one more seven is 42.

Practice Activity 1

It is expected that at the Grade 5 level, pupils have developed the concept of multiplication as repeated addition and have mastered the multiplication facts to an extent that they are able to carry out the basic multiplications mentally. However, learning mental skills is an ongoing process and should be revisited whenever necessary for further consolidation.

Prepare separate lessons to teach your class mental skills for learning each of the following multiplication facts:

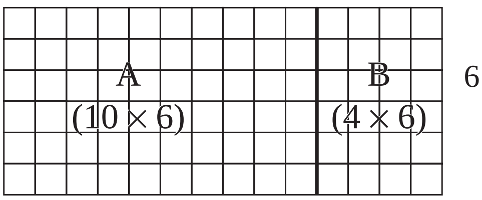
(i) Facts with 4 (ii) Facts with 3 (iii) Facts with an even factor

(iv) Any fact

Multiplication Algorithms

Multiplying a two-digit number by a two-digit number: the array method

The array shows 14 × 6.



We can write

$$\begin{array}{r} 14 \\ \underline{\times 6} \end{array} \quad \text{as} \quad \begin{array}{ccc} A & & B \\ 10 & + & 4 \\ & & \underline{\times 6} \\ 60 & + & 24 = 84 \end{array} \quad \text{or} \quad \begin{array}{r} 14 \\ \underline{\times 6} \\ 24\ B \\ \underline{+\ 60\ A} \\ \underline{84} \end{array} \quad \text{or} \quad \begin{array}{r} 14 \\ \underline{\times 6} \\ 84 \end{array}$$

Multiplying a two-digit number by a two-digit number

Split method 1

To multiply 32×28 using the split method, you proceed as follows:

32×28

	32		32
	$\underline{\times 28}$		$\underline{\times 28}$
8×2	16		256
8×30	240	→ this can be shortened to	$\underline{+ 640}$
20×2	40		896
20×30	$\underline{600}$		
	896		

Study these steps carefully as you will be expected to explain them to your class. With your pupils, you should take time to work through the steps. Repeat the steps with many more examples.

Split method 2

This split method uses diagrams. Place the 32 along the top and the 28 along the left side as shown. Then multiply at the intersections:

32

×	10	10	10	2	= 100 + 100 + 100 + 100 + 100 + 100 = 600
10	100	100	100	20	20 + 20 = 40
10	100	100	100	20	80 + 80 + 80 = 240
8	80	80	80	16	16 = 16
					896

28

Practice Activity 2

Prepare multiplication lessons for your class on each of the three methods of multiplication algorithms: Array, Split method 1 and Split method 2.
For each of the methods, show pupils how to multiply using the short method.

Common mental methods in multiplication

1. Multiplying by breaking up a number (using the Distributive Law)

   Example 1: Treat 28 as 20 + 8

   28×3 is 20×3 and 8×3 is 60 and 24 which is 84

   Example 2: Treat 28 as 30 – 2

   28×3 is 30×3 subtract 2×3 is 90 – 6 which is 84
2. Multiplying by using factors

   Example: 8×15 is $8 \times 3 \times 5$… Is 24×5… is 120.

3. Multiplying by 10, 100, 1000

   Example: 85×10 is 850

   85×100 is 8500

   8.51×100 is 851

   85×1000 is 85 000

4. Multiplying by a multiplier of 10, 100, 1000

   Example: 42×200: multiply 42 by 2 is 84 then by 100 is 8400

5. Multiplying by doubling and halving.

   Example: 11×12 is 22×6 which is 132

   8×15 is 4×30 is 2×60 which is 120

6. Multiplying by 50, 25

   Example 1: 13×50: multiply 13 by 100 gives 1300, then divide by 2 which is 650.

   Example 2: 13×25: multiply 13 by 100 gives 1300, then divide by 2 twice (650, 325) or by 4 to give 325.

7. Special methods for decimal numbers that are one decimal short of being integers.

   Example: 4×2.99 is about 4×3 which is 12, subtract $4 \times .01$ … gives 11.96, etc.

Application of multiplication

You can make up story problems to give situations where multiplication can be used.

Example:

1. I planted a row of tomato seedlings. There were 45 seedlings in a row and I made 12 rows. How many seedlings did I plant?
2. Muleya's mother makes 75 cakes every day to sell.

   How many does she make in 19 days?

   If she sells each for 20 cents, how many dollars does she get in 19 days? (1 dollar = 100 cents).

Unit Activity 2

Make up story questions for the application of multiplication.

Copy and complete the multiplication table for numbers from 1 to 12 which is shown below:

×	1	2	3	4	5	6	7	8	9	10	11	12
1												
2												
3												
4												
5												
6												
7												
8												
9												
10												
11												
12												

Practice Activity 3

1. Ask your pupils to make the same multiplication table of numbers from 1 to 12 as you made in the Unit Activity 2.
2. In pairs, your pupils will be using the table to practice the multiplication facts. Alternating, one calls out a multiplication, say 4×6, and the other should give the product without looking at the table.
3. Devise some activities that will encourage your pupils to practice multiplication. For example, look ahead to the Unit Activity 3 and prepare a set of multiplication flash cards for your class.

Division

The concept of division, introduced by Grade 2, revolves around sharing and grouping things. Thereafter, the concept is linked to the multiplication operation as its reverse process. It is essential that these processes are emphasised from the time division is introduced and throughout subsequent courses so children can build on the ideas and consolidate their understanding of the operation.

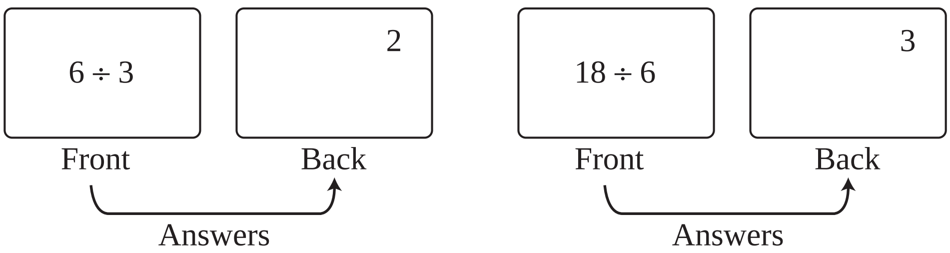
Division facts

Division is usually thought of in terms of multiplication. Therefore, division facts are multiplication facts.

For example, to think of $36 \div 9$, the problem is really to solve the equation $\ldots. \times 9 = 36$

Unit Activity 3

1. Use the table in Unit Activity 2 to make division flash cards for practicing division facts. Make the cards as follows:
   (i) Take a card and write a division fact without the answer (the **quotient**).
   (ii) On the back of the card in the upper right hand corner write the quotient.
   (iii) On the back write a different division fact.
   (iv) On the front write the answer to this new fact.
   (v) Make 78 cards, repeating steps (i) to (iv).



Long division

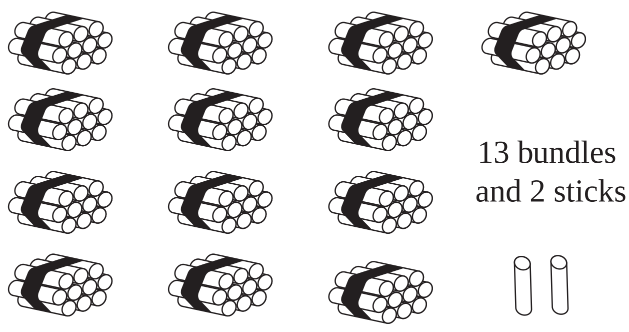
Before looking at long division, use concrete objects to divide 132 by 12, then by 13.

Materials you need for these activities

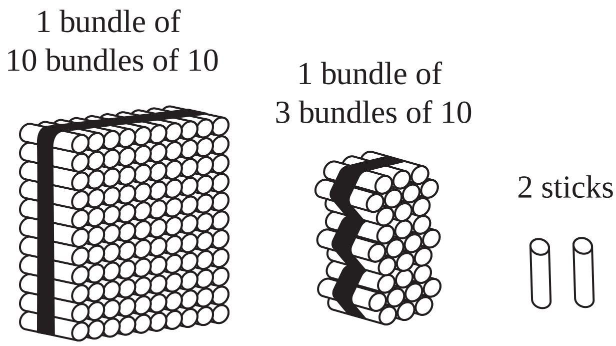
132 short sticks
strings

What to do

- First make 13 bundles of 10 sticks each, with 2 loose sticks.



- Then make 1 bundle of 10 bundles of 10 sticks, 1 bundle of 3 bundles of 10 sticks, and 2 loose sticks.

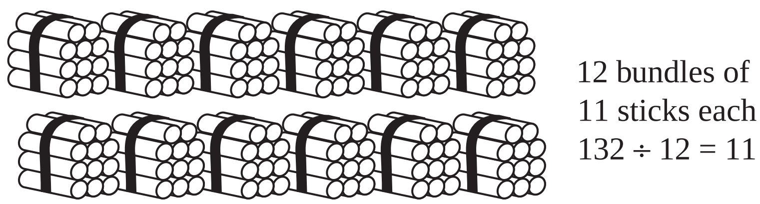


132 ÷ 12

This is the same as putting the 132 sticks into 12 bundles. How many sticks are there in each group?

To answer this question, do the groupings, as follows:

- There is only one bundle of 100 sticks. What do you do with this bundle to form 12 bundles?
- Unbundle the 100 sticks into 10 bundles of 10. Add the three bundles of ten sticks in the tens place value. How many bundles of 10 sticks do you get? ***Answer: 13 bundles of 10 sticks***
- You can now form 12 bundles of 10 sticks with one remaining.
- To this remaining bundle of 10 sticks add the 2 sticks in the ones place value. How many sticks do you have? ***Answer: 12 sticks***
- You can add one stick to each of the 12 bundles. How many sticks do you finally have in each of the 12 bundles? ***Answer: 11 sticks***



- This activity has given you the answer to 132 ÷ 12 as 11.
- Now carry out this calculation (without the sticks), using long division, as follows:

$\begin{array}{r} 11 \\ 12\overline{)132} \end{array}$	12 into 1. Can't.
12	12 into 13 goes 1
12	1 × 12 is 12
– 12	13 – 12 is 1 ten, plus 2 is 12
	12 into 12 is 1
	1 × 12 is 12
	12 – 12 is 0

Unit Activity 4

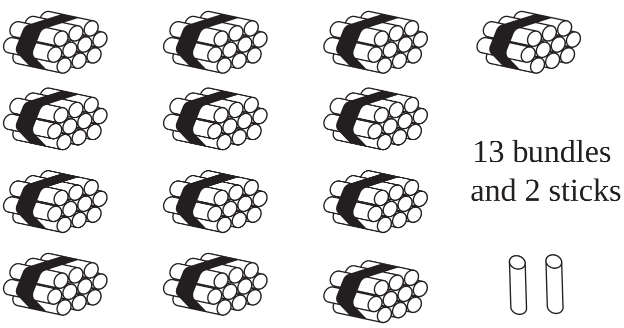
1. Use sticks to illustrate the number 154.
2. Use your sticks to explain the following divisions. Illustrate the steps with diagrams.

   (a) 154 ÷ 14 (b) 154 ÷ 12

Practice Activity 4

1. Devise an activity for your pupils to carry out division of a three-digit number by a two-digit number, using sticks or any other suitable materials.
2. Prepare examples of long division that are suitable for each of the Grades 5 to 7. Use these examples in your teaching.
3. Use the story questions you made up in Unit Activity 4. Make more if need be.
4. Using a 1–12 multiplication table, practice division facts using flash cards.

You can now do 132 ÷ 13, using your sticks, following the same steps as in the first example.



This activity gives you the answer to 132 ÷ 13 as 10 remainder 2
Here is the layout of the long division algorithm:

10 r 2	3 into 1 can't
13) 132	13 into 13 goes 1
– 13	1 × 13 is 13
2	13 – 13 is nothing
– 0	13 into 2 is 0
2	0 × 13 is 0
	2 – 0 is 2
	Remainder 2

Some mental methods in division

1. Dividing by 10, 100, 1000
   Examples: 850 ÷ 10; 245 ÷ 100; 4200 ÷ 1000
2. Dividing by multiples of 10
   Example: 720 ÷ 90 divide by 10 is 72, divide by 9 is 8
3. Dividing by multiplying up
   Example 1: 56 ÷ 4, 10 lots of 4 is 40; 4 lots of 4 is 16. So 56 ÷ 4 is 14
   Example 2: 120 ÷ 15, 4 × 15 is 60, 2 × 60 is 120, so 120 ÷ 15 is 8
4. Dividing by 50, 25 using divide by 100 then multiply
   Example: 2500 ÷ 50 is 2500 ÷ 100 is 25, to give 25 × 2 which is 50

Reflection

Do you analyse computational errors made by pupils? Why?

Error analysis

As a teacher of primary school mathematics, you should be alert to error problems. Some teachers are only interested in either putting a tick ✓ or a cross (✗) against a pupil's answer. That approach is not too helpful. A teacher should give individual attention by pausing at an error, analysing it, and giving appropriate remedial work. If a teacher is not able to identify the cause of an error then he/she cannot provide an effective remedy. Be careful not to decide on the error pattern too quickly. Look at other work by the pupil before making a decision. Errors can be grouped into four categories:

1. **Wrong operation.** The pupil confuses the meaning of the operations and carries out the subtraction process instead of addition:

$$\begin{array}{r} 47 \\ \underline{+\ 15} \\ 32 \end{array} \quad \text{instead of} \quad \begin{array}{r} 47 \\ \underline{+\ 15} \\ 62 \end{array}$$

2. **Obvious computational error**. The pupil applies the correct operation, but fails to recall the basic number fact, e.g., $9 \times 6 = 56$.
3. **Defective algorithm**. The pupil applies the correct operation but uses incorrect procedure: for instance, the pupil forgets to carry or carries when there is nothing to carry.
4. **Random response**. This is mostly a wild guess, where it is difficult to figure out how a pupil arrived at the answer.

Now, let us look at Moyo's work. Identify and find the cause of the error, if any, and provide an appropriate remedy.

a) $\begin{array}{r} 24 \\ \underline{\times 2} \\ 48 \end{array}$ b) $\begin{array}{r} 52 \\ \underline{\times 3} \\ 156 \end{array}$ c) $\begin{array}{r} 18 \\ \underline{\times 6} \\ 308 \end{array}$ d) $\begin{array}{r} 67 \\ \underline{\times 5} \\ 455 \end{array}$

Moyo has no difficulty with multiplication problems that do not involve carrying, but he has difficulty with multiplication problems that do involve carrying. This type of error falls under "defective algorithm". Moyo adds the number of tens he carried to the tens before multiplying. The cause of the error could be in the addition Moyo used to first add the number he carried before he did the rest of the addition.

Moyo may be helped by using partial products, or by recording the number carried in the correct place value, but below the product. For instance:

Using partial products

$$\begin{array}{r} 18 \\ \underline{\times 6} \\ 48 \\ \underline{60} \\ 108 \end{array}$$

Recording below product

$$\begin{array}{r} 18 \\ \underline{\times 6} \\ \underline{108} \\ 4 \end{array}$$

Unit Activity 5

1. Find Liseli's pattern of error and provide remedial work.

1\. A. $\begin{array}{r} 1 \\ 24 \\ \underline{+ 9} \\ \mathbf{123} \end{array}$ B. $\begin{array}{r} 1 \\ 69 \\ \underline{+ 5} \\ \mathbf{134} \end{array}$

2\. A. $\begin{array}{r} 45 \\ \underline{- 18} \\ \mathbf{33} \end{array}$ B. $\begin{array}{r} 54 \\ \underline{- 7} \\ \mathbf{53} \end{array}$

Unit Activity 6

In a classroom situation, identify errors made by pupils. This should be a teaching life habit. Ask pupils how they arrive at the wrong answers. Then identify the cause of error and remember to provide effective remedial work.

Estimation

A clear distinction is made between computational estimation and mental computation. In computational estimation, the aim is to get a 'rough' or approximate result. Mental computation means doing calculations mentally.

Sometimes, an approximate result may be more desirable than an exact one. In everyday life, estimation skills can be more valuable and time-saving than exact calculations. When teaching estimation, it is important to use words and phrases like about, close to, just about, a little more or less than, or between.

You probably have done estimation of one kind or another in your life, as this is inevitable.

1. Give an estimate answer to each of the following:
   (a) $29 + $99 + $15.78
   (b) Time is 1512 hours
   (c) 98×16
   (d) marks in a test: 78, 87, 96, and 39.

2. In which of the following situations would an estimate value be sufficient?
   (a) A waiter figures tax at 5%.
   (b) A waiter totals the bill.
   (c) A customer figures out a tip for 15%.
   (d) The customer checks the bill.
   (e) An accountant figures out how much money was made on ticket sales.
   (f) A newspaper reports the number of people who attended the game.

Estimation strategies

Estimation strategies are based on the idea of using 'nice' numbers that are close to the numbers in the computation. For example, the cost of 25 drinks at 99 pence each can be estimated by using £1 instead of 99 pence, which gives an estimated cost of £25.

From this, we see a hierarchy in the development of estimation strategies. Estimation depends on mental skills, which depends on mastery of number facts, which in turn depends on the acquisition of the concepts of number operations.

Front-end methods

These involve the use of the leading or leftmost digits in numbers and ignoring the rest. This is followed by an adjustment on what has been ignored. This front-end method is a very easy estimation strategy for addition and subtraction. The approach is appropriate when all or most of the numbers have the same number of digits.

Example:

(a) Add numbers in columns.

$$\begin{array}{r} 489 \\ 37 \\ 651 \\ +\ 208 \end{array}$$

Front-end column

Adjust

The front-end gives 4 + 6 + 2 = 12 – estimation is about 12 hundreds.

Adjust the middle 8 + 3 + 5 is 16, therefore about 160, plus 1200 from the front-end column. The estimate is **1360**

(b) Numbers not in columns.

Add: $4.98 + $53.50 + $36.25

Front-end numbers (tens position): 0 + 5 + 3 = 8, or about $80.

Adjust in the ones position: 4 + 3 + 6 = $13 or $14, considering the fractions.

The estimate is $94.

Multiplication and division

For multiplication and division, the front-end method uses the first digit in each of the two numbers involved. The computation is then done using zero in the other positions.

For example, 48 × 7 is 40 × 7 or, since 48 is nearer 50 than 40, adjust the first digit in 48 upwards and use 50 × 7.

When both numbers have more than one digit, the front-ends of both are used.

For example, 347 × 74 gives 300 × 70 or 21 000.

For division, first determine where the first digit of the quotient belongs.

For example, $7\overline{)3482}$, the first digit is 4 and it belongs in the hundredths column over the 4. Therefore, the front-end estimate is 400. This method always produces an underestimate. In this example, the answer is closer to 500 and therefore 480 or 490 is a good adjustment.

Rounding Methods in addition and subtraction

The most common form of estimation is rounding off to an easier number to work with. In an addition, you add cumulatively as you round each number.

For example, to add \$48.27, \$1.89, 85¢, \$7.10 and \$24.95, you can use two possibilities:

1. Rounding \$48.27 to tens gives \$50, ignore the next two small numbers. Rounding \$7.10 to \$10 gives \$60 then round off \$24.95 to \$20 which gives \$80.
2. By rounding to the nearest dollar, \$48.25 gives \$48, \$1.89 gives \$2, 85¢ gives \$1, \$7.10 gives \$7, and \$24.95 gives \$25. Adding \$48 plus \$2 plus \$1 plus \$7 plus \$25 gives an estimate of \$83.

In subtraction, there are only two numbers so the strategy is to only round the number you are subtracting.

For example, 6724 – 1863. Round off 1863 to 2000. This gives 6724 – 2000, which gives 4624. Now you can adjust this answer to about 4800.

For 624 + 385, you can only round one number. This will give either 600 + 385 or 624 + 400.

In multiplication and division, rounding is done the same way as in addition and subtraction.

For example, 7×836 gives 7×800 which is 5600. In this case the product is down by 7×36. To get a more accurate result, round 836 to 840 so that $7 \times 840 = 7(800 + 40) = 5600 + 280$ which is 5880.

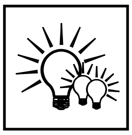
If possible you should round only one factor and select the largest. For example 47×7821 would give 47×8000 which is 376 000, but 50×8000 would give 400 000.

The other method in multiplication is to round up by one factor and down by the other, even if that is not the closest round number.

For example, 86×28 would give 80×30 giving 2400. Compare with the actual product of 2408. The estimated answer is quite close.

In the case of division, rounding is done by looking for the closest compatible number.

For example, $4325 \div 7$ would give $4200 \div 7$ which is 600.



Practice Activity 5

Devise activities for teaching estimation strategies to your class.

Self Assessment 1

Do the following tasks:

1. (a) Make separate arrays to illustrate the following multiplications:

   (i) 17×5 (ii) 18×7

   (b) Do the following multiplications using the split 1 and 2 methods:

   (i) 63×41 (ii) 243×27

2. Explain, using illustrative examples, any three of the mental methods used for each of the multiplication and division operations.
3. Use the shortest method in long division to explain the procedure for carrying out the following divisions:

   (a) $346 \div 12$ (b) $2587 \div 19$ (c) $36\ 608 \div 18$

Unit Activity 7

1. Describe the strategies you would use to ensure that your pupils consolidate the basic facts of:

   (a) Multiplication (b) Division

2. Using illustrative examples, explain how you would teach the following:

   (a) dividing a one-digit number by a one-digit number
   (b) dividing a two-digit number by a one-digit number
   (c) dividing a three-digit number by a two-digit number
   (d) dividing a four-digit number by a three digit number

Summary

In this unit, you:

- consolidated multiplication and division facts
- learned multiplication and division algorithms
- were introduced to methods for carrying out mental calculations for multiplication and division

Reflection

- Do you have difficulty remembering multiplication facts?
- How would you use your own experience in learning multiplication facts to enable your pupils learn them better?
- What are the difficulties related to using sticks or any other objects, in explaining division of a three-digit number by a two-digit number? Is there a better way of doing this?
- Did you have difficulty relating the activities with sticks to the actual calculations? Is there a better way of explaining this?
- Can these activities be extended to the division of a four-digit number by a two-digit number? A four-digit number by a three-digit number? If not, what can you do to explain these to pupils?
- Discuss your methods with other teachers.

Unit 5: Problem Solving

Introduction

Problem solving is one of the most emphasized skills in the teaching of Mathematics today. Problem solving plays a significant role in the mental development of the child. It equips the child with techniques of problem solving which he/she will find useful in adult life. Problem solving is one of the core purposes for teaching Mathematics.

Although problem solving permeates the teaching and learning strategies in these modules, it is formally introduced in this unit to provide the teacher with a variety of techniques to apply in teaching. This unit is based on a package of lecture materials from the Faculty of Education at Craigie College, University of Paisley, Scotland.

Objectives

After working through this unit, you should be able to teach problem solving skills to your upper primary school pupils using the following strategies:

- guess and check
- looking for patterns
- listing

Reflection

Try the following problems:

1. 45 pupils each pay US$7 for a Christmas party. How much is this altogether?
2. In the multiplication given below, a pair of two-digit numbers are multiplied to give the product 1035. The units digits are missing in both numbers that are being multiplied. Find the missing digits:

   $4\square \times 2\square = 1035$

As an experienced person in solving word problems that lead to a standard method of finding the solution, you probably had no difficulty solving the first problem. The first problem is a routine word problem that translates into the standard multiplication process.

You probably found that the second problem was not as straightforward as the first. The process is still multiplication, or division, but mathematical thinking and some sort of strategy is required to find the answer.

Problem solving is *not* confined to the traditional “mathematical problem”, which refers to word problems where the process of solution is standard. The similarity of wording in “maths problem” and “problem solving” is unfortunate. This table shows the key difference between the concepts:

Traditional maths problems	The answer to the word problem is not obvious, but the method for finding the answer is predetermined—either by the problem itself or by the type of maths that students are learning this year.
Maths “problem solving”	The answer to the word problem is not obvious, and neither is the solution. The student must devise a solving method, then solve the problem.

In the upper primary classroom, it is best not to define or teach either term, but rather have the students do problem solving without learning the definitions.

Pupils should be ready to learn problem solving at the upper primary level, by which time they should have mastered the basic facts of addition, subtraction, multiplication, and division as well as the necessary computational skills.

Problem Solving Strategies

Part I: The guess and check strategies

“Guess-and-check” is much like “trial-and-error”, but avoids using the negative word *error*. Pupils guess at possible answers until they find, by checking, one that works. Not long ago, the guess-and-check method would not have been considered a proper maths technique to be learning in school! However, the utility of this strategy outweighs any dissimilarity to traditional mathematics instruction.

Students benefit from having more structure than just “go ahead and guess”. This subsection introduces a number of guess-and-check activities to try with your students.

Guess-and-check activities consist of the following: operation grids; the ‘super adder’; magic squares; arithmogons; ‘clever’ triangles; targets; missing numbers; and cards.

Operation grids

	4	7
	8	3

	32	21
28	4	7
24	8	3

Figure 5.1

Use grids of the type in *Figure 5.1* to give your pupils problems on operation grids.

The rows and columns are multiplied and the answers are written in the top row and the left hand column.

The guess and check ideas can be introduced by using a similar example. This time the numbers at the top and left are given, as shown in *Figure 5.2*, and pupils are asked to complete grid.

	20	48
24		
40		

Figure 5.2

What two whole numbers can multiply to give 24? Guess 3×8 and check to see if this works. The discussion should show that 20 cannot be found by either 3 or 8 multiplied by any whole number. Therefore, the original guess is wrong, shown in *Figure 5.3*.

	20	48
24	3	8
40		

Figure 5.3

	20	48
24	4	6
40	5	8

Figure 5.4

Further guessing and checking will produce the correct grid as shown in *Figure 5.4*.

Self Assessment 1

1. Formulate five problems on operation grids.
2. Work out solutions to the problems in Worksheet 1 given below:

Worksheet 1

	3×5 ↓	4×6 ↓
	15	24
12	3	4
30	5	6

		15	24
3×4 →	12	3	4
5×6 →	30	5	6

The numbers at the top are found by multiplying the numbers in the columns. The numbers at the left are found by multiplying the numbers in the rows.

Now complete grids (a) to (f) below:

	6	15
10		
9		

(a)

	21	24
28		
18		

(b)

	36	48
54		
32		

(c)

	56	66
48		
77		

(d)

	48	65
52		
60		

(e)

	84	72
63		
96		

(f)

The numbers 0 to 4 have been fitted into the 'L' shape so that the total of the row (7) is the same as the total of the column.

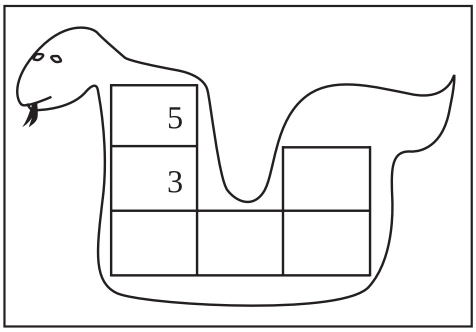
1		
2		
4	3	0

Now fit the numbers 0 to 6 into this 'L' shape so that the row and the column have the same total.

Practice Activity 1

Prepare a problem-solving lesson for your class based on the problems you formulated in Unit Activity 1 and Worksheet 1.

The 'Super Adder'



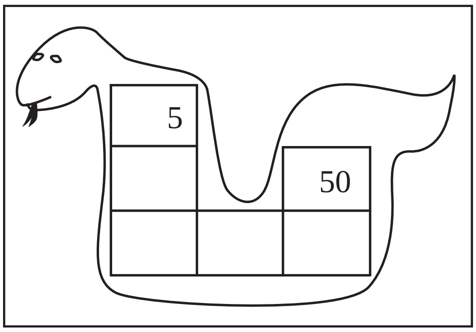
5		
3		30
8	11	19

Figure 5.5

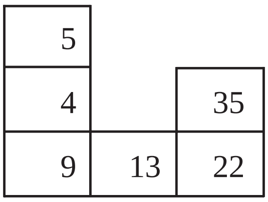
The super adder idea can be introduced in a similar way to the operation grid. The problems are written on work cards as shown in *Figure 5.5*.

Starting from the head, each pair of numbers adds together to give the next number on the snake.

The guess-and-check strategy is introduced through the example illustrated in *Figure 5.6*.



Guess 4
Check: Does not give final total of 50.
4 is too small



Guess 7
Check it

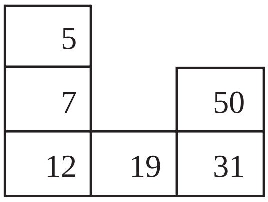


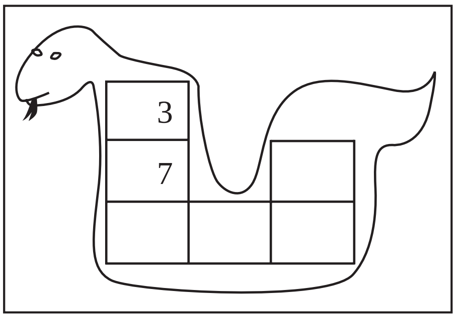
Figure 5.6

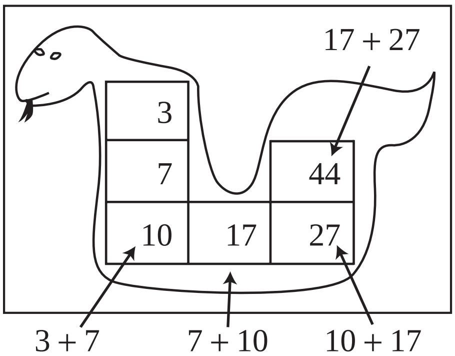
A number can be guessed for the second box and then checked to see if it gives the final total.

Practice Activity 2

Work out the solutions to the super adders in Worksheet 2 given below. Prepare a lesson and a worksheet on super adders and give it to your class.

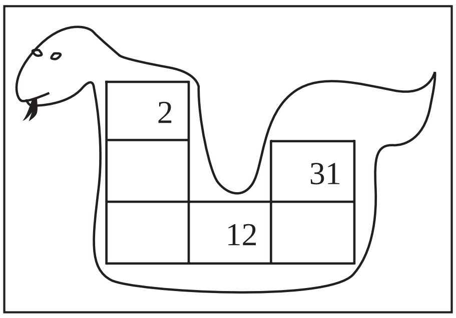
Worksheet 2: Super Adders

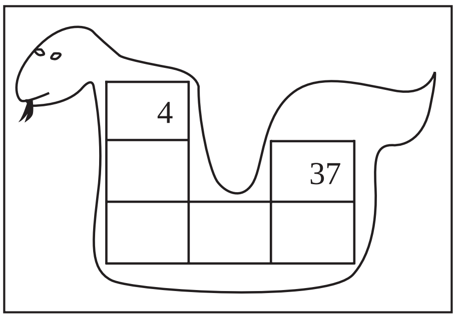


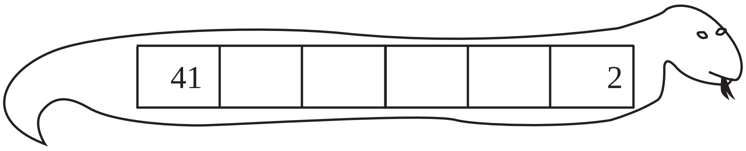


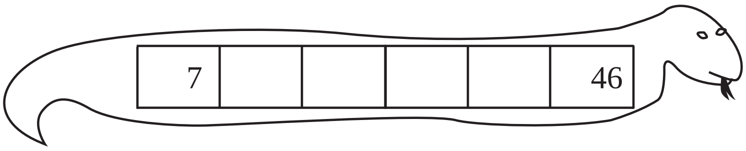
Pairs of numbers are added to give the next number in the snake.

Now, complete these super adders:









Magic squares

Practice Activity 3

Give this activity to your class.

Draw on the board a square as shown in *Figure 5.7*. Tell the pupils that the trick in this magic square is that the rows, columns, and diagonal totals are equal, in this case 15.

4	9	2
3	5	7
8	1	6

Figure 5.7

Draw another square with several numbers missing, as shown in *Figure 5.8*. Write the missing numbers on separate cards and give the cards to seven pupils. Ask your pupils to complete the square.

	7	
	11	

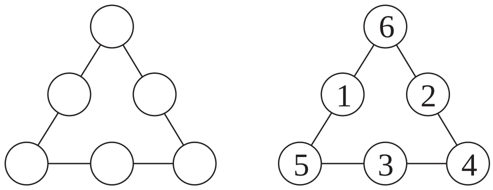
Figure 5.8

If the process takes too long, add the number 3 to the top middle box which still leaves them quite a bit of guessing and checking to complete the grid.

Arithmogons

Arithmogons are similar to magic squares. *Figure 5.9* shows an example of arithmogon.

Place the numbers 1 to 6 in *Figure 5.9(a)* so that each line of three along a side adds up to 12 and the three corners add up to 15.



(a) (b)

Figure 5.9

Clever triangles

Figure 5.10 shows an example of clever triangles. The numbers in the circles at each end of a side add up to the number in the middle of that side.

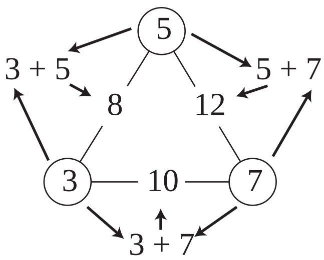
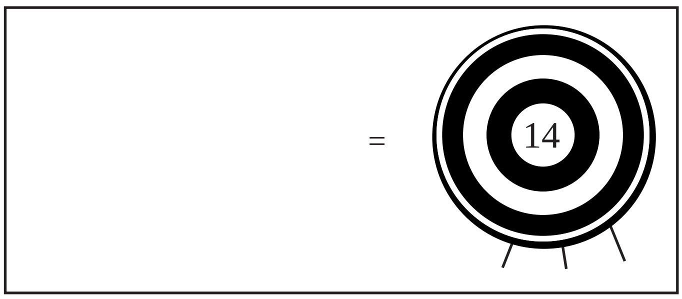


Figure 5.10

Targets

The idea is to make the target using the three numbers given and some of the signs +, –, × and ÷. Each of the numbers and the operation signs are given on separate cards.



Cards $\boxed{+}$ $\boxed{-}$ $\boxed{\times}$ $\boxed{\div}$ $\boxed{3}$ $\boxed{5}$ $\boxed{7}$

Guessing and checking will go on until a solution is found.

$$\left(\boxed{5}\ \boxed{-}\ \boxed{3}\right)\ \boxed{\times}\ \boxed{7} = \boxed{14}$$

Practice Activity 4

1. Formulate five problems on targets.
2. Prepare four separate problem-solving lessons, one each for magic squares, arithmogons, clever triangles, and targets. Try them out in your class.

Self Assessment 2

Work out the solutions to the problem in Worksheet 3 on magic squares, arithmogons, and clever triangles.

Worksheet 3

Magic Squares

1. Fill in the missing numbers in these magic squares:

(a)

4		2
	5	
8		6

(b)

41	113	
81	71	
	29	101

(c)

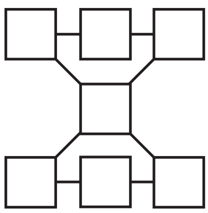
	7	
	10	
	13	

2. Use the numbers 3, 5, 6, 7, 10, and 11 to complete this magic square:

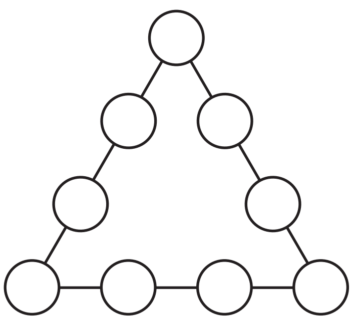
8	9	4

Arithmogons (same total)

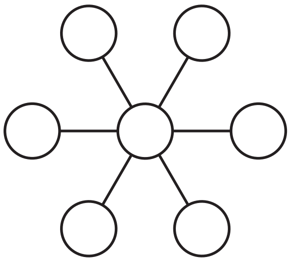
1. Write the numbers 1 to 7 in the boxes so that each row, column, and diagonal of three boxes adds up to 12.



2. Fit the numbers 1 to 9 into the diagram so that each side adds up to 20.

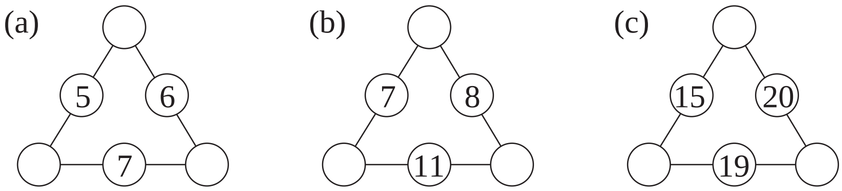


3. Write the numbers 1, 3, 5, 7, 9, 11, and 13 in the circles so that each line of three numbers adds up to 21.



Clever triangles

4. Complete the circles so that the two numbers at the end of each side add up to the number in the middle of that side.



Missing numbers

This requires finding missing numbers in an expression. The use of a calculator may be helpful but is not necessary.

Start by asking pupils to find answers to the following multiplications:

$19 \times 19 = \square$

$24 \times 24 = \square$

$71 \times 71 = \square$

Then introduce an example like the one given below:

$\square \times \square = 2209$

Explain that the numbers in the two boxes should be the same. The number must be greater than 24 but less than 71. Make a guess, say 63.

Checking $63 \times 63 = 3969$ gives a number too large. Continue guessing and checking until they narrow down to the solution.

$\square \times \square = 2209$

This idea can then be extended to consecutive whole numbers.

$15 \times 16 = \square$ $\qquad$ $122 \times 123 = \square$

Which two consecutive whole numbers will fit the boxes?

$\overset{38}{\square} \times \overset{39}{\square} = 1482$ $\qquad$ $\square \times \square = 8372$

The idea can be further extended to the missing digits:

$73 \times \square 6 = 4088$

Cards

Number cards can be used to create problem-solving questions. Some examples are given below:

Example 1

4		5	1		2	8	
	6			3		7	9

Move one card so that the sum of each set is the same. (There need not be three cards in each set).

Example 2

Arrange the digits $\boxed{1}$ $\boxed{9}$ $\boxed{3}$ to make a three-digit number that divides exactly by 7, leaving no remainder.

Self Assessment 3

Work out the solutions to the problems in Worksheet 4.

Practice Activity 5

Prepare a problem-solving, group-work lesson based on Worksheet 4.
Try the lesson in your class.

Worksheet 4

Missing numbers

1. The numbers in each pair of boxes must be the same. Which numbers, multiplied by themselves, give these answers? Complete:

   (a) $\square \times \square = 81$ (b) $\square \times \square = 256$

   (c) $\square \times \square = 625$ (d) $\square \times \square = 5041$

2. Which two consecutive numbers, multiplied together, give these answers?

   (a) $\square \times \square = 132$ (b) $\square \times \square = 702$

   (c) $\square \times \square = 380$ (d) $\square \times \square = 2756$

3. Fill in the missing digits:

   (a) $73 \times \square 6 = 4088$ (b) $\square \times \square 7 = 4042$

4. Find the missing digits by completing the boxes:

(a) $\begin{array}{r} 27\square \\ +\,1\square 3 \\ \hline \square 20 \end{array}$ (b) $\begin{array}{r} 46 \\ \times\,\square \\ \hline \square\square 2 \end{array}$ (c) $\begin{array}{r} 3\square \\ 4\overline{)\square 52} \end{array}$

5. Nine cards are grouped into three sets, as shown. Move one card into a different set so that the sum of each set is the same. There need not be three cards in each set.

(a) $\boxed{1}\ \boxed{2}\ \boxed{3}$ (b) $\boxed{4}\ \boxed{5}\ \boxed{6}$ (c) $\boxed{8}\ \boxed{7}\ \boxed{9}$

Part II: Look for Patterns

You have already been using patterns to teach multiplication and addition facts; they are a natural way to teach maths at this age. Under this heading, we are encouraging students to look for patterns, as a strategy, when they encounter unusual math problems.

Number patterns I

Start by showing pupils how patterns develop.
Write on the board pairs of numbers with a rule (an operation) given.

Example 1

Rule $\boxed{\text{add 8}}$

4 $\longrightarrow$ 12
6 $\longrightarrow$ 14
3 $\longrightarrow$ 11
18 $\longrightarrow$ ....
54 $\longrightarrow$ ....

Example 2

Rule $\boxed{\text{multiply by 4}}$

3 $\longrightarrow$ 12
6 $\longrightarrow$ 24
7 $\longrightarrow$ ....
18 $\longrightarrow$ ....

The next step is to introduce the idea of looking at the pattern of numbers to determine what the rule is.

Example 3

Find the rule

$\boxed{\phantom{\text{subtract 5 subtract}}}$

10 $\longrightarrow$ 5
18 $\longrightarrow$ 13
40 $\longrightarrow$ 35

The first attempt to guess the rule may be to 'half it'; but one must disprove or confirm this through several examples. After going through several examples, the rule in this case is to 'subtract 5'.

After pupils have had a lot of practice on finding rules involving one operation, they can be introduced to two operations, one following another.

Example 4

	add 3		Multiply by 4	
3	⟶	6	⟶	12
8	⟶	. . . .	⟶	. . . .
4	⟶	. . . .	⟶	. . . .

Give examples for other combinations of operations following one after the other. Identify operations that follow one after the other:

	$\boxed{\quad\quad}$		$\boxed{\quad\quad}$	
13	⟶	9	⟶	3
7	⟶	3	⟶	1
16	⟶	12	⟶	4

Through a discussion of the pattern, the operations will be identified as "subtract 4" followed by "divide by 3". Give pupils lots of practice on identifying the operations in combined operations problems.

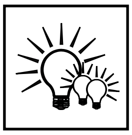
The next step is to try examples in which the operations are combined in a single rule.

Example 5

Find the rule in the following pattern:

	$\boxed{\quad\quad}$	
3	⟶	7
8	⟶	17
4	⟶	9

This is the most difficult stage and only some of the more able pupils will see the pattern. Discuss this with pupils so they understand that the rule is 'double and add 1' or 'multiply by 2 and add 1'.



Practice Activity 6

1. Work out the solutions to problems in Worksheet 5.
2. Prepare a problem-solving lesson for group work and try it out in your class.

Worksheet 5: What is the rule?

1. Write the rule for each of these tables.

(a)

	$\boxed{\quad\quad}$	
15	⟶	3
5	⟶	1
30	⟶	6

(b)

	$\boxed{\quad\quad}$	
5	⟶	17
30	⟶	42
9	⟶	21

2. Complete the missing numbers.

7	add 5 →	12	divide by 2 →	6
7	multiply by 4 →		subtract 10 →	
7	add l →		multiply by 8 →	

3. (a) Write the rule in each of the boxes A and B.

Input	A	Output	B	Output
7	→	28	→	18
3	→	12	→	2
11	→	44	→	34
18	→	72	→	62

(b) Using the rule in (a), find the missing numbers.

15 ⟶ ⟶

20 ⟶ ⟶

4. Fill in the missing numbers.

3	multiply by 5 →		add 33 →	
	subtract 30 →		divide by 5 →	4
	add l5 →		multiply by 3 →	108
	divide by 4 →		subtract 12 →	13

Number patterns II

These patterns make use of, or imply, two dimensions.
Begin by building a pattern with pupils.

(1) $\begin{array}{r} 1 \\ \underline{\times 1} \\ 1 \end{array}$ (2) $\begin{array}{r} 11 \\ \underline{\times 11} \\ 121 \end{array}$ (3) $\begin{array}{r} 111 \\ \underline{\times 111} \\ 12\,321 \end{array}$

Ask pupils to find the patterns in (4) and (5).

(4) $\begin{array}{r} 1111 \\ \underline{\times 1111} \\ 1234321 \end{array}$ (5) $\begin{array}{r} 11111 \\ \underline{\times 11111} \\ 123454321 \end{array}$ (6) $\begin{array}{r} 111111 \\ \underline{\times 111111} \\ ? \quad . \end{array}$

Ask pupils to predict the answer to number (6).

Then ask them for the answer to the multiplication below, which is not the next in the sequence. $\begin{array}{r} 11111111 \\ \underline{\times 11111111} \\ .\,.\,.\,. \end{array}$

There are 9 ones, so the middle digit of the answer is 9. Go right and left, reducing the numbers by 1 until you get the last number on either side, which is 1.

$$\begin{array}{r} 111111111 \\ \underline{\times\, 111111111} \\ 9 \\ \textit{middle digit} \end{array} \qquad \begin{array}{r} 111111111 \\ \underline{\qquad\quad \times\, 111111111} \\ 12345678987654321 \end{array}$$

You can then follow the above multiplication sequences with the following addition sequences, which you can leave to pupils to do on their own:
Find the following additions:

$$\begin{array}{r} \\ \\ 1 \\ \underline{+\ 11} \\ 12 \end{array} \qquad \begin{array}{r} \\ 1 \\ 11 \\ \underline{+\ 111} \\ 123 \end{array} \qquad \begin{array}{r} 1 \\ 11 \\ 111 \\ \underline{+\ 1111} \\ 1234 \end{array}$$

After supplying the answers to the above additions, ask pupils to predict the answer to the following addition:

$$\begin{array}{r} 1 \\ 11 \\ 111 \\ 1111 \\ 11111 \\ 111111 \\ 1111111 \\ \underline{+\ 11111111} \end{array}$$

The 8 ones in the right column give 8 as the last digit of the answer 12345678.

Self Assessment 4

1 Work through the problems given in Worksheet 6.
2. Prepare a problem-solving lesson based on Worksheet 6.

Practice Activity 7

1. Formulate five problems on the basis of number patterns II.

Worksheet 6: Number patterns

1. (a) Complete the sequence below.

$1 \times 1089 = 1089$

$2 \times 1089 = 2178$

$3 \times 1089 =$ ____

$4 \times 1089 =$ ____

$5 \times 1089 =$ ____

(b) Look at the pattern of your answers. Use the pattern to extend the table up to 9×1089 without multiplying. Check your answers by doing the multiplications.

2. (a) Complete the following patterns.

$$1 \times 9 - 1 = 8$$
$$21 \times 9 - 1 = \_\_\_\_$$
$$321 \times 9 - 1 = \_\_\_\_$$
$$4321 \times 9 - 1 = \_\_\_\_$$

(b) Using the pattern in (a), guess the following:

$$54321 \times 9 - 1$$
$$654321 \times 9 - 1$$
$$7654321 \times 9 - 1$$

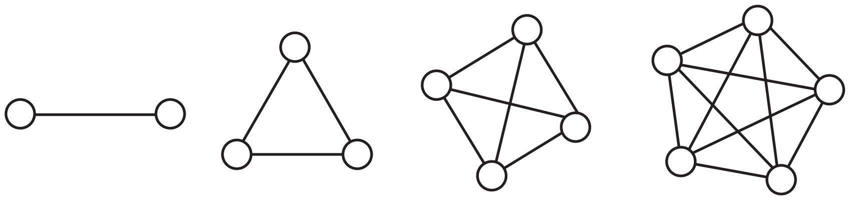
3. (a) Choose any three-digit number, for example, 365.
Repeat it to give a new number, 365365.
Divide by 13. Then divide by 11. And then by 7.
What do you notice?

(b) Try this for several three-digit numbers.
Can it be done with two-digit numbers?
Can it be done with four-digit numbers?

Investigating patterns

These patterns usually require the use of objects or sketches. Encourage pupils to investigate or explore a pattern or a relationship that can sometimes be extended to a general solution.

Example
How many lines can be drawn to join n points?



2 points 3 points 4 points 5 points

Number of points	2	3	4	5	6	7	8	…	n
Number of lines	1	3	6	?	?	?	?	…	?

Primary school pupils may use this problem to predict the next results in the sequence.

Secondary pupils may even derive the general formula for n points as:

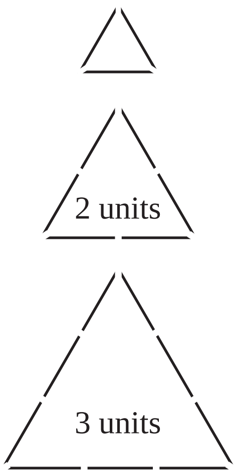
Number of lines = $\dfrac{n(n-1)}{2}$ where n stands for the number of points.

Example
By how much does the perimeter of an equilateral triangle increase as the sides increase by 1?

You need match sticks for this problem. Any sticks or straws of equal size can be used.

Put your pupils in groups. Each group should have its own bunch of sticks.

Make a triangle with sides of one unit and draw a table.



Length of side	Number of sticks
1	3
2	6
3	9
4	12

Discuss the pattern when you reach the triangle with four units. Pupils will see that the pattern goes up in threes. That is, you add on three each time to give the next one in the sequence. Draw the attention of your pupils to the right hand column where this pattern emerges.

Ask them to predict the number of sticks required for 7, 8, etc., in that order.

4	12
5	15
6	18
7	?
8	?

The next step for your pupils is more difficult. Predict the answer for a triangle which is not the next one in the sequence. For example, a triangle of 14 units per side. This exercise may be appropriate for more able pupils.

Give pupils a hint to work across the table to see if they can find a rule. Ask them the questions:

What do you do to the side length to get the number of sticks each time?

What do you do to 4 to get 12? To 5 to get 15? To 6 to get 18?

Draw arrows across the table.

The rule ‘multiply by 3’ should lead to 14 $\longrightarrow$ 42

Point out the similiarity to the “find the rule” problems they did previously.

Length of side	Number of sticks
1 $\longrightarrow$	3
2 $\longrightarrow$	6
3 $\longrightarrow$	9
14 $\longrightarrow$	?

Practice Activity 8

1. Work through the problems in Worksheet 7.
2. Prepare a problem-solving, group-work lesson based on Worksheet 7.

Worksheet 7: Patterns

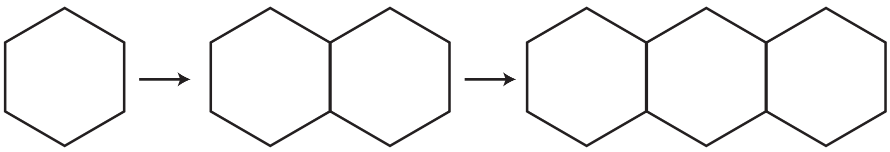
1. (a) Copy this pattern.
   (b) What do you think the total for row 5 will be? Check your answer.
   (c) Which row do you think will total 36?

row 1					1					1
row 2				1	2	1				4
row 3			1	2	3	2	1			9
row 4		1	2	3	4	3	2	1		16
row 5										

2. For this problem, you will need a hexagon cut-out or template, paper, and a pencil for drawing. Make a copy of the table below, and use the template to draw a hexagon.

Number of hexagons	1	2	3	4	5	6
Number of edges on perimeter						

   (a) Use your hexagon to draw a chain like this:



   Each time you draw a hexagon at the end of the chain, complete the correct column in your table.

   (b) How many edges would there be on the perimeter of a chain of 100 hexagons?

   Explain how to calculate this without drawing the chain.

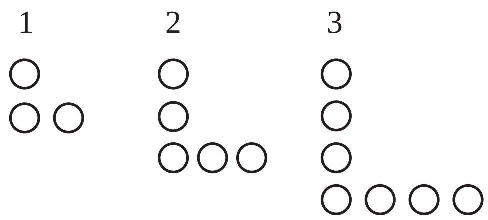
3. (a) For each of these number patterns, write the next two lines.

(i)	(ii)	(iii)
$(1 \times 7) + 3 = 10$	$(1 \times 9) - 4 = 5$	$(1 \times 8) - 3 = 5$
$(2 \times 7) + 3 = 17$	$(2 \times 9) - 4 = 14$	$(2 \times 8) - 3 = 13$
$(3 \times 7) + 3 = 24$	$(3 \times 9) - 4 = 23$	$(3 \times 8) - 3 = 21$

   (b) What is the increase in the answers from one line to the next?

   (i) …………….. (ii) ……………… (iii) …………..

4. (a) Complete drawing the dot patterns for 4 and 5:



(b) Complete the table:

Dot pattern	1	2	3	4	5
Number of dots	3				

The increase in the number of dots each time is ……

You can calculate the number of dots in any of these dot patterns like this:

Multiply the dot pattern number by 2, then add 1.

The number of dots in pattern 100 is $(100 \times 2) + 1 = 201$

Part III: Listing strategies

Listing the possible answers to a problem comes naturally to students at this age. The aim in your teaching is to impart ways of organizing those lists—like placing them in a table—so the patterns and answers are more evident.

Combinations

This strategy involves combining the choices to determine the total number of possible combinations. Use a simple timetable to introduce the idea. Four subjects have to be timetabled so that two are in the morning and two are in the afternoon.

Mathematics and Language are in the morning while Games and Drama are in the afternoon.

Timetable

Morning	Afternoon
Maths or Languages	Games or Drama

Pupils are required to choose two subjects, one in the morning and the other in the afternoon. How many possible choices are there?

Discuss this with your pupils and show them how to make a systematic list in so they do not miss any. There are four possible choices:

Maths or Games
Maths or Drama
Language or Games
Language or Drama

Introducing a third choice in the morning leads to two more possibilities.

Morning	Afternoon
Maths or Languages or Art	Games or Drama

The possible choices now are:
Maths, Games
Maths, Drama
Language, Games
Language, Drama
Art, Games
Art, Drama

At this stage, ask your pupils to do problems 1 and 2 in Worksheet 8.

Practice Activity 9

1. Do problems 1 and 2 in Worksheet 8.
2. Introduce your pupils to another form of recording the same list by using abbreviations or in a table as shown below:

Abbreviations

The choices of subjects are indicated by abbreviations. M-Maths, L-Language, A-Art, G-Games, D-Drama, T-Television, S-Swimming

Choices are:			
	MGT	LGT	AGT
	MGS	LGS	AGS
	MDT	LDT	ADT
	MDS	LDS	ADT

Table
A table can also be used to determine the number of choices.

Morning			Afternoon		Evening	
Maths	Language	Art	Games	Drama	TV	Swimming
■			■		■	
■			■			■
■				■	■	
■				■		■
	■		■		■	
	■		■			■
	■			■	■	
	■			■		■
		■	■		■	
		■	■			■
		■		■	■	
		■		■		■

Practice Activity 10

Prepare problem-solving lessons based on Worksheet 8. Plan your lessons in a variety of ways: pupils working in pairs, in groups, or individually.

Worksheet 8: Listing – Shopping choices

1. (a) A shop sells pens in three sizes: Fine(F), Medium(M), Broad(B).

   Each size is available in five colours: red(r), green(g), blue(b), orange(o), purple(p).

   Complete this list to show all the different pens the shop sells.

Fr	Fg	Fb	Fo	Fp
Mr				

   (b) Notebooks are sold in three sizes: Large(L), Medium(M), Small(S)

   Each of the three sizes is sold in three styles: plain (p), lined(l), graph(g).

   Make a list similar to the one in (a) above, to show all the different notebooks that are sold.

2. A clothes shop sells school uniforms: Blouse(B), Jersey(J), Skirt(S), Trousers(T).

   Each item is sold in three colours: grey(g), blue(b), maroon(m).

   Find how many different items of school uniform are sold in the shop by:

   (a) Completing this list:

Bg	Bb	Bm

   (b) Completing this table: (three have been done for you).

Blouse	B	B	B									
Jersey												
Skirt												
Trousers												
grey	■											
blue		■										
maroon			■									

3. A shop sells school bags in sizes: small, medium, large.

   Each size is sold in three colours: red, blue, green.

   Each colour is in two materials: canvas, plastic.

(a) Which school bag would you choose?

(b) Complete the table to find the number of different school bags in the shop. Six have been done for you.

Small (S)	S	S	S	S	S	S												
Medium (M)																		
Large (L)																		
Red (r)	r	r																
Blue (b)			b	b														
Green (g)					g	g												
Canvas	■		■		■													
Plastic		■		■		■												

Arrangements

Arrangements is another form of listing. The formal maths term for arrangements is “Permutations”. Consider two pupils, one boy and one girl, sitting in a row. In how many ways can they sit?

Sitting arrangement: either BG or GB

There are two ways the pupils can sit.

In how many ways can three pupils, Sianga(S), Banda(B), and Mutinta(M) sit in a row?

When Sianga sits on the left, there are two possible arrangements:

S B M	or	S M B

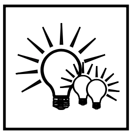
When it is Banda on the left, there are two possible arrangements:

B S M	or	B M S

Similarly, when Mutinta sits on the left, there are two possible arrangements:

M S B	or	M B S

Altogether there are six possible sitting arrangements.



Practice Activity 11

1. Work through Worksheet 9.
2. Plan a problem-solving lesson for your pupils based on Worksheet 9.

Worksheet 9 – Arrangements

1. Four pupils, Bweupe, Moyo, Tsepo, and Kumalo have to sit in a row.
   (a) List all their possible sitting arrangements.
   (b) What is total number of possible arrangements?
   (c) How many possible arrangements are there if Moyo can only sit at the end of the row, either right or left?
2. Sixteen cups, six blue, five yellow and five red have to be packed in boxes with two different coloured cups in each box.
   (a) By lettering the cups B, Y, and R, list the possible ways in which the cups can be packed.
   (b) How many boxes are there altogether?

Summary

This unit formally introduced problem solving. The strategies covered were guess-and-check, looking for patterns, and listing.

Within the guess-and-check strategy, we discussed a number of techniques to enhance your teaching:

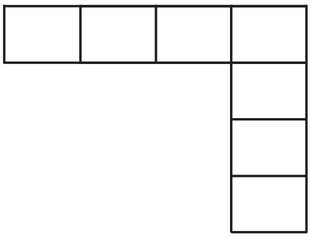
- Operation grids
- Super adders
- Magic squares
- Arithmogons
- Clever triangles
- Targets
- Missing numbers

These practical activities are intended to make learning more meaningful and enjoyable for your pupils.

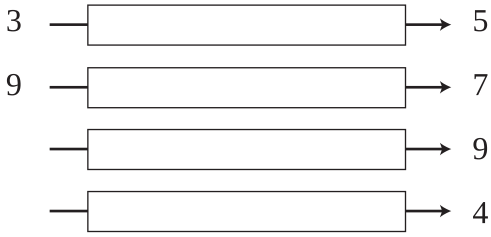
Unit 5 Test

Solve the following problems and state the strategy you use for each one:

1. Fit the numbers 0 to 8 into the ‘L’ to give equal row and column totals.



2. The rule used in the pattern below combines two operations. Find:
   (a) the rule
   (b) the missing numbers.



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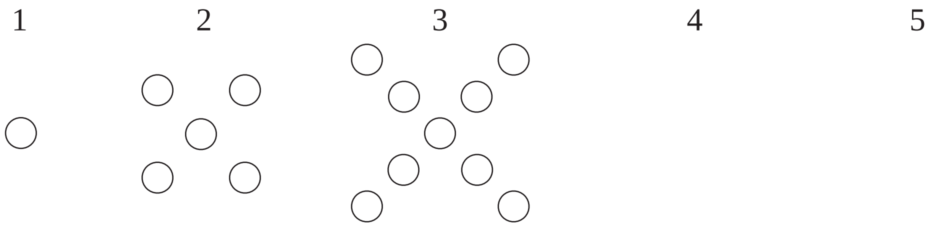


   (a) Copy and complete the table:

Dot pattern	1	2	3	4	5
Number of dots	1				

   (i) Find the number by which the dots increase each time.
   (ii) Find the rule by which the dots increase.
   (iii) Find the number of dots in pattern 100.

4. Eighteen pens, five yellow, five red, four blue, and four green have to be put in packs.
   (a) They can be put in packs of three, so that each pen in the pack is a different colour. By using the letters Y, R, B, and G, list all the possible ways in which the packing can be done.
   (b) The pens can also be put in packs of six containing at least one pen of each colour, and not more than two pens of the same colour. Use the letters Y, R, B, and G to list all the possible ways in which the packing can be done.

Answers to Worksheet Questions

Worksheet 1

1.

	6	15
10	**2**	**5**
9	**3**	**3**

(a)

	21	24
28	**7**	**4**
18	**3**	**6**

(b)

	36	48
54	**9**	**6**
32	**4**	**8**

(c)

	56	66
48	**8**	**6**
77	**7**	**11**

(d)

	48	65
52	**4**	**13**
60	**12**	**5**

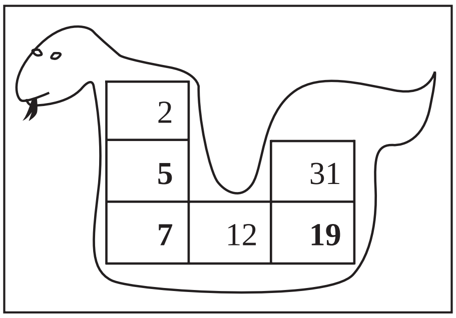
(e)

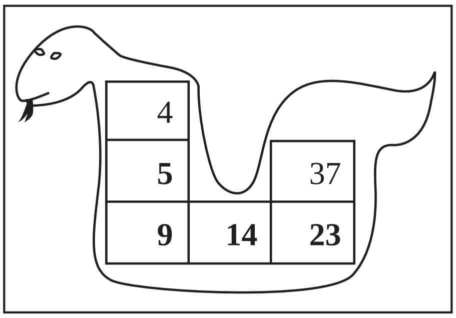
	84	72
63	**7**	**9**
96	**12**	**8**

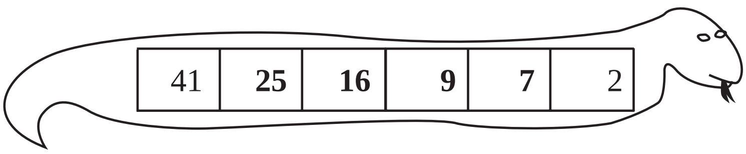
(f)

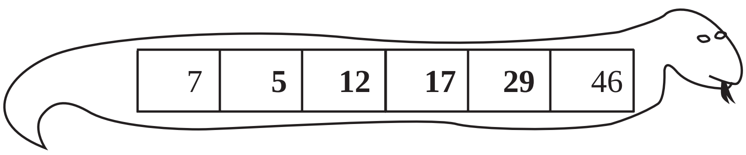
2		
3		
1	5	0

Worksheet 2









Worksheet 3

Magic squares

1. (a)

4	**9**	2
3	5	**7**
8	**1**	6

(b)

41	113	**59**
81	71	**53**
83	29	101

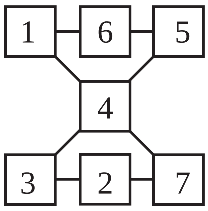
(c)

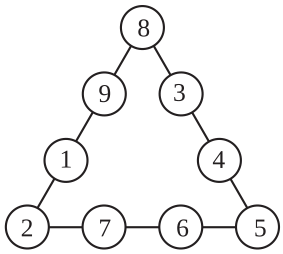
15	7	**8**
3	10	**17**
12	13	**15**

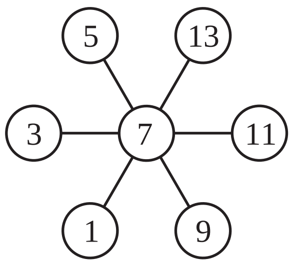
2.

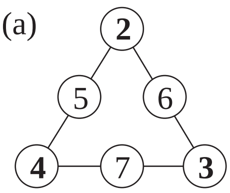
10	**5**	**6**
3	**7**	**11**
8	9	4

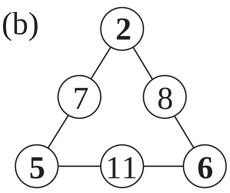
Arithmogons

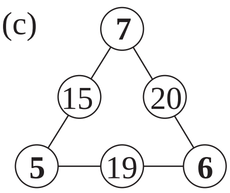
1. 

2. 

3. 

4. (a) 

(b) 

(c) 

Worksheet 4

Missing numbers

1. (a) $\boxed{9} \times \boxed{9} = 81$ (b) $\boxed{16} \times \boxed{16} = 256$

(c) $\boxed{25} \times \boxed{25} = 625$ (d) $\boxed{71} \times \boxed{71} = 5041$

2. (a) $\boxed{11} \times \boxed{12} = 132$ (b) $\boxed{26} \times \boxed{27} = 702$

(c) $\boxed{19} \times \boxed{20} = 380$ (d) $\boxed{52} \times \boxed{53} = 2756$

3. (a) $73 \times \boxed{5}6 = 4088$ (b) $\boxed{86} \times \boxed{4}7 = 4042$

4. (a) $\begin{array}{r} 27\boxed{7} \\ +\,1\boxed{4}3 \\ \hline \boxed{4}20 \end{array}$ (b) $\begin{array}{r} 46 \\ \times\boxed{7} \\ \hline \boxed{3}\boxed{2}2 \end{array}$ (c) $\begin{array}{r} 3\boxed{8} \\ 4\,\overline{)\,\boxed{1}52} \end{array}$

Cards

5. Move the “9” into the set (a) so that each set adds up to 15.

Worksheet 5

1. (a) divide by 5 (b) add 12

2.

7	add 5 →	12	divide by 2 →	6
7	multiply by 4 →	**28**	subtract 10 →	**18**
7	add l →	**8**	multiply by 8 →	**64**

3. (a)

	A		B	
7 →	multiply by 4	→ 28 →	subtract 10	→ 18
3 →		→ 12 →		→ 2
11 →		→ 44 →		→ 34
18 →		→ 72 →		→ 62

(b) 15 ⟶ 60 ⟶ 50

20 ⟶ 80 ⟶ 70

4.

3	multiply by 5 →	**15**	add 33 →	**48**
50	subtract 30 →	**20**	divide by 5 →	4
21	add l5 →	**36**	multiply by 3 →	108
100	divide by 4 →	**25**	subtract 12 →	13

Worksheet 6

2. (b) $54321 \times 9 - 1 = 434\ 568$

$654321 \times 9 - 1 = 5\ 234\ 568$

$7654321 \times 9 - 1 = 61\ 234\ 568$

Worksheet 7

1. (b)

row 1					1					1
row 2				1	2	1				4
row 3			1	2	3	2	1			9
row 4		1	2	3	4	3	2	1		16
row 5	**1**	**2**	**3**	**4**	**5**	**4**	**3**	**2**	**1**	**25**

(c) Row 6

2. (a)

Number of hexagons	1	2	3	4	5	6
Number of edges on perimeter	**6**	**10**	**14**	**18**	**22**	**26**

(b) 402

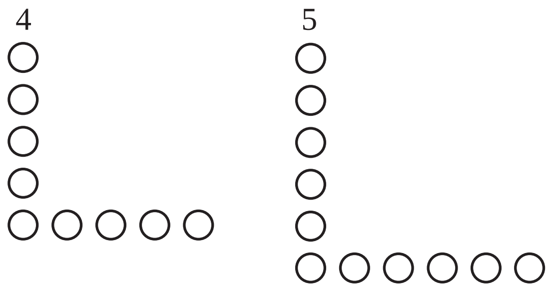
3. (a) (i) (ii) (iii)

(i)	(ii)	(iii)
$(4 \times 7) + 3 = 31$	$(4 \times 9) - 4 = 32$	$(4 \times 8) - 3 = 29$
$(5 \times 7) + 3 = 38$	$(5 \times 9) - 4 = 41$	$(5 \times 8) - 3 = 37$

(b) What is the increase in the answers from one line to the next?

(i) + 7 (ii) + 9 (iii) + 8

4. (a)



(b)

Dot pattern	1	2	3	4	5
Number of dots	3	5	7	9	11

The increase in dots is by + 2.

Worksheet 8

1. (a)

Fr	Fg	Fb	Fo	Fp
Mr	**Mg**	**Mb**	**Mo**	**Mp**
Br	**Bg**	**Bb**	**Bo**	**Bp**

(b)

Lp	Ll	Lg
Mp	Ml	Mg
Sp	Sl	Sg

2. (a)

Bg	Bb	Bm
Jg	**Jb**	**Jm**
Sg	**Sb**	**Sm**
Tg	**Tb**	**Tm**

(b)

Blouse	B	B	B									
Jersey				**J**	**J**	**J**						
Skirt							**S**	**S**	**S**			
Trousers										**T**	**T**	**T**
grey	■			■			■			■		
blue		■			■			■			■	
maroon			■			■			■			■

12 uniform iitems

3. (b)

Small (S)	S	S	S	S	S	S												
Medium (M)							**M**	**M**	**M**	**M**	**M**	**M**						
Large (L)													**S**	**S**	**S**	**S**	**S**	**S**
Red (r)	r	r					**r**	**r**					**r**	**r**				
Blue (b)			b	b					**b**	**b**					**b**	**b**		
Green (g)					g	g					**g**	**g**					**g**	**g**
Canvas	■		■		■		■		■		■		■		■		■	
Plastic		■		■		■		■		■		■		■		■		■

18 varieties of bags.

Worksheet 9

1. Bweupe(B), Moyo(M), Tsepo(T), Kumalo(K)
   (a) Their possible sitting arrangements:

BMTK	MBTK	TBMK	KBMT
BTMK	MBKT	TBKM	KBTM
BMKT	MKBT	TKMB	KTMB
BKTM	MKTB	TKBM	KTBM
BTKM	MTBK	TMBK	KMBT
BKMT	MTKB	TMKB	KMTB

(b) 24

(c) 12

2. (a) 3B + 3Y, 3B + 3Y, 2Y + 2R

   (b) 3 boxes.

Answers to Unit 5 Test

1.

7	3	1	0
			2
			4
			5

2. 3 → add 12, divide by 3 → 5

   9 → add 12, divide by 3 → 7

   15 → add 12, divide by 3 → 9

   0 → add 12, divide by 3 → 4

3.

Dot pattern	1	2	3	4	5
Number of dots	1	**5**	**9**	**13**	**17**

(i) by + 4

(ii) 4n – 3

(iii) 397

4. (a) RGB, RGY, RGY, RBY, RBY, YBG

   (b) GBYRRY, GBYRRY, YRGBBG or
   GBYRGY, GBYRRY, GBYRRB or
   GBYRBY, GBYRRY, GBYRRG

Unit 6: Number Bases

Introduction

In Units 2–5, you worked with whole numbers in base ten. In this unit, you will extend your knowledge of base ten to counting, addition, and subtraction in other bases.

A note of caution! This unit extends your knowledge of arithmetic to other bases, and your syllabus may require that other-base arithmetic be taught to students in your grade. However, current thinking about maths pedagogy is strongly against the general teaching of other-base arithmetic. It is now considered an enrichment topic only, and just for maths-adept students.

Arithmetic in other bases became a common teaching topic in the early 1960s, the thought being that it would prepare students for the new computer age. However, experience has shown this not to be the case for the majority of students. A comparison can be made with the use of the abacus and proportional models that were used to teach base-ten arithmetic in Unit 2. The models' benefit is that they tend to include the less able students. With the tactile assistance of an abacus, more students grasp the place-value concept on the first pass than would be the case if they learned just through numbers on a page.

Teaching other bases, even with appropriate models, seems to have the opposite effect. It excludes the less maths-able students, presenting concepts which only the brighter students can merge into their growing sense of numeracy. For this reason, and also because other-base arithmetic has not proven to be a required general skill in the computer age, those North American jurisdictions which pioneered other number bases in upper primary in the 1960s have mostly abandoned them now.

Objectives

After working through this unit, you should be able to:

- demonstrate how to count in other bases and convert from one base to another
- illustrate addition in other bases using models
- illustrate subtraction in other bases
- demonstrate how to add and subtract using different algorithms
- analyse errors and provide appropriate remedial work

Counting in Other Bases

The earliest form of counting was done with fingers. As such, number names in African languages are in base five and ten.

In the lower grades in the primary education, pupils concentrate on base ten. After pupils have mastered working in base ten, at a later stage in upper primary, they are exposed to counting, adding, and subtracting in base five and eight. At the secondary level, other bases are introduced, including base two.

Base ten is the most important base, followed by base sixteen (hexadecimal or "hex"). The latter is often used to write the numbers used by computers. If your students go on to design Web pages, for example, they may use hexadecimal numbers to specify the screen colours. Since each hex digit represents exactly four binary (base two) digits, a hex digit effectively is a shorthand for the binary digits used inside the computer. Another number base system is the base twelve or duodecimal system. Known popularly as the "dozens" system, it is used only for teaching the concepts of other number bases in school.

The base name indicates the number of digits that each base uses. For instance:

- Base ten uses ten digits—0, 1, 2, 3, 4, 5, 6, 7, 8, and 9
- Base five uses five digits—0, 1, 2, 3, and 4
- Base eight uses eight digits—0, 1, 2, 3, 4, 5, 6, and 7

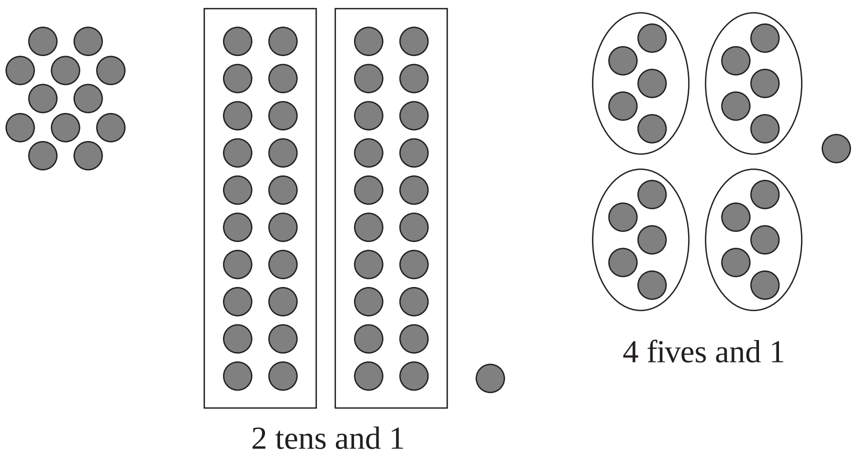
When the base is not stated, we assume it is base ten.

Reflection

Why do we not introduce base two at the primary level?

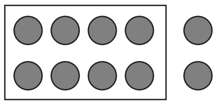
Counting from base ten to base five or eight using sets

For this activity, you will need a collection of small stones. Group the stones in tens and later in fives, then in eights. Count how many groups and leftovers in each situation and record your findings.

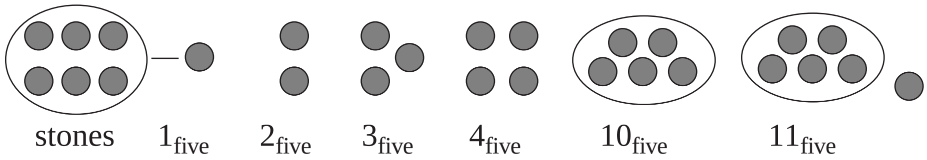


There is a standard way of recording sets. Four fives and 1 is written as 41_{five} (read "four one base five"). Two eights and 5 is written as 25_{eight} (read "two five base eight"). It is very important to state the base, otherwise we will not be able to tell the value of the number (meaning of digits).

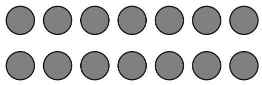
Now look at the following picture. What base is being used? What number does the picture represent? (12_{eight}).



Now let us count in base five. What digits are used in base five? (0, 1, 2, 3, 4). How many stones are in this picture? Count in base five.



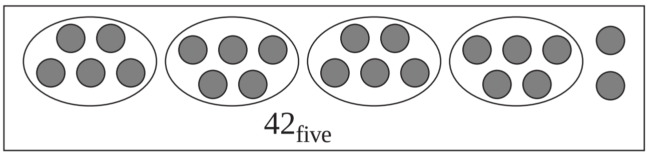
There are 11_{five} stones.
What digits are used in base eight? Now you count these in base ten then in base eight.



In base ten: 1, 2, 3, 4,…, 13, 14
In base eight: 1_{eight}, 2_{eight}, …, 15_{eight}, 16_{eight}

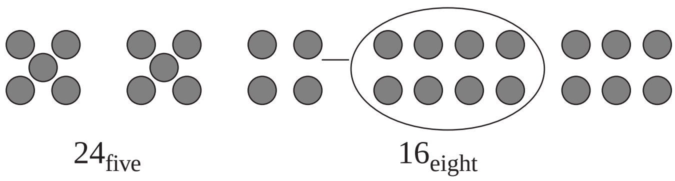
Change from base five or eight to base ten:

- Change 42_{five} to base ten
- Show 42_{five} using pictures



Now count in base ten. How many stones are in the box? There are twenty-two stones.

Change 24_{five} to base eight. How can you change from base five to eight in early stages? Show 24_{five} using models. Then count in base eight.



Change the stones to base eight by grouping.

Counting in base twelve

Can you tell the number of digits in base twelve and base two?

In base twelve, new symbols are created for two extra digits: 10_{ten} is T, 11_{ten} is E and 12_{ten} is 10_{twelve}. Why are there new symbols for these numbers?

Self Assessment 1

Counting in other bases

1. Describe visual representations of thirty counters in base twelve and base two. Record the number in standard form in each base. Include the base when you write the number.
2. Find the numbers preceding and succeeding each of the following:

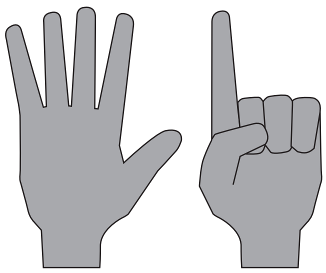
   (a) $EE0_{twelve}$ (b) 555_{six} (c) 10000_{two}
3. Count from 1 to 17 in base sixteen and record the numerals as you count.

Practice Activity 1

After pupils have acquired the concept of counting in other bases have them sing this base five song.

One finger is working (three times)
Chorus
Urah urah urah
Two fingers are working (three times)
Three fingers are working
Four fingers are working
One hand is working
One hand, one finger are working (three times)
One hand two fingers are working
One hand three fingers are working
One hand four fingers are working
Two hands are working (now clap to the beat of the song)

As you sing, show the numbers. For example:



You can extend the song as far as it makes sense to you.

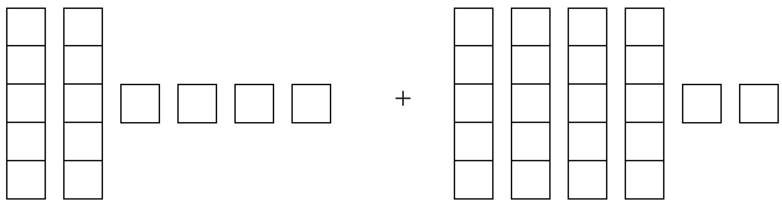
- Let pupils tell you the number of claps for four hands.
- Have the pupils tell you the mathematical concepts involved in this song.
- Ask pupils how far they can extend this song before it loses meaning.

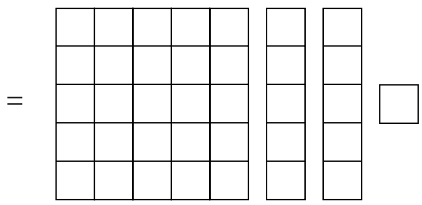
Developing Addition in Other Bases

In developing an algorithm for other bases, use concrete teaching aids such as multibase blocks and bean sticks before using the abstract approach of numerals only.

Remember that translating into base ten is against the concept of performing the four operations in other bases. Students should try to think in the given base.

(a) Using multibase blocks (flats, sticks, single blocks)





$$\begin{array}{r} 24_{five} \\ +\ 42_{five} \\ \hline 121_{five} \end{array}$$

(b)

Five-fives	Fives	Ones
	2	4
	4	2
1	2	1

(c)

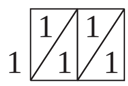
$$\begin{array}{r} 24_{five} \\ +\ 42_{five} \\ \hline 121_{five} \\ \hline 1 \end{array}$$

Algorithms in other bases

Transfer the algorithms you learned in base ten to other bases.

(a) Lattice method, or "carry one" diagonally

$$\begin{array}{rrr} & 2 & 4_{five} \\ + & 4 & 2_{five} \end{array}$$



$$\begin{array}{rrr} 1 & 2 & 1_{five} \end{array}$$

(b) Intermediate algorithms

$$
\begin{array}{rccc}
 & & 2 & 4_{\text{five}} \\
+ & & 4 & 2_{\text{five}} \\
\hline
 & & 1 & 1 \\
 & 1 & 1 & 0 \\
\hline
 & 1 & 2 & 1_{\text{five}}
\end{array}
$$

(c) Scratch

$24_{\text{five}} + 42_{\text{five}} + 34_{\text{five}}$

$$
\begin{array}{rccc}
 & & \overset{2}{\cancel{2}} & 4_{\text{five}} \\
+ & & \overset{3}{\cancel{4}} & \overset{1}{\cancel{2}}_{\text{five}} \\
+ & & \overset{1}{\cancel{3}} & \overset{0}{\cancel{4}}_{\text{five}} \\
\hline
 & 2 & 1 & 0_{\text{five}}
\end{array}
$$

Self Assessment 2

Adding in other bases:

1. Illustrate $23_{\text{twelve}} + 42_{\text{twelve}}$ using multibase blocks.
2. Jane wrote:

$$
\begin{array}{r}
22_{\text{five}} \\
33_{\text{five}} \\
\hline
55_{\text{five}}
\end{array}
$$

What is Jane's error? What is the cause of this error? How can you help her?

Practice Activity 2

1. Addition table for base five.
   - Pair the pupils
   - Have them build up the addition table for base five
2. Snaps Game
   - After pupils have built the addition table for base five, let them play snaps game in pairs
   - Give each player a set of plus name cards for addition facts for base five
   - Have players shuffle the cards
   - Each player turns the cards face down in front of herself or himself

Continues on next page.

- Players turn over their top cards at the same time. If the cards name the same number, the first player to say "snap" and state the correct sum takes the two matching cards
- If the cards do not match and a player says "snap" the other player takes the two cards. A player who takes the two cards puts them at the bottom of his/her pile
- If no player says "snap" each player puts the card at the bottom of her/his pile
- The game ends when a player has no cards
- The player with cards is the winner

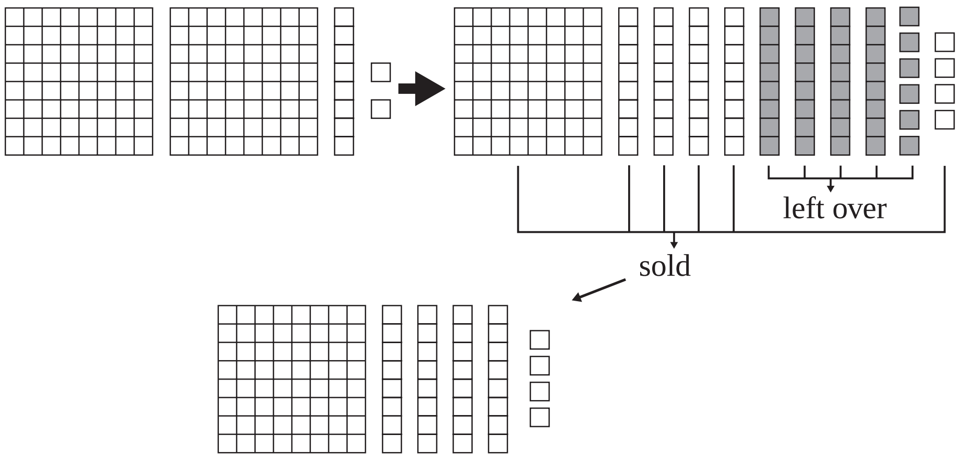
N.B. You can make variations to this game or design your own.

Subtraction in other Bases

Sibeso has 212_{eight} bananas. She told her brother to sell the bananas. When Sibeso came back she found only 46_{eight} bananas. How many bananas did her brother sell? ($212_{eight} - 46_{eight}$)

Visualise the unsold bananas in base ten ($212_{eight} - 46_{eight}$)

(a) Subtraction using multibase blocks



144_{eight} bananas were sold.

(b)

Eight-eights	Eights	Ones
2	1	2
	4	6

Eight-eights	Eights	Ones
2	0	12
	4	6
		4

Eight-eights	Eights	Ones
1	10	12
	4	6
1	4	4

(c) Subtracting from the base

$$\begin{array}{r} 2\ 1\ 2 \\ -\quad 4\ 6 \\ \hline 1\ 4\ 4 \end{array}$$

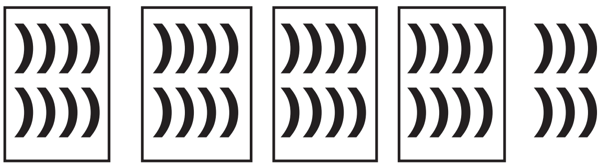
First step - $10_{eight} - 6_{eight} + 2_{eight}$

Second step - $100_{eight} - 40_{eight} + 0_{eight}$

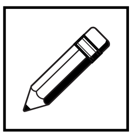
Third step - $100_{eight} - 0_{eight}$

Now you can use the algorithms you encountered in base ten to subtract in other algorithms.

Visualising 46_{eight} bananas



$46_{eight} = 38$



Self Assessment 3

Subtraction in other bases

1. Illustrate subtraction of $22_{three} - 11_{three}$ on a number line.
2. What is Andrew's error? How can you help him?

$$\begin{array}{r} \overset{5}{\cancel{6}}\ \overset{1}{\cancel{3}}_{eight} \\ -\ 1\ \ 7_{eight} \\ \hline 4\ \ 6_{eight} \end{array}$$

3. Use the complementary method and equal addition to subtract $32_{five} - 14_{five}$.

Practice Activity 3

Subtracting in base five and eight:

- In co-operative learning groups, let pupils use a model of their choice to represent $45_{eight} - 26_{eight}$
- Have them verbalise and record their procedures
- Demonstrate how to subtract in other bases using equal additions

Self Assessment 4

1. When we read numerals aloud in base ten, we indicate place value. Why don't we do the same when we read other bases? For instance, we read 245 in base ten as two hundred and forty five, but when we read 245_{eight}, we simply say "Two four five base eight".
2. Illustrate using a number line $101_{two} + 11_{two}$.
3. Fill in the missing number in the following:

$$
\begin{array}{rccccc}
 & 2 & 0 & 0 & 1 & 0_{three} \\
- & & 2 & \_ & 2 & \_{}_{three} \\
\hline
 & 1 & \_ & 2 & \_ & 1_{three}
\end{array}
$$

4. Your friend finds a sheet of paper on your table and thinks you made a computational error. What base did you use?

$$
\begin{array}{r}
32 \\
-13 \\
\hline
15
\end{array}
$$

5. Perform this operation

7 gross	2 dozens	1 ones
– 1 gross	6 dozens	8 ones

Hint: gross = 144 ones

Summary

Working with other bases is an enrichment activity for the upper grades—use it cautiously with your less math-adept students. Working with other bases also allows teachers to appreciate the problems faced by the pupils when they learn base ten for the first time.

Place-value materials, such as counters and flats, sticks, and multibase blocks are useful for clarifying the meaning of other bases.

Pupils need to engage in activities that will help them add and subtract in other bases.

The decomposition method of subtraction is emphasised because it is easily modelled with any place-value devices. The mental processes for subtraction includes subtracting the partial subtrahend from 10 and adding the partial minuend where decomposition is involved. The use of various algorithms in other bases promote creativity.

Unit 6 Test: Number Bases

1. Use models to express the following in the base given. State your process.
   a) 27 in base eight
   b) 103_{five} to base eight
2. Fill in the missing number in the stated base:
   a) Christmas comes on December ___________ (base five).
   b) In a leap year, February has ___________ days (base twelve).
3. Describe how to subtract 1 2 3 from 2 1 4 in base eight, using models, then the abstract approach.
4. Use scratch method (algorithms) to add:
   $14_{\text{five}} + 24_{\text{five}} + 43_{\text{five}} + 22_{\text{five}} + 32_{\text{five}} + 40_{\text{five}}$

Unit 6: Answers to Self Assessments

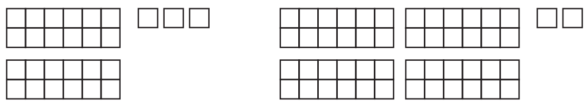
Self Assessment 1

1. Thirty counters in base twelve is 2 groups of 12 counters and 6 more counters, or 26_{twelve}. Thirty counters in base two is 15 groups of 2 or 150_{two}.
2. (b) 554_{six}, 556_{six}

   (c) 1001_{two}, 10001_{two}
3. $1_{sixteen}$, $2_{sixteen}$, $3_{sixteen}$, $4_{sixteen}$, $5_{sixteen}$, $6_{sixteen}$, $7_{sixteen}$, $8_{sixteen}$, $9_{sixteen}$, $10_{sixteen}$, $11_{sixteen}$, $12_{sixteen}$, $1_{sixteen}3$, $14_{sixteen}$, $15_{sixteen}$, $100_{sixteen}$, $101_{sixteen}$

Self Assessment 2

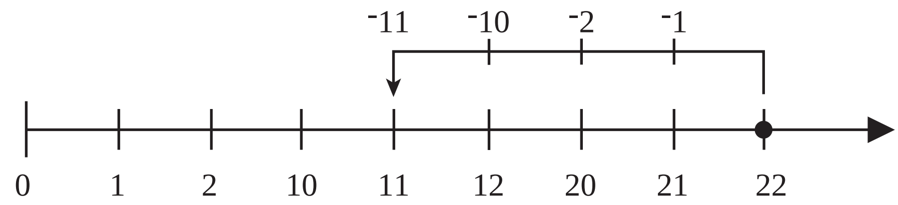
1.



2. 110_{five}

Self Assessment 3

1.



answer: 11_{three}

2. 44_{eight}
3.

$$\begin{array}{r} 3\ 2 \\ -\ 1\ 4 \\ \hline 1\ 3 \end{array}$$

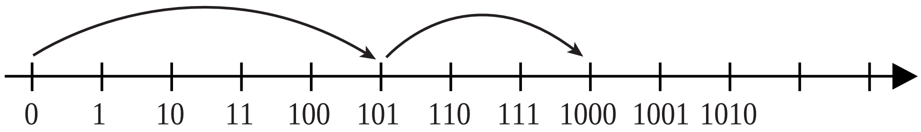
$10_{five} - 6_{five} + 2_{five}$

$20_{five} - 10_{five}$

Self Assessment 4

1. It seems we don't have appropriate power names in other bases. If we try to show meanings of digits, we end up translating the powers in base ten. For example, 245_{eight} could be read as two sixty-fours, four eights, and five. This type of reading is more confusing, unless we come up with meaningful power names within each base.

2.



$$101_{two} + 11_{two} = 1000_{two}$$

3.

$$\begin{array}{cccccc}
 & 2 & 0 & 0 & 1 & 0_{three} \\
- & & 2 & \underline{0} & 2 & \underline{2}_{three} \\
\hline
 & 1 & \underline{0} & 2 & \underline{1} & 1_{three}
\end{array}$$

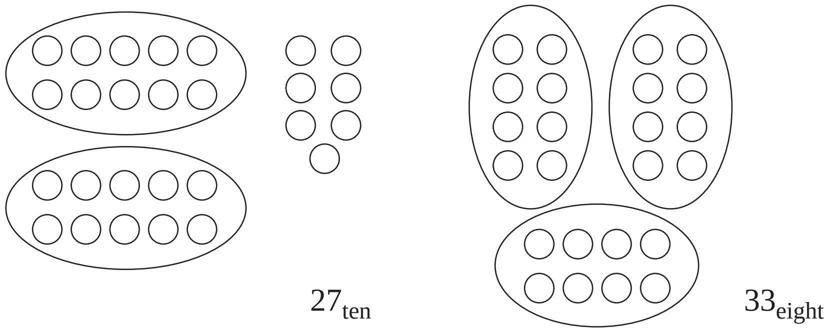
4. Base six

5.

$$\begin{array}{lll}
\overset{6}{\cancel{7}} \text{ gross} & \overset{12}{\cancel{2}} \text{ dozens} & \overset{12}{\cancel{1}} \text{ ones} \\
1 \text{ gross} & 6 \text{ dozens} & 8 \text{ ones} \\
\hline
5 \text{ gross} & 7 \text{ dozens} & 11 \text{ ones}
\end{array}$$

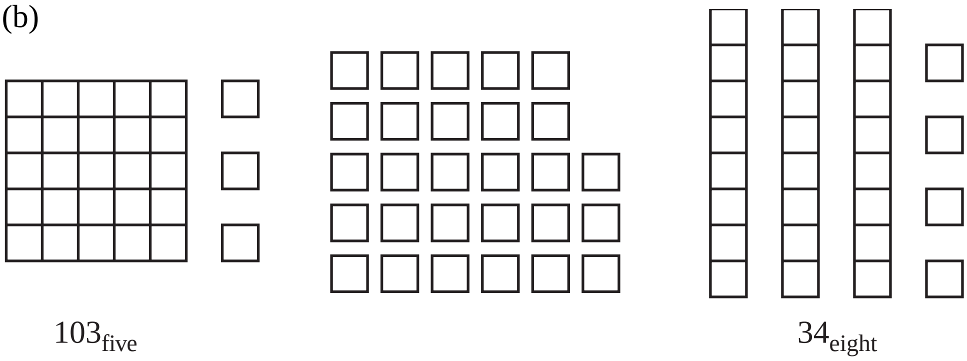
Answers for Unit 6 Test

1. (a)



Identify 27 by having two groups of tens and seven ones. Then group into eight by counting 1, 2, 3, 4, 5, 6, 7, 10 and circle the set. Count the number of circles and number of leftovers.

(b)



Make a diagram representation of 100_{five} + 3. Then break up 100_{five} into ones and add.

Recount in base eight and when you make a set of eight (10_{eight}), draw a stick. Count the number of sticks left over.

2. (a) 100_{five} (b) 24_{twelve}

3.

$$\begin{array}{rrrl} & 2 & 1 & 4_{eight} \\ - & 1 & 2 & 3_{eight} \\ \hline & & 7 & 1_{eight} \end{array}$$

$4_{eight} - 3_{eight}$

$10_{eight} - 2_{eight} + 1_{eight}$

$100_{eight} - 100_{eight} = 0$

References

Burger B. F. and Musser, G.L., *Mathematics for Elementary Teachers*, 1988, New York, Macmillan Publishing Company.

Billstein, R. Libeskind, S, Lott, J.W., *Mathematics for Elementary School Teachers*, 1990, Redwood City, The Benjamin/Cummings Publishing Company Inc.