Module 2

# Upper Primary Mathematics

## Fractions

The Commonwealth of Learning

**Science, Technology and Mathematics Modules**

**for Upper Primary and Junior Secondary School Teachers of Science, Technology and Mathematics by Distance in the Southern African Development Community (SADC)**

**Developed by**
**The Southern African Development Community (SADC)**

**Ministries of Education in:**

- **Botswana**
- **Malawi**
- **Mozambique**
- **Namibia**
- **South Africa**
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## SCIENCE, TECHNOLOGY, AND MATHEMATICS MODULES

This module is one of a series prepared under the auspices of the participating Southern African Development Community (SADC) and The Commonwealth of Learning as part of the Training of Upper Primary and Junior Secondary Science, Technology and Mathematics Teachers in Africa by Distance. These modules enable teachers to enhance their professional skills through distance and open learning. Many individuals and groups have been involved in writing and producing these modules. We trust that they will benefit not only the teachers who use them, but also, ultimately, their students and the communities and nations in which they live.

The twenty-eight Science, Technology, and Mathematics modules are as follows:

**Upper Primary Science**
Module 1: *My Built Environment*
Module 2: *Materials in my Environment*
Module 3: *My Health*
Module 4: *My Natural Environment*

**Junior Secondary Science**
Module 1: *Energy and Energy Transfer*
Module 2: *Energy Use in Electronic Communication*
Module 3: *Living Organisms' Environment and Resources*
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Module 1: *Teaching Technology in the Primary School*
Module 2: *Making Things Move*
Module 3: *Structures*
Module 4: *Materials*
Module 5: *Processing*

**Junior Secondary Technology**
Module 1: *Introduction to Teaching Technology*
Module 2: *Systems and Controls*
Module 3: *Tools and Materials*
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**Upper Primary Mathematics**
Module 1: *Number and Numeration*
Module 2: *Fractions*
Module 3: *Measures*
Module 4: *Social Arithmetic*
Module 5: *Geometry*

**Junior Secondary Mathematics**
Module 1: *Number Systems*
Module 2: *Number Operations*
Module 3: *Shapes and Sizes*
Module 4: *Algebraic Processes*
Module 5: *Solving Equations*
Module 6: *Data Handling*

## A MESSAGE FROM THE COMMONWEALTH OF LEARNING

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*Dato' Professor Gajaraj Dhanarajan*
President and Chief Executive Officer

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## CONTACTS FOR THE PROGRAMME

The Commonwealth of Learning
1285 West Broadway, Suite 600
Vancouver, BC V6H 3X8
Canada

Ministry of Education
Private Bag 005
Gaborone
Botswana

Ministry of Education
Private Bag 328
Capital City
Lilongwe 3
Malawi

Ministério da Eduação
Avenida 24 de Julho No 167, 8
Caixa Postal 34
Maputo
Mozambique

Ministry of Basic Education,
Sports and Culture
Private Bag 13186
Windhoek
Namibia

National Ministry of Education
Private Bag X603
Pretoria 0001
South Africa

Ministry of Education and Culture
P.O. Box 9121
Dar es Salaam
Tanzania

Ministry of Education
P.O. Box 50093
Lusaka
Zambia

Ministry of Education, Sport and Culture
P.O. Box CY 121
Causeway
Harare
Zimbabwe

## MODULE WRITERS


**Mr. Yamboto Mumbula:** *Writing Team Leader*
Senior Inspector of Schools
Ministry of Education
Lusaka, Zambia

**Ms. Patricia Mbumwae:** Lecturer (Primary Mathematics)
NISTCOL, Chalimbana
Lusaka, Zambia

**Mr. Zanzini B. Ndhlovu:** Lecturer (Maths Methods)
University of Zambia
Lusaka, Zambia

**Mr. Bentry Nkhata:** Lecturer (Maths Methods)
University of Zambia
Lusaka, Zambia


## FACILITATORS/RESOURCE PERSONS


**Mr. Simon Chiputa:** Co-ordinator, Teacher Education
Ministry of Education
Lusaka, Zambia


## PROJECT MANAGEMENT & DESIGN


**Ms. Kgomotso Motlotle:** Education Specialist, Teacher Training
The Commonwealth of Learning (COL)
Vancouver, BC, Canada

**Mr. David Rogers:** *Post-production Editor*
Open Learning Agency
Victoria, BC, Canada

**Ms. Sandy Reber**: *Graphics & desktop publishing*
Reber Creative
Victoria, BC, Canada

## UPPER PRIMARY MATHEMATICS PROGRAMME

### Introduction

Welcome to the programme in Teaching Upper Primary Mathematics. This series of five modules is designed to help you strengthen your knowledge of mathematics topics and acquire more instructional strategies for teaching mathematics in the classroom.

Each of the five modules in the mathematics series provides an opportunity to apply theory to practice. Learning about mathematics entails the development of practical skills as well as theoretical knowledge. Each topic includes examples of how mathematics is used in practice and suggestions for classroom activities that allow students to explore the maths for themselves.

Each module also explores several instructional strategies that can be used in the mathematics classroom and provides you with an opportunity to apply these strategies in practical classroom activities. Each module examines the reasons for using a particular strategy in the classroom and provides a guide for the best use of each strategy, given the topic, context, and goals.

The guiding principles of these modules are to help make the connection between theory and practice, to apply instructional theory to practice in the classroom situation, and to support you, as you, in turn, help your students to apply mathematics to practical classroom work.

### Programme Goals

This programme is designed to help you:

- strengthen your understanding of mathematics topics
- expand the range of instructional strategies that you can use in the mathematics classroom

### Programme Objectives

By the time you have completed this programme, you should be able to:

- guide students as they work in teams on practical projects in mathematics, and help them to work effectively as members of a group
- use questioning and explanation strategies to help students learn new concepts and to support students in their problem solving activities
- prepare your own portfolio of teaching activities

### *The relationship between this programme and the mathematics curriculum*

The content presented in these modules includes some of the topics most commonly covered in the mathematics curricula in southern African countries. However, it is not intended to comprehensively cover all topics in any one country's mathematics curriculum. For this, you need to consult your national or regional curriculum guide. The curriculum content presented in these modules is intended to:

- provide an overview of the content in order to support the development of appropriate teaching strategies
- use selected parts of the curriculum as examples of the application of specific teaching strategies
- explain those elements of the curriculum that provide essential background knowledge, or that address particularly complex or specialised concepts
- provide directions to additional resources on the curriculum content

## How to work on this programme

As is indicated in the goals and objectives, this programme requires you to participate actively in each module by applying instructional strategies when exploring mathematics with your students and by reflecting on that experience. There are several ways to do this.

### *Working on your own*

You may be the only teacher of mathematics in your school, or you may choose to work on your own so you can accommodate this programme within your schedule. If this is the case, these are the recommended strategies for using this module:

1. Establish a schedule for working on the module. Choose a date by which you plan to complete the first module, taking into account that each unit will require between six and eight hours of study time and about two hours of classroom time to implement your lesson plan. For example, if you have two hours a week available for study, then each unit will take between three and four weeks to complete. If you have four hours a week for study, then each unit will take about two weeks to complete.
2. Choose a study space where you can work quietly without interruption, such as a space in your school where you can work after hours.
3. If possible, identify someone who is interested in mathematics or whose interests are relevant to it (for example, a science teacher in your school) with whom you can discuss the module and some of your ideas about teaching mathematics. Even the most independent learner benefits from good dialogue with others. It helps us to formulate our ideas—or as one learner commented, "How do I know what I'm thinking until I hear what I have to say?"

### *Working with colleagues*

If there are other teachers of mathematics in your school or in your immediate area, then it may be possible for you to work together on this module. You may choose to do this informally, perhaps having a discussion group once a week or once every two weeks about a particular topic in one of the units. Or, you may choose to organise more formally, establishing a schedule so that everyone is working on the same units at the same time, and you can work in small groups or pairs on particular projects.

Your group may also have the opportunity to consult with a mentor, or with other groups, by teleconference, audioconference, letter mail, or e-mail. Check with the local coordinator of your programme about these possibilities so you can arrange a group schedule that is compatible with these provisions.

### *Colleagues as feedback/resource persons*

Even if your colleagues are not participating directly in this programme, they may be interested in hearing about it and about some of your ideas as a result of taking part. Your head teacher or the local area specialist in mathematics may also be willing to take part in discussions with you about the programme.

### *Working with a mentor*

As mentioned above, you may have the opportunity to work with a mentor, someone with expertise in maths education who can provide feedback about your work. If you are working on your own, communication with your mentor may be by letter mail, telephone, or e-mail. If you are working as a group, you may have occasional group meetings, teleconferences, or audioconferences with your mentor.

### *Resources available to you*

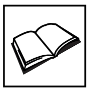
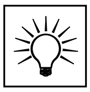
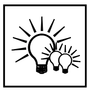
Although these modules can be completed without referring to additional resource materials, your experience and that of your students can be enriched if you use other resources as well. A list of resource materials is provided at the end of each module. You might also find locally available resource material that will enhance the teaching/learning experience. These include:

- manipulatives, such as algebra tiles, geometry tiles, and fraction tiles
- magazines with articles about maths
- books and other resources about maths that are in your school or community library

# ICONS

Throughout each module, you will find some or all of the following icons or symbols that alert you to a change in activity within the module.

Read the following explanations to discover what each icon prompts you to do.

| | | |
|---|---|---|
|  | **Introduction** | Rationale or overview for this part of the course. |
|  | **Learning Objectives** | What you should be able to do after completing this module or unit. |
|  | **Text or Reading Material** | Course content for you to study. |
|  | **Important—Take Note!** | Something to study carefully. |
|  | **Self-Marking Exercise** | An exercise to demonstrate your own grasp of the content. |
|  | **Individual Activity** | An exercise or project for you to try by yourself and demonstrate your own grasp of the content. |
|  | **Classroom Activity** | An exercise or project for you to do with or assign to your students. |
|  | **Reflection** | A question or project for yourself—for deeper understanding of this concept, or of your use of it when teaching. |
|  | **Summary** | |
|  | **Unit or Module Assignment** | Exercise to assess your understanding of all the unit or module topics. |
|  | **Suggested Answers to Activities** | |

# CONTENTS

## Module 2: Fractions

# Module 2
# Fractions

## Introduction to the Module

This module is about teaching fractional numbers, fractional numerals, and their manipulation. Algorithms for manipulating fractional numbers and numerals only became feasible with the advent of the Hindu-Arabic numeral system. Just try to add one-third plus one-fourth using Roman numerals! Learning these fraction algorithms, and the reasoning behind them, is among the most important tasks in Upper Primary Mathematics.

## Aim of the Module

The aim of the series of modules is to attempt to guide teachers to make teaching of mathematics simple, enjoyable, and understandable. In particular, this module explores ways to consolidate the manipulation of fractions in the early stages of learning. It also explores interesting and practical ways of using fractional numbers and their operations.

## Structure of the Module

There are six units in this module. Unit 1 is a review of common fractions. Unit 2 discusses the two operations of addition and subtraction. In Unit 3 we look at multiplication and division operations. Units 4 through 6 extend the basic concepts to decimals, ratios, and percentages. The module offers a series of unit and practice activities to help you consolidate the content and teaching strategies. The self-assessment exercises will help you evaluate your methods.

## Objectives of the Module

The specific objectives of the module are to:

- help teachers consolidate the basic concepts of fractional numbers
- explore interesting and practical ways of teaching fractional numbers
- introduce teachers to problem-solving skills

# Unit 1: Common Fractions

## Introduction

In Module 1, we worked with whole numbers. But whole numbers alone cannot help us to solve mathematical problems which involve parts of whole numbers. To do this, we need to extend the number system to include rational numbers. In this module, we will work with rational numbers. Under rational numbers, we have fractions such as common fractions, ratios, decimals, and percentages. In this unit, we will deal with types of common fractions, equivalent fractions, and comparing common fractions. Later units deal with the types of fractions and the four operations on common fractions.

## Objectives

At the end of this unit you should be able to:

- distinguish between a common fraction and a rational number
- demonstrate meaning of fractions using various models
- develop the concept of fractional parts
- investigate the effect of an increasing denominator on the value of a fraction
- illustrate types of common fractions
- explain how to compare common fractions using concepts and not rules
- demonstrate how to find equivalent fractions using models
- develop an equivalent fraction algorithm

## Common Fractions and Rational Numbers

The word *fraction* comes from the Latin word *frangere* which means ‘to break’. A common fraction is written in the $\frac{a}{b}$ where $a$ and $b$ are whole numbers; $a < b$ and $b \neq 0$. A rational number takes the same form but $a$ and $b$ are integers and $b \neq 0$ (for instance $\frac{3}{4}$, $\frac{^-5}{8}$, $\frac{16}{^-32}$, $\frac{7}{1}$).

### Reflection

Is every common fraction a rational number? How do you know?

### Unit Activity 1

Investigate the following problem. Knowing that 2 is not an integer, is $\frac{2}{4}$ a common fraction or rational number?

### *Meaning of Common Fractions*

To help pupils grasp the concept of common fractions, you should expose pupils to more than one definition. A common fraction may be looked at as a part of a whole, as an expression of division, or as a ratio.

**Fraction as a part of whole:** a unit in the form of a continuous shape or as a discrete set is partitioned into equal sized parts.

**Fraction as an expression of division:** if a woman walks 2 km per hour, how many hours will it take to walk 5 km? She takes $5 \div 2$ or $\frac{5}{2}$ hours.

A fraction $\frac{a}{b}$ is another way of writing $a \div b$.

**Fraction as an expression of ratios:** ratios are expressions of a relationship between two quantities. If two out of five pupils are boys, then the ratio of boys to girls can be expressed as 2 to 5 or 2:5 or $\frac{2}{5}$. The fraction notation of ratios is found in proportion.

### *Fraction Models*

Besides the numerical interpretation of common fractions, you should use a variety of models such as area, length, and set models. As you go through this unit, think of other model presentations.

**Area Models**

Area models can be used to demonstrate visually that a fraction is part of a whole. *Figure 1.1* illustrates a variety of area models.

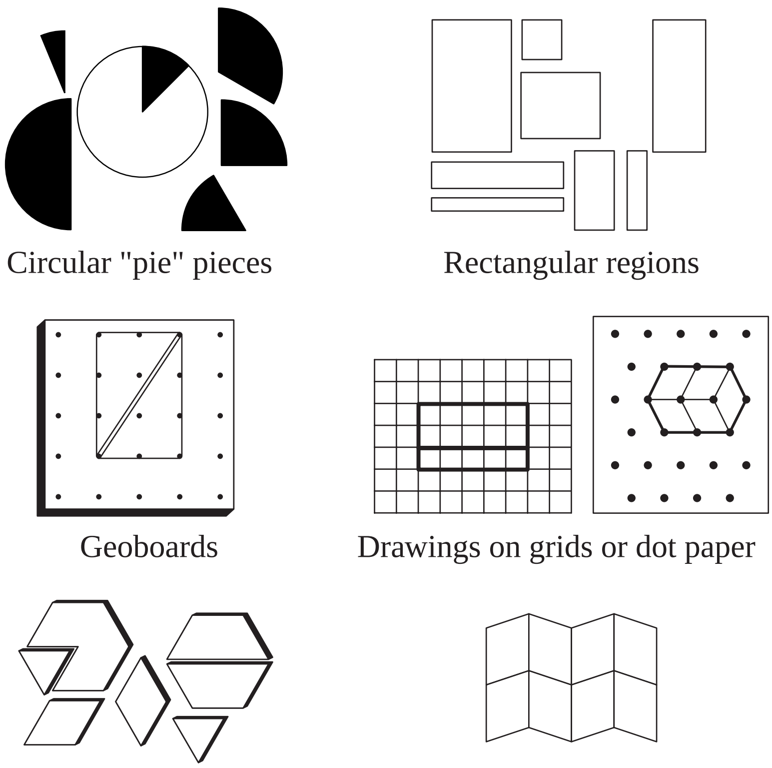


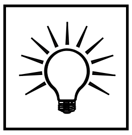

*Figure 1.1: Area or region models for fractions*

**Variety of Examples**

When you demonstrated area models, did you develop the concept of models with parts of the same size but not necessarily the same shape?

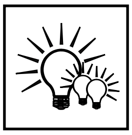
Look at the following chart illustrating 'creative' halves. Do you think the shaded part in each shape represents one-half? Which shapes represent halves that are not necessarily the same shape? How do you know?

*Figure 1.2: Creative halves*



## Unit Activity 2

Design your own "creative halves".



## Practice Activity 1

1. Have your pupils make 'creative halves'.
2. Display their models in the classroom.
3. Let pupils explain the process they used to create their "creative halves". How do they know that the parts are halves?

**Non examples**

Give pupils non-examples or counterexamples to test their conceptual understanding of common fractions. What fraction makes up the shaded part in *Figure 1.3*? Do you remember asking pupils this type of a problem? If not, why?

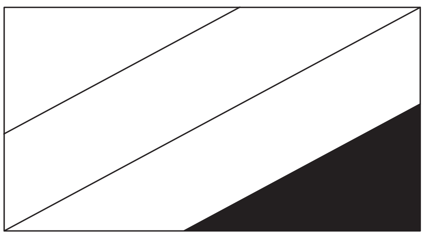


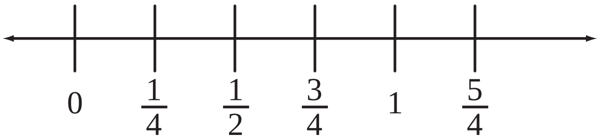
*Figure 1.3: Non-examples*

> **Reflection**
>
> When illustrating an interpretation of a common fraction on a chalk board, do you find it convenient to use a circle model? Why?

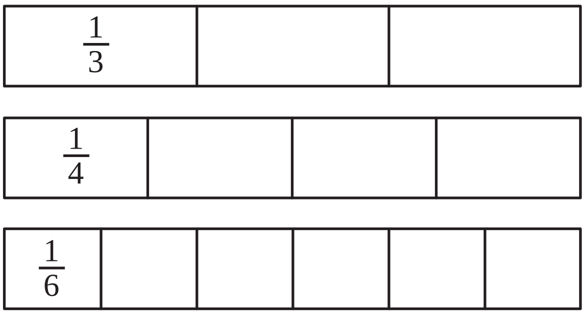
**Length Model**

Length models are similar to area models. The only difference is that lengths are compared instead of area. In length models, we can compare number lines and physical materials on the basis of length.

In the length models in *Figure 1.4*, common fractions are looked upon as parts of a whole.




Number line




Folded paper strips

*Figure 1.4 – Length models for fractions*

**Set Models**

Set models also illustrate common fractions as part of a whole. The set of objects make a whole, and subsets make up parts of the whole. The idea of looking at a *set* of elements as a *single* entity contributes to making set models difficult for primary pupils. Despite the difficulties faced by pupils, we cannot do away with the set model interpretation of fractions because it links real life situations to using fraction and ratio concepts. For instance, four objects are two-thirds of six objects.

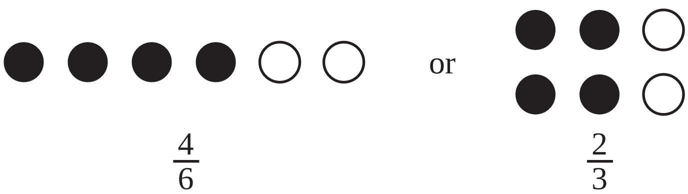



*Figure 1.5: Set models for fractions*

## Reflection

Mary went to visit her friend and found her friend's family about to share a pumpkin pie. This was Mary's favourite kind of pie, so she was disappointed when her friend's mother gave her some sweet potatoes instead. Why do you think Mary wasn't given a piece of the pie? What mathematical concept is involved in this scenario?

## Practice Activity 2

1. Investigate the effect of increasing denominators (numerator remaining constant) on the value of a common fraction.
2. Find a fraction between each of the following pairs:

   a) $\frac{3}{4}, \frac{3}{5}$ b) $\frac{a}{b}, \frac{c}{d}$

3. Illustrate $\frac{5}{6}$ using the following models:

   a) set model b) numberline

## Comparing Fractions: Use Concepts, Not Rules

It is not until the students begin to compare that they think about the relative size of fractional parts. You may know various approaches for comparing two fractions. Before pupils are introduced to rules or algorithms such as finding the common denominator or cross multiplication, they should be given an opportunity to envision the relative size of various fractions.

## Practice Activity 3

Use reflective thought to compare fractions using concepts. In *Figure 1.6*, which fraction in each pair is greater? Give one or more reasons. Do not use drawing or models or common denominators or cross-multiplication.

$\frac{4}{7} \quad \frac{3}{7}$ $\qquad$ $\frac{3}{8} \quad \frac{3}{5}$ $\qquad$ $\frac{1}{4} \quad \frac{2}{3}$ $\qquad$ $\frac{3}{4} \quad \frac{4}{5}$

*Figure 1.6: Comparing fractions using concepts.*

### Conceptual Thought Patterns for Comparison

Let us now look at the four ways of comparing fractions.

*Different number of same-size parts*: Since $\frac{4}{7}$ and $\frac{3}{7}$ have same-size parts, and four parts are greater than three parts, therefore $\frac{4}{7} > \frac{3}{7}$.

*Same number of different sized parts*: $\frac{3}{8}$ and $\frac{3}{5}$ have the same number of parts, but they are different sizes. The greater the denominator, the smaller the parts. Since $\frac{3}{8}$ has a greater denominator, $\frac{3}{5} > \frac{3}{8}$.

*More and less than an easy fraction*: $\frac{1}{4}$ is less than $\frac{1}{2}$ but $\frac{2}{3}$ is greater than $\frac{1}{2}$. Therefore, $\frac{2}{3} > \frac{1}{4}$.

*Closer to an easy fraction*: $\frac{3}{4}$ is one-quarter from one and $\frac{4}{5}$ is one-fifth from one.

But $\frac{1}{4} > \frac{1}{5}$, therefore $\frac{4}{5}$ is closer to one than $\frac{3}{4}$. Therefore, $\frac{4}{5} > \frac{3}{4}$.

## Practice Activity 4

Have a "why we know it is more" discussion. Arrange the class in cooperative groups or pairs of students. Provide them with one or more models for fractions. Give the class a pair of fractions to compare. The task is to find as many good explanations for their choice as possible within an allotted time. Explanations can be written down and then discussed as a full class.

## Reflection

A group of children are invited to share several chocolate bars. Biggie is offered the choice of one-third or one-half of a chocolate bar. Since he wants more chocolate, he chooses a half. He is disappointed, though, when he sees that Irene, who chose one-third, has bigger piece of chocolate.

What assumption did Biggie make when he made his choice?

Biggie assumed that the chocolate bars were all the same size. However, a piece representing one-third can be bigger than one-half if it is part of a larger whole, as shown in *Figure 1.7*.


*Fig. 1.7: "Chocolate Fallacy"*

## Reflection

Is it right when introducing proper fractions or improper fractions to liken them to pairs or objects on top of each other? For instance, a big monkey carrying a small monkey as representing a proper fraction, and vice versa as an improper fraction.

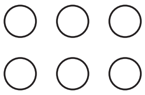
In your teaching of mixed numbers and improper fractions, did you at one time ask pupils the other names of a mixed number, for example $4\frac{1}{3}$?

Most pupils only know that $4\frac{1}{3} = \frac{13}{3}$. They do not know that $4\frac{1}{3} = 4 + \frac{1}{3}$.

You should extend this idea that $4 + \frac{1}{3}$ is also $3\frac{4}{3}$, $2\frac{7}{3}$ and $1\frac{11}{3}$. This idea is met later when subtracting fractions, for instance $4\frac{1}{3} - 1\frac{2}{3}$.

## Practice Activity 5

1. If 6 beads are a whole set, how many make up one-third of a set?



2. If this rectangle is three halves, draw a shape that could be whole.

## Practice Activity 6

1. Use models to display collections such as 11 quarters. Have pupils orally count the displays and give two names for each. Then discuss how they could write the different names using numbers. Pupils already know how to write $\frac{11}{4}$. For 2 wholes and 3 quarters a variety of alternatives might be suggested: 2 whole and $\frac{3}{4}$ or 2 and $\frac{3}{4}$ or $2 + \frac{3}{4}$. After doing this with other collections, explain that $2 + \frac{3}{4}$ is usually written as $2\frac{3}{4}$ with the "+" left out.
2. Now reverse the process. Write mixed numbers on the chalkboard and have pupils make model representations using only one kind of fractional part. Let pupils write the improper fraction name.
3. Have pupils figure out how to convert a mixed number to an improper fraction. Do not tell them any rule or procedure.

## Equivalent Fraction Concepts

Pupils should have a sound knowledge of equivalent fractions before learning the four operations on common fractions especially addition and subtraction.

Two fractions are equivalent if they are representations for the same amount. To acquire the conceptual understanding of equivalent fractions pupils should use models such as area, length, and sets to discover different names for models of fractions.

## Practice Activity 7: Area Models

Paper folding can be used to effectively model the concept of equivalent fractions. Fold a sheet of paper into halves or thirds. Unfold and colour a fraction of the paper. Write the fraction. Now refold and fold one more time (*Figure 1.7*). Before opening, guess how many parts will be in the whole sheet and how many will be coloured. Open the paper. What fraction names can be given to the shaded region? Is the name still the same? Why? What about the amount of the shaded part, is it still the same? Why? Record the equivalent fractions you have generated.

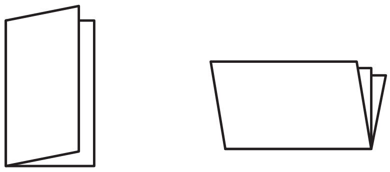


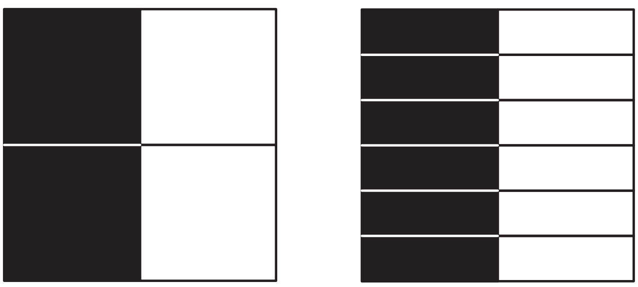
*Figure 1.7*

**Rectangle Slicing: Multiply-by-1-method**

We would like you to develop an equivalent fraction algorithm using rectangle slicing with your pupils.

Put pupils in cooperative learning groups. Give them papers. Have them draw several equal squares whose sides are 8 cm. Have them shade the same fraction in several different squares vertically subdividing lines. Next, pupils slice each rectangle horizontally into different fractional parts, as shown in *Figure 1.8*. Help pupils focus on the products involved by having them write top and bottom numbers as a product in the fraction.

Start with each square showing $\frac{1}{2}$



$\frac{1}{2} = \frac{1 \times \boxed{2}}{2 \times \boxed{2}} = \frac{2}{4}$ $\qquad$ $\frac{1}{2} = \frac{1 \times \boxed{6}}{2 \times \boxed{6}} = \frac{6}{12}$

*Figure 1.8: A model for the equivalent fractions*

*Continues on next page*

- What product tells how many parts are shaded?
- What product tells how many parts are in the whole?
- Is there a pattern in the multiplication of finding equivalent fractions?
- Help pupils to discover the Multiply-by-1-method.

## Unit Activity 2

**"Have fun with fractions"**

Have pupils in pairs discover equivalent fractions using fraction-equivalent cards. How many $\frac{1}{4}$ pieces make $\frac{1}{2}$?

<table>
<tr><td colspan="2">$\frac{1}{2}$</td></tr>
<tr><td>$\frac{1}{4}$</td><td>$\frac{1}{4}$</td></tr>
</table>

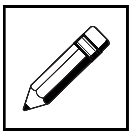
Example of a fraction-equivalent card. Concept can be extended to $\frac{1}{3}$, $\frac{1}{6}$, and $\frac{1}{8}$.

Have pupils discover as many relationships as they can. Have them draw and record the relationships. Observe the strategies they are using in finding the relationships.

**Equivalent Fraction Concentration Game**

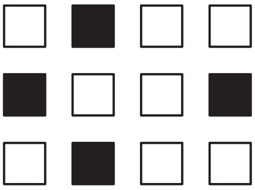
Again give pupils equivalent fractions to identify. Two or four pupils can play the game. Cards are turned over with the side carrying the name down. Players take turns to identify a set of equivalent fractions. Then the pupil turns the cards over and arranges the equivalent cards while others are watching. If the cards are equivalent, the pupil names them. For instance, if the cards contain one $\frac{1}{4}$ and two $\frac{1}{8}$, a pupil says "$\frac{1}{4}$ and $\frac{2}{8}$ are equivalent" and keeps the cards. That pupil continues playing until he/she picks fractions which he/she cannot prove equivalent. If the player fails to prove the fractions equivalent, he/she puts them back and turns them over mentioning each fraction. Then the next player plays.

A player loses a turn if equivalent fractions cannot be found. The winner is the player who has the greatest number of cards at the end of the game.

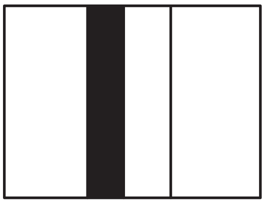


## Self Assessment

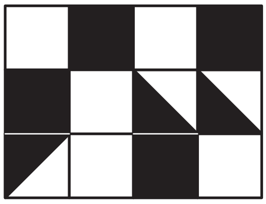
1. Is $\frac{10}{10}$ a proper or an improper fraction or neither?
2. Why does the rule (multiplying the whole number by the denominator and adding the numerator) work for changing a mixed number? For instance, $4\frac{2}{3}$ as $\frac{14}{3}$ involves multiplying 4 and 3 and adding 2.
3. Arrange this set in three different ways.



4. What fraction is shaded?

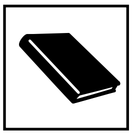


(a)



(b)

5. If $\frac{x}{m} = \frac{z}{m}$, what must be true about $m$, $x$ and $z$?



## Summary

The concept of common fractions has been developed through various meanings, use of a variety of models, and a variety of examples to promote flexibility of thought. An investigative approach is used to promote problem solving and games to consolidate the concepts.

The frequent use of models and diagrams has meant that most fractions dealt with are the very simple ones: quarters, thirds, and the like. This may seem like a restriction, but for dealing with fractions in real life, it is not so. The common fractions one encounters in work and everyday life rarely extend beyond the 8ths and 10ths. More complex fractions are almost always expressed as decimals.

## Unit 1 Test: Common Fractions

1. Illustrate $\frac{5}{6}$ using the following models:
   a) set model
   b) area model
   c) length model

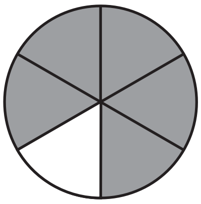
2. Find a fraction less than $\frac{1}{10}$. Find another fraction less than the fraction you found. Can you continue this process? Is there a smallest fraction greater than 0?

3. What fraction is represented by the shaded portion?

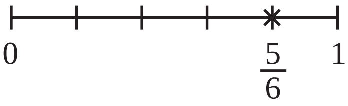
## Answers to Practice Activity 2

2. a) $\frac{4}{5}$ b) $\frac{b}{d}$

3. a) set model b) number line



$\frac{5}{6}$



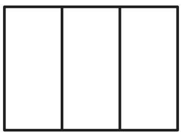
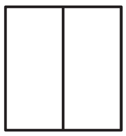

## Answer to Reflection *(Just before Practice Activity 5)*

Normally we do not provide answers to reflections, since their purpose is for you to think freely. This time we do, to help you avoid the teaching trap that this reflection suggests. *Do not* use a physical analogy to justify the upper and lower parts of a fraction—it confuses many pupils, some of them permanently. A baby monkey is *not* a fraction of its mother and you will shatter your pupils' developing concept of fractions if you suggest that it is. Since you used "carry" when you taught addition, you should not suggest that the denominator "carries" the numerator. If you do, some pupils will forever confuse division and addition.

## Answers to Practice Activity 5

1. $\frac{2}{6} = \frac{1}{3}$

2.

 $= \frac{3}{2}$  $= \frac{2}{2}$ = a whole

## Answers to Self Assessment

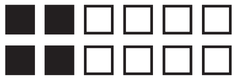
1. $\frac{10}{10}$ is neither a proper or improper fraction.

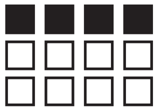
2. Rename the mixed number as a whole number and a fraction. Then find an equivalent fraction of the whole number whose denominator is the denominator of the fraction part. This is what brings in the multiplication of a whole number by a denominator. Now the two numbers have the same denominators. So we add the numerators.

   For instance, $4\frac{2}{3} = 4 + \frac{2}{3} = (4 \times \frac{3}{3}) + \frac{2}{3}$

   $= \frac{12}{3} + \frac{2}{3} + \frac{12 + 2}{3} = \frac{14}{3}$

3.  $\frac{4}{12}$

 $\frac{2}{6}$

 $\frac{1}{3}$

4. It is difficult to name the shaded parts because they are not divided equally.

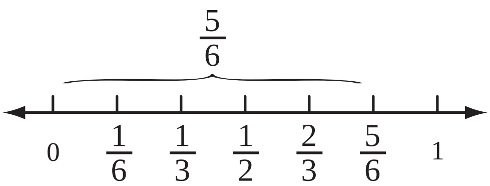
5. $x = z$

## Answers to Unit 1 Test

1.

a)  $\frac{5}{6}$ Set model

b)  $\frac{5}{6}$ Area model

c) 

Length model

2. $\frac{1}{1000} > \frac{1}{1\,000\,000} > \frac{1}{100\,000\,000}$

   There is no smallest fraction because the set of common fractions is infinite.

3. $\frac{3}{4}$

# Unit 2: Addition and Subtraction of Common Fractions

## Introduction

In Unit 1, you were exposed to different meanings of fractions and various ways of representing fractions. This unit extends those concepts to the addition and subtraction of common fractions.

## Objectives

By the end of this unit, you should be able to:

- illustrate addition of common fractions using a variety of models
- investigate activities forming fractions and fractional patterns
- demonstrate subtraction of common fractions using a variety of models
- develop puzzles to consolidate addition or subtraction concepts of common fractions
- identify causes of errors and provide an effective remedy
- solve problems involving addition and subtraction of common fractions

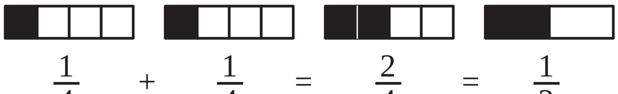
## Adding Common Fractions Using Models

In everyday life we see problems involving common fractions though people rarely express values by means of common fractions. Besides, most of the objects in everyday life are not regular or identical. However, you are encouraged to find realistic and practical examples in everyday life. This will make common fractions more meaningful.

**Like Denominators**

Mary and John bought a loaf of bread. Mary ate one quarter and John ate another quarter. How much bread did they eat altogether? ($\frac{1}{4} + \frac{1}{4}$)

Think of what $\frac{1}{4}$ means. It means one part out of four equal parts. If you put together $\frac{1}{4}$ and $\frac{1}{4}$ what do you get? What is $\frac{2}{4}$ in its simplest form?



$$\frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

*Figure 2.1*

## Reflection

When modelling addition, is it wrong to show the addends as rectangular slabs as in *Figure 2.1*?

If this is a cause of some errors, then you can use the following rectangular slab representation.

$\frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

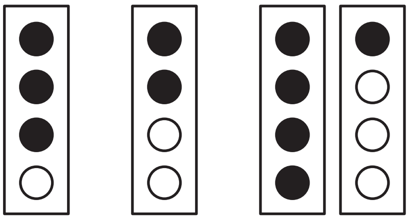
*Figure 2.2*

What error of conceptualization in *Figure 2.1* is avoided by using *Figure 2.2*?

**Unlike denominators using set models**

Alice brought some pre-packed packets of milk to school. She drank $\frac{3}{4}$ *L* of milk at break time. After school, she drank $\frac{1}{2}$ *L* of milk. How much milk did she drink altogether?

What set size can be used for the whole? Since $\frac{3}{4}$ and $\frac{1}{2}$ have different denominators, you need to find a common fraction equivalent to $\frac{1}{2}$ whose denominator is 4. So $\frac{1}{2} + \frac{2}{4}$. Therefore, the smallest whole set we can use is 4, as in *Figure 2.3*:



$\frac{3}{4} + \frac{2}{4} = \frac{5}{4}$ or $1\frac{1}{4}$

*Figure 2.3*

Combine (add) the shaded elements 4 + 1 = 5. Out of sets of 4, that is $\frac{5}{4}$ or $1\frac{1}{4}$.

## Unit Activity 1:

**Adding Mixed Numbers Using Length Model**

Now it is your turn to make a realistic and practical story problem involving $1\frac{2}{3} + 1\frac{1}{2}$. Solve this problem using length models.

## Practice Activity 1

In a classroom situation, use various models to present addition of common fractions. Put pupils in cooperative learning groups. Let them come up with a realistic story problem involving $\frac{1}{3} + \frac{1}{6} + \frac{1}{2}$. Let them use two different models to add the fractions. Let them record each step as they go. Have pupils discuss the story problems they made. Let pupils tell you why they picked the models they used.

## Subtracting Common Fractions Using Models

The meaning of subtraction in fractions is just the same as for whole numbers. Pupils should be exposed to more than one meaning, as opposed to always looking at subtraction as take-away.

Group work and teacher directed activities should continue in subtraction just as in addition. To introduce subtraction of common fractions, use story problems and models.

**Subtraction may mean:**

1. **Take-away**

   My mother gave me $\frac{1}{2}$ of a chocolate bar. I ate one-quarter of the chocolate bar. How much chocolate is remaining? ($\frac{1}{2} - \frac{1}{4}$)

   - In this situation $\frac{1}{2} - \frac{1}{4}$ means take away $\frac{1}{4}$ from $\frac{1}{2}$.
   - Shade half. Think how you can get $\frac{1}{4}$ from $\frac{1}{2}$. Slice half into two equal parts. Now you have two quarters. Taking away one quarter, you are left with another quarter, as in *Figure 2.4*.

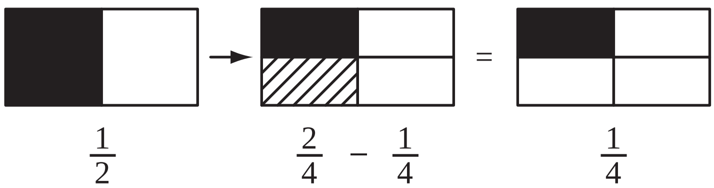
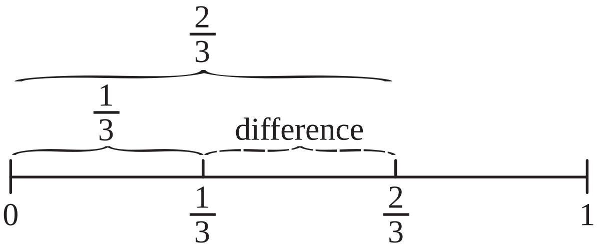



*Figure 2.4: Subtracting using area model.*

2. **Comparison**

   Stephen gave two-thirds of a metre of cloth to his elder sister and his young brother one-third of a metre of cloth. The young brother complained that he was given less than his sister. How much more was the sister given than the brother?

   How can you find the difference between $\frac{2}{3}$ and $\frac{1}{3}$? Draw $\frac{2}{3}$ and $\frac{1}{3}$ in a comparative format. How much more is $\frac{2}{3}$ than $\frac{1}{3}$? By comparing, $\frac{2}{3}$ is more than $\frac{1}{3}$ by $\frac{1}{3}$.



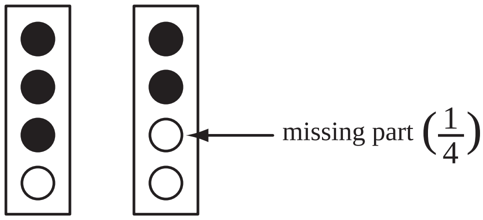

$\frac{2}{3} - \frac{1}{3} = \frac{1}{3}$

*Figure 2.5: Comparison: subtraction using length model*

3. **Part-part-whole**

   Shanzuwa left $\frac{3}{4}$ kg of meat with his uncle. When he went to collect his meat, he only found $\frac{1}{2}$ kg of meat. How much meat was missing? $(\frac{3}{4} - \frac{1}{2})$

   - Use set models to illustrate the story problem.
   - You need the smallest set that can be divided into both quarters and halves. The set is 4.
   - Use the comparative layout to find the missing part.




$\frac{3}{4} - \frac{2}{4} = \frac{1}{4}$

*Figure 2.6: Part-part-whole subtraction using set models*

## Unit Activity 2

Write a story problem about each of the following mathematical sentences. Use different meanings of subtraction and find the answers using different models.

a) $\frac{4}{5} - \frac{2}{5}$ b) $2\frac{1}{3} - 1\frac{3}{4}$

Record your process. Make sure parts in a whole are equal.

## Practice Activity 2

1. Demonstrate subtraction of common fractions involving different meanings of subtraction using a variety of models.
2. Give pupils problems, such as $2\frac{1}{2} - \frac{2}{3}$. Have them write two story problems of different meanings of subtraction. Let them model the subtractions using different models. Lastly, have them verbalise their procedures.

## Reflection

Which model do pupils find difficult to use—area model, set model, or length model? Why?

**Consolidation Activities**

After giving pupils enough practice, you can use games or puzzles to consolidate the concept they are learning. For instance, after addition of common fractions using the abstract approach, you can use the puzzle “Fraction Spokes”.

## Unit Activity 3

**Investigative Approach**

**Fraction spokes**

1. Place fractions $\frac{1}{6}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{6}$ in the circles so that each diagonal adds to the same number.

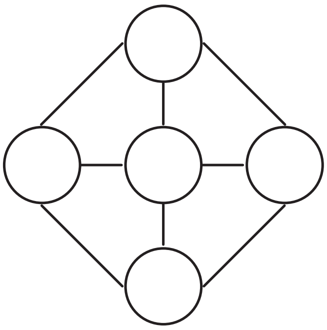
   

   *Figure 2.7: Fraction spokes*

   - What procedure did you see?
   - What mathematical concept is involved in this puzzle?
   - Is there a relationship between the number of circles and the highest denominator?

2. Make a similar new puzzle. See if you can devise variations and extensions to the puzzle.

**Optional Activities**

1. A unit fraction can be written as the sum of two unit fractions. Study the following examples and try to discover a general pattern.

$$\frac{1}{4} = \frac{1}{5} + \frac{1}{20}$$
$$\frac{1}{6} = \frac{1}{7} + \frac{1}{42}$$
$$\frac{1}{2} = \frac{1}{3} + \frac{1}{6}$$

*Continues on next page*

2. You are given two measuring cups. One holds one-quarter of a litre, the other holds one-third of a litre. How would you use these two cups to measure exactly $\frac{11}{4}$ litres, $\frac{5}{12}$ litre, and $\frac{5}{6}$ litre?

## Practice Activity 3

After your pupils have learned addition of common fractions using the abstract approach, give them the "Fraction spokes" number puzzle below.

**Number Puzzle**

Let them work in pairs. Can they place these fractions in the circles so that each diagonal adds up to the same number?

$\frac{1}{12}, \frac{1}{6}, \frac{1}{4}, \frac{1}{3}, \frac{5}{12}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}, \frac{11}{12}, \frac{1}{2}, \frac{7}{12}.$

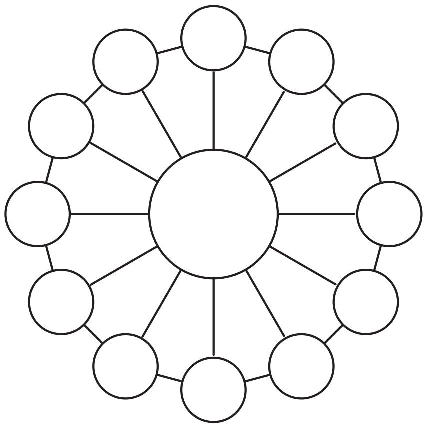


*Figure 2.8: Fraction spokes*

## Reflection

Do you analyse computational errors made by pupils? Why?

## Error Analysis

As a teacher of primary school mathematics, you should be alert to error patterns. Most teachers are only interested in either putting a tick (✓) or a cross (✗) against a pupil's answer. That is not being helpful. A teacher should give individual attention by pausing at an error, analysing it, and giving appropriate remedial work. If a teacher is not able to identify the cause of an error, then he/she cannot provide an effective remedy.

Be careful not to decide on the error pattern too quickly. Look at other work by the same pupil before making a decision. Now, let us look at Peter's work. Can you identify the error, the cause of the error, and provide an appropriate remedy?

a) $2\frac{5}{6} - 1\frac{1}{6} = 1\frac{4}{6}$ b) $4\frac{1}{4} - 2\frac{3}{4} = 2\frac{2}{4}$ c) $9 - 3\frac{1}{2} = 6\frac{1}{2}$

Peter simply looks for a greater fraction then subtracts the lesser one from it. He does not care whether the greater fraction belongs to the minuend or to the subtrahend. Peter thinks subtraction of common fractions is commutative.

**Remedial**

Interpret $4\frac{1}{4} - 2\frac{3}{4}$ as take-away. Use models to illustrate the problem. In your process of modelling, ask Peter if he can subtract $\frac{3}{4}$ from $\frac{1}{4}$. Also ask him for other names for $4\frac{1}{4}$ such as $4 + \frac{1}{4}$ and $3 + \frac{5}{4}$.

## Unit Activity 4

Shanzuwa has difficulty when adding common fractions. Can you find his pattern of error? Why might Shanzuwa be using such a procedure? How can you help him?

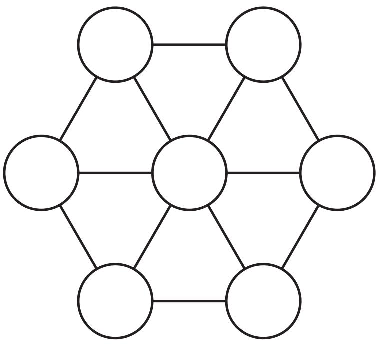
a) $\frac{2}{3} + \frac{1}{4} = \frac{4}{7}$ b) $\frac{3}{4} + \frac{1}{4} = \frac{4}{8}$

## Practice Activity 4

In your classroom, identify errors made by pupils. At times ask them how they arrived at their wrong answers. Then identify causes of the error and remember to provide effective remedial work.

## Self Assessment

1. Formulate and write a word problem involving real life situations for $1\frac{1}{2} + \frac{5}{6}$. Use set models to arrive at the answer.
2. Find common fractions to fill in the circles so that each diagonal adds up to the same number. Describe the procedure you used.



*Continues on next page*

3. Write three story problems of real life situations about $\frac{3}{4} - \frac{2}{3}$. Give the meaning of subtraction in each case.

4. Lovemore is having difficulty subtracting common fractions. What is he doing? What might be causing his difficulty? Can you help Lovemore?

$$4\frac{2}{3} - 1\frac{5}{6} = 4\frac{4}{6} - 1\frac{5}{6} = 3\frac{10}{6} - 1\frac{5}{6} = 2\frac{10-5}{6} = 2\frac{5}{6}$$

5. If you place one full container of flour on one pan of a balance scale and a similar container $\frac{3}{4}$ full and a $\frac{1}{3}$ kg weight on the other pan, the pans balance.

   How much does the full container of flour weigh?

## Summary

When introducing a new operation, pupils should be exposed to various meanings of the operation through story problems. A variety of models also enable pupils to make a choice of representation. Since model representations promote conceptual knowledge, rules should be delayed. The use of puzzles promotes problem solving and critical thinking. Mathematical games should be encouraged because games promote mental work. Besides, children naturally like playing games.

## Unit 2 Test: Addition and Subtraction of Common Fractions

1. Illustrate the following problems by using a model:

   a) The village is $3\frac{1}{2}$ km away. Gladys has walked $1\frac{3}{4}$ km. How much further does she need to walk to reach the village? (use the length model).

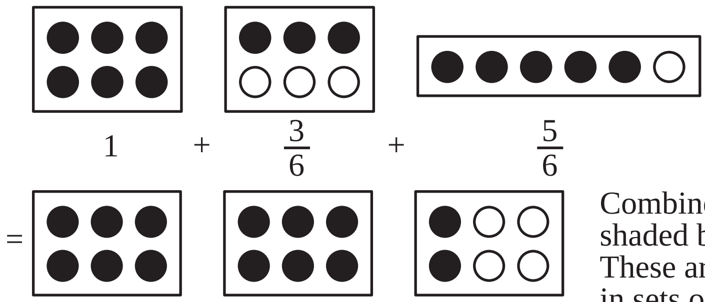
   b) My aunt gave me some biscuits and my sister gave me $2\frac{1}{2}$ kg of biscuits. Now, I have $4\frac{1}{4}$ kg of biscuits. How many biscuits did my aunt give me? (use set model)

2. Separate the set of fractions given below into two so that the sum of each set is the same.

   $\frac{1}{6}, \frac{1}{12}, \frac{1}{3}, \frac{1}{18}, \frac{1}{9}, \frac{1}{36}$

## Answers for Self Assessment

1. Addition of common fractions using set models. Danny has one and a half packets of chocolate beads. His sister gives him five-sixth of a packet of chocolate beads. How many packets does he have altogether?



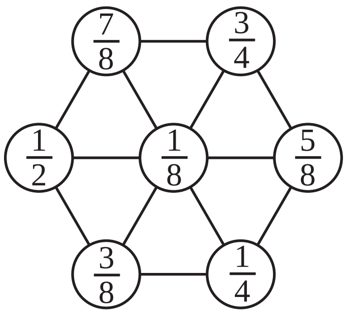
$1 + \frac{3}{6} + \frac{5}{6}$

$= \frac{14}{6}$ or $2\frac{1}{3}$

Combine shaded beads. These are all in sets of six.

2. **"Fractions spokes"**

   The number of circles is one less the lowest common denominator. In this puzzle, the number of circles is 7. Therefore, the lowest common denominator is 8. In this situation the greatest proper fraction is $\frac{7}{8}$ and the unit fraction is $\frac{1}{8}$. The fractions in an ordered manner are $\frac{1}{8}, \frac{2}{8}, \frac{3}{8}, \frac{4}{8}, \frac{5}{8}, \frac{6}{8}, \frac{7}{8}$. Put the smallest fraction in the centre. Then fill the outside circle by pairing two fractions and putting them in the two circles which are diagonal. Pair the greatest fraction with the next smallest fraction and so on. Convert all fractions to their simplest form before putting them in the circle.




3. (i) Boyd has three-quarters of a metre of cloth. When he comes back from school he finds only two-thirds of a metre of cloth. How much cloth is missing?

   (ii) Boyd has three-quarters of a metre of cloth. He gives away two-thirds of a metre to Carol. How much cloth is left?

   (iii) Boyd has three-quarters of a metre of cloth. Ellen has two-thirds of a metre of cloth. How much longer is Boyd's cloth than Ellen's cloth?

4. Lovemore is always thinking in base ten. When he renamed one instead of using the denominator (6) to get the equivalent fraction, Lovemore uses base ten place value. As such, $1 = \frac{10}{6}$ instead of $1 = \frac{6}{6}$.
   This pupil can be helped by re-introducing the concept of changing mixed numbers to improper fractions using models.

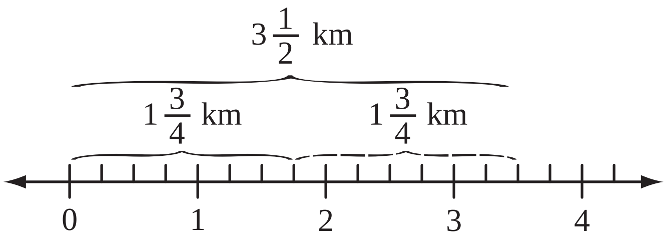
5. Let $x$ be the weight, in kg, of full a container of flour.

$$x = \frac{3x}{4} + \frac{1}{3}$$
$$x - \frac{3x}{4} = \frac{1}{3}$$
$$3(4x - 3x) = 4$$
$$12x - 9x = 4$$
$$3x = 4$$
$$x = \frac{4}{3}$$

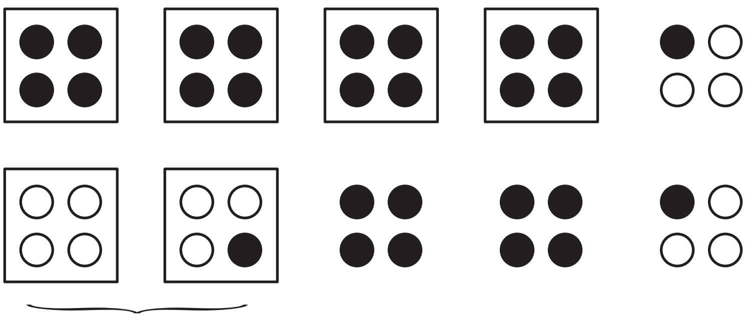
$\therefore$ Full container of flour weighs $1\frac{1}{3}$ kg

## Answers to Unit 2 Test

1. a)




b)



$2\frac{1}{2}$

unshaded in here from aunt ($1\frac{3}{4}$)

$\boxed{1\frac{3}{4}} + 2\frac{1}{2} = 4\frac{1}{4}$

My aunt gave me $1\frac{3}{4}$ kg of biscuits.

2. $\frac{1}{6}, \frac{1}{12}, \frac{1}{3}, \frac{1}{18}, \frac{1}{9}, \frac{1}{36}$

$\frac{6}{36}, \frac{3}{36}, \frac{12}{36}, \frac{2}{36}, \frac{4}{36}, \frac{1}{36}$

$\frac{2}{36} + \frac{12}{36} = \frac{6}{36} + \frac{3}{36} + \frac{4}{36} + \frac{1}{36}$

The two sets are of $\{\frac{1}{3}, \frac{1}{18}\}$ and $\{\frac{1}{6}, \frac{1}{12}, \frac{1}{9}, \frac{1}{36}\}$

# Unit 3: Multiplication and Division of Common Fractions

## Introduction

In Unit 2, you explored the concept of addition and subtraction of common fractions. In this unit you will explore concepts and develop algorithms for the remaining two operations, multiplication and division of common fractions. Then you will diagnose common computational errors.

## Objectives

By the end of this unit, you should be able to:

- illustrate multiplication of common fractions using models
- demonstrate division of common fractions using models
- formulate problems involving multiplication and division of fractions
- illustrate division of common fractions using a ratio table below
- analyse error patterns involving multiplication and division of common fractions

## Multiplication of Common Fractions: Concept Exploration

The meaning of multiplication of common fractions is the same as for whole numbers. To some people, $2 \times 3$ means two sets with three elements in each set, while to others it might mean three sets with two elements in each. In this module, the multiplicand gives the number of sets and the multiplier consists of the number of elements in a set.

Your task is to help the pupils understand multiplication of common fractions and help them to develop the algorithm. Let them explore multiplication of fractions using models and let them discover the rules.

Let us look at the four categories involving multiplication of common fractions: multiplication of proper fractions by whole numbers, multiplication of a whole number by a proper fraction, multiplication of proper fractions, and multiplication of mixed numbers.

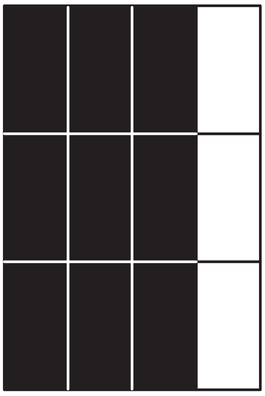
### *Multiplication of a Proper Fraction by a Whole Number Using Rectangular Slab Models*

We start with multiplication of proper fractions by a whole number because it is easiest. In this case, we look at multiplication as repeated addition.

Elizabeth buys three-quarters of a metre of cloth each month. How many metres of cloth does she buy in three months? ($3 \times \frac{3}{4}$)

The meaning of $3 \times \frac{3}{4}$ is three $\frac{3}{4}$s.

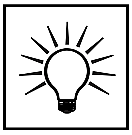
$\frac{3}{4}$



$3 \times \frac{3}{4}$

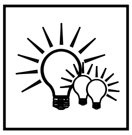
*Figure 3.1*

In the area model representation of $3 \times \frac{3}{4}$, there are nine shaded parts. So $3 \times \frac{3}{4} = \frac{9}{4}$.



## Unit Activity 1

1. Multiply $2 \times \frac{1}{6}$ and $4 \times \frac{1}{2}$ using rectangular slabs.
2. Find a rule of multiplying proper fractions by a whole number.



## Practice Activity 1

1. Discuss with your pupils the meaning of $4 \times \frac{1}{3}$
2. Have pupils tell story problems about $4 \times \frac{1}{3}$
3. Have pupils draw representations of $4 \times \frac{1}{3}$
4. Let pupils find the product using their models.
5. Give them similar problems to solve using model representation.
6. Have they seen a pattern? Can they tell the product of $10 \times \frac{2}{3}$ without using the diagram representation?

### *Multiplication of a Whole Number by a Common Fraction*

*Using set model*

Here we look at multiplication as an array. In everyday life, problems arising from multiplication of a whole number by a proper fraction are many. For instance, $\frac{1}{2}$ of one litre is $\frac{1}{2}$ of 1000 ml.

*Example*: In a family of 10 children, $\frac{3}{5}$ of the children are boys. How many are boys? $\frac{3}{5}$ of 10 is the same as $\frac{3}{5} \times 10$. The product of $\frac{3}{5} \times 10$ and $10 \times \frac{3}{5}$ is the same, but the meaning and process is quite different.

**Solution**

To solve $\frac{3}{5}$ of 10 using models, it is convenient to use set models which take the form of an array. In set models, the denominator tells us the number of elements in each row. The number of rows is determined by the total number of elements divided by the number of elements in each row (denominator).

We will make a rectangular array showing $\frac{3}{5}$ of 10. The number of elements in each row is five and the number of rows is two. In an array of 2 by 5, there are 3 boys in each row.

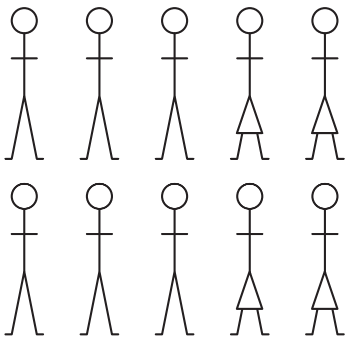


*Figure 3.2: Rectangular array*

There are six boys. How many are girls?

## Unit Activity 2

1. Verify that $\frac{1}{4} \times 8$ and $8 \times \frac{1}{4}$ gives rise to the same answer.
2. Using models, generate the rule of multiplying a whole number by a common fraction.
3. Can you find the error made by Jelita. Why would someone compute in this way? $\frac{1}{4} \times 8 = \frac{8}{32}$
4. In a cost saving measure, Judith’s company reduced all salaries by $\frac{1}{8}$.
   If Mulenga’s monthly salary was $2400.00, what will he receive now? If another employee’s salary is $2800.00, what was the old salary?

## Practice Activity 2

1. Discuss with the pupils the meaning of $\frac{2}{3} \times 9$
2. Have them make a story problem involving $\frac{2}{3} \times 9$
3. Have them use set models to illustrate $\frac{2}{3} \times 9$
4. Give them another problem to enable them to generate the rule of multiplying a whole number by a fraction.
5. Have the pupils try $9 \times \frac{2}{3}$ using the models. Is the product the same as for $\frac{2}{3} \times 9$? Is the meaning and process same?

### *Multiplication of Fraction by a Fraction*

Explain the meaning of $\frac{1}{4} \times \frac{1}{2}$. The meaning is still the same: $\frac{1}{4}$ of a set of $\frac{1}{2}$.

## Unit Activity 3

1. Use rectangular slicing to find the product of $\frac{1}{4} \times \frac{1}{2}$ and generate the rule after examining multiplication of other proper fractions using models.
2. Solve $\frac{2}{5}x = \frac{3}{7}$

## Reflection

When you were generating the rule, did you notice that the product of two proper fractions is less than either fraction? Why is it so?

## Practice Activity 3

1. Ask pupils to explain the meaning of $\frac{2}{3} \times \frac{1}{2}$.
2. Have pupils make a story problem involving $\frac{2}{3} \times \frac{1}{2}$ and use rectangle slicing to find the product.
3. Let them discover the rule of multiplying proper fractions.
4. Have pupils explain why a product of two fractions is smaller than either fraction.

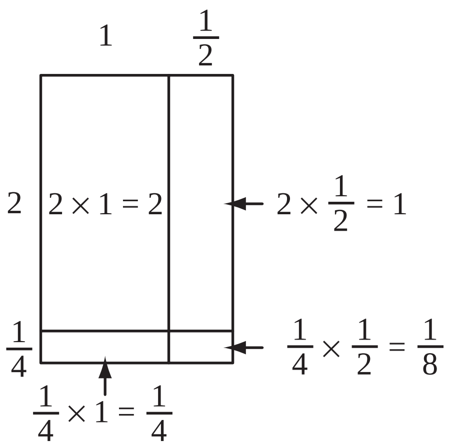
### *Multiplying Mixed Numbers*

Pupils find it difficult to visualise and model multiplication of mixed numbers.

The meaning of $2\frac{1}{4} \times 1\frac{1}{2}$ is $2\frac{1}{4}$ of a set of $1\frac{1}{2}$. Here we have to use the distributive properly over addition for multiplication.

$2\frac{1}{4} \times 1\frac{1}{2} = (2 + \frac{1}{4})(1 + \frac{1}{2})$.

**Using Area Models**




$$2\frac{1}{4} \times 1\frac{1}{2} = 2 + 1 + \frac{1}{4} + \frac{1}{8} = 3\frac{3}{8}$$

*Figure 3.3*

You may have noticed that if we use the area model for the multiplication of mixed numbers, we need to use both a concrete approach and an abstract approach. For pupils to be able to use this model, they should be able to multiply proper fractions.

## Unit Activity 4

Examine this other approach:

$2\frac{1}{4} \times 1\frac{1}{2} = (2 + \frac{1}{4})(1\frac{1}{2}) = (2 \times 1\frac{1}{2}) + (\frac{1}{4} \times 1\frac{1}{2})$ in area model this is:

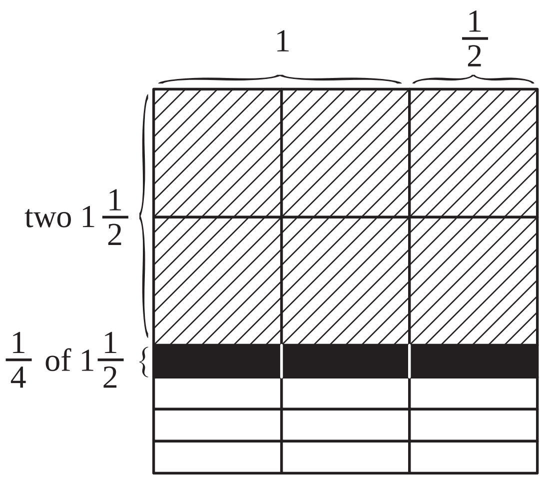



*Fig 3.4*

- For a primary pupil, what are the advantages of using the first approach?
- What are the disadvantages of using the second approach?
- Which model is the most appropriate to use of the two and why?

## Practice Activity 4

- Arrange pupils in cooperative learning groups.
- Have pupils use models to determine the products of mixed numbers and come up with a rule.
- Observe if the pupils are finding it difficult to model the multiplication to mixed numbers.

## Division of Common Fractions Using Models

Some teachers find it difficult to give pupils the conceptual knowledge of division of common fractions. Thus, they are tempted to tell pupils the rule, "invert-and-multiply", since pupils already have the procedural knowledge to multiply common fractions.

You are encouraged to teach the meaning of division of common fractions through story problems and models. Let the pupils discover the rules. Try to use both partive and measurement division, and use simple situations.

### *Partitive Division (sharing)*

Dividing a proper fraction by a whole number. James has $\frac{1}{2}$ metre of cloth and wants to share it equally among his three friends. How many metres will each get? ($\frac{1}{2} \div 3$)

Use models of your choice to come up with a solution to the above problem. Describe the process you used.

### *Measurement Division (Repeat Subtraction)*

Dividing a whole number by a proper fraction. Martha has 2 metres of cloth. If she cuts it into pieces that are each $\frac{3}{4}$ metres long, how many pieces will she have? ($2 \div \frac{3}{4}$)

Let us explore the problem together. Have two strips of paper to represent two metres. We need to divide each strip into four equal parts. Now join the strips end to end ($\frac{8}{4}$) as in *Figure 3.5*. Count the number of three-quarters in both strips. There are two three-quarters with a left over of two-quarters. Compare two-quarters with three-quarters. Two-quarters is two-thirds of three-quarters. Remember your unit of measure is three-quarters.

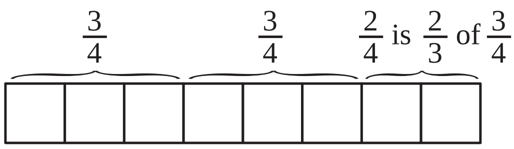



*Figure 3.5: Division using length model*

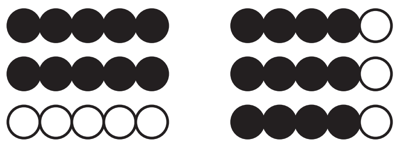
Focusing on $\frac{8}{4} \div \frac{3}{4}$, can you find the quotient to $2 \div \frac{3}{4}$?

### *Division using Set Model*

Jane has $\frac{2}{3}$ metres of cloth. How many $\frac{4}{5}$ metre pieces can she cut from her cloth?

$(\frac{2}{3} \div \frac{4}{5})$

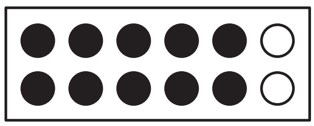
$\frac{2}{3} \div \frac{4}{5}$ – The whole set is 15.



$\frac{2}{3}$ is 10 counters $\frac{4}{5}$ is 12 counters

How many sets of $\frac{4}{5}$ are in $\frac{2}{3}$?

How many sets of 10 in set of 12?



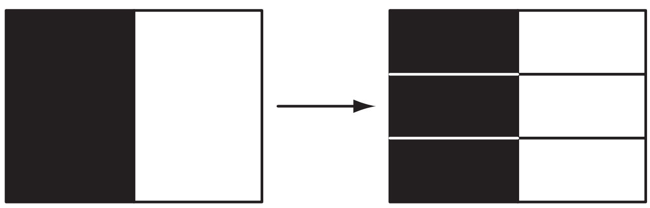
$\frac{5}{6}$ of a set of 12

Are you able to follow?

## Reflection

Modelling of multiplication and division of common fractions seem to be difficult if not confusing. Do you think it is worth spending time to model these two operations in common fractions?

Draw $\frac{1}{2} \div 3$ by slicing a rectangular whole vertically into two equal parts. Divide the half into three equal parts.



$$\frac{1}{2} \div 3 = \frac{1}{6}$$

*Figure 3.6: Division using area model*

Rather than being given algorithms for division of common fractions, students should be helped to discover the rules on their own.

Algorithms such as ‘common denominator’ and ‘multiplicative inverse’ can be modelled. Others are difficult to model.

### *Common Denominator Algorithms*

You developed this algorithm when you were exploring measurement division. The fractions to be divided are first changed to equivalent fractions with the same denominator, then the numerators are divided.

Example:

$$2 \div \frac{3}{4} = \frac{8}{4} \div \frac{3}{4} = \frac{8 \div 3}{4 \div 4} = \frac{\frac{8}{3}}{\frac{4}{4}} = \frac{\frac{8}{3}}{1} = \frac{8}{3} = 2\frac{2}{3}$$

### *Multiplicative Inverse (invert-and-multiply) Algorithms*

**Step 1:** Through guided discovery, pupils are to find that a fraction multiplied by its reciprocal always gives 1.

Questions on finding reciprocal are set:

$\frac{2}{5} \times \frac{5}{2} = \frac{10}{10}$ $\qquad$ $\frac{1}{2} \times \_\_\_\_ = \frac{2}{2} = 1$

$10 \times \_\_\_ = \frac{10}{10}$ $\qquad$ $\frac{3}{4} \times \_\_\_\_ = \_\_\_\_ = 1$

In general $\frac{a}{b} \times \frac{b}{a} = 1$ $(a \neq 0,\ b \neq 0)$

**Step 2:** The above ‘discovery’ is applied to ensure 1 appears in the denominator.

Example:

$$\frac{3}{4} \div \frac{2}{3} = \frac{3}{4} = \frac{\frac{3}{4} \times \frac{3}{2}}{\frac{2}{3} \times \frac{2}{3}} = \frac{\frac{9}{8}}{1} = \frac{9}{8} = 1\frac{1}{8}$$

### *Ratio Table*

If the division is interpreted as a ratio, the problem can be stated as $\frac{a}{b} \div \frac{c}{d}$ as (fraction) ÷ 1. In a ratio table, the division $\frac{3}{4} \div \frac{1}{2} = 1\frac{1}{2}$, $\frac{3}{4} \div \frac{1}{3} = 2\frac{1}{4}$, $2\frac{1}{2} \div 4\frac{1}{4} = \frac{10}{17}$ will appear as shown.

$\frac{3}{4} \div \frac{1}{2} = 1\frac{1}{8}$

| $\frac{3}{4}$ | $\frac{6}{4}$ | $1\frac{1}{2}$ |
|---|---|---|
| $\frac{1}{2}$ | 1 | 1 |

$\frac{3}{4} \div \frac{1}{3}$

| | | |
|---|---|---|
| $\frac{3}{4}$ | $\frac{9}{4}$ | $2\frac{1}{4}$ |
| $\frac{1}{3}$ | 1 | 1 |

$2\frac{1}{2} \div 4\frac{1}{4}$

| | | | |
|---|---|---|---|
| $2\frac{1}{2}$ | $\frac{6}{4}$ | 10 | $\frac{10}{17}$ |
| $4\frac{1}{4}$ | $\frac{17}{4}$ | 17 | 1 |

*Figure 3.7: Ratio Table*

These are all algorithms for division of common fractions. You can discover some more.

## Unit Activity 5

1. (a) Make a story problem involving $1\frac{2}{3} \div \frac{1}{2}$

   (b) Model $1\frac{2}{3} \div \frac{1}{2}$ using strips and counters.
2. Why does the invert-and-multiply rule work?
3. Prove that the common denominator algorithm is a valid algorithm.
4. In the ratio table, write the operations carried out to move from one column to the next in each of the workings for $\frac{2}{3} \div \frac{3}{4} = \frac{8}{9}$
5. Sibeso has difficulty with division. What is the cause of her error? How can you help her? $\frac{3}{4} \div \frac{1}{2} = \frac{3}{4}x \div \frac{1}{2} = \frac{3}{8}$

## Reflection

Which of these two algorithms is ‘best’—the common denominator algorithm or the invert-and-multiply algorithm. Why?

## Practice Activity 5

1. Have pupils model partitive division and measurement division.
2. Help pupils generate common denominator algorithms and invert-and-multiply algorithms.

## Self Assessment

1. Find Misheck's error pattern. How can you help him?

   a) $\frac{1}{4} \times \frac{2}{3} = 83$ b) $\frac{3}{5} \times \frac{2}{7} = 121$

2. Write a story problem about each of the following. Let the stories carry different meanings of division. Illustrate the products using number lines.

   a) $\frac{1}{4} \div 5$ b) $2\frac{1}{2} \div \frac{2}{3}$

3. What is the reciprocal of $\frac{0}{1}$?

4. Neo is having difficulty with division. As indicated below, identify the error and its cause.

   $$\frac{4}{5} \div \frac{3}{5} = \frac{1\frac{1}{3}}{5}$$

## Summary

Though multiplication and division operations are difficult to model, you have seen that modelling gives more meaning to the operations. Story problems should be used because they give meaning and direction to the process. Pupils should be helped to discover the algorithm, otherwise, they will have the procedural knowledge without understanding what they are doing.

By using models to introduce each operation, *more* pupils will develop a *clearer* understanding of the rules than if you taught the rules alone. Your job of teaching fraction operations will be complete when your pupils are confidently using the rules without having to rely on the models.

Many of your students will encounter these operations and rules again when they study rational expressions (fractions containing variables) in algebra. A solid understanding of the rules will be a valuable asset for that future encounter.

## Unit 3 Test

1. Nawa has difficulty with division. What is the cause of the error?

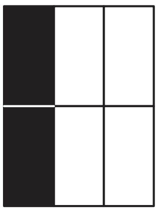
   $\frac{2}{3} \div \frac{1}{4} = \frac{3}{2} \times \frac{1}{4} = \frac{3}{8}$

2. Demonstrate how to divide $\frac{3}{4} \div \frac{1}{8}$ using sets.

3. A teacher left 17 hens to her three daughters, Lungowe, Inutu and Nosiku. Lungowe was to receive $\frac{1}{2}$, Inutu $\frac{1}{3}$ and Nosiku $\frac{1}{9}$. They could not decide how to divide up the hens until Howard loaned them one more hen, making 18. Then Lungowe took $\frac{1}{2}$ of the 18, Inutu $\frac{1}{3}$, and Nosiku $\frac{1}{9}$ of the 18. This left Howard's hen, which was returned to him. Why did this procedure seem to work?

4. A bus passenger fell asleep halfway to her destination. When she woke up, the distance remaining was one-tenth the distance travelled while she slept. How much of the entire trip was she asleep?

5. Joyce names this fraction as $\frac{2}{4}$. What is the cause of error?

6. Write a story problem for each sentence. Then use length model representations to illustrate the problems.

   (a) $1\frac{1}{2} \div \frac{2}{3}$ (b) $3 \div \frac{2}{3}$

7. There was half of some nshima covered in a bowl. George ate half of the remaining half. Judith decided to eat half of the nshima George left. Who ate the biggest piece of nshima, George or Judith? What part of nshima was left after Judith had eaten?

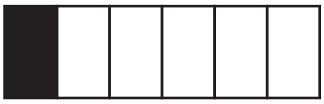
8. Lazarus added proper fractions in the following way. What is the cause of the error? How can you help Lazarus?

   $\frac{1}{4} \div \frac{3}{4} = \frac{4}{8}$ $\frac{2}{3} + \frac{1}{6} = \frac{3}{9}$

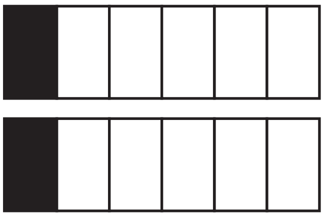
## Answers for Unit Activities

***Unit Activity 1***

1.

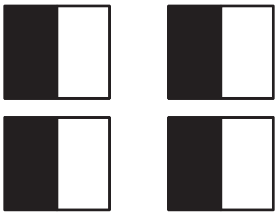


$\frac{1}{6}$



$2 \times \frac{1}{6}$

$4 \times \frac{1}{2} = \frac{4}{2} = 2$



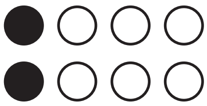
$4 \times \frac{1}{2}$

2. Find a rule for multiplying proper fractions by a whole number.
   Add the fraction the number of times designated by the whole number.
   For example, in $2 \times \frac{1}{6}$, add $\frac{1}{6}$ twice, resulting in $\frac{2}{6}$.
   Alternatively, multiply the numerator, but not the denominator, by the whole number.

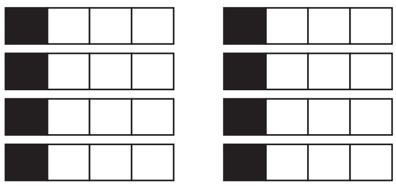
***Unit Activity 2***

1.

$\frac{1}{4} \times 8 = 2$



$8 \times \frac{1}{4}$



$\frac{8}{4} = 2$

2. The product of a whole number multiplied by a fraction is that fraction's amount of the whole number. As in the first example, $\frac{1}{4}$ of eight objects is two objects.
3. Jelita multiplied both the numerator and denominator by the whole number, which is incorrect. Multiply only the numerator by the whole number.

4. $\frac{1}{8} \times 2400 = \frac{2400}{8} = 300$

Mulenga's salary is now $2400.00 – $300 = $2100 per month.

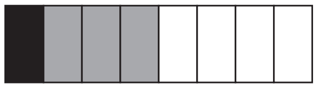
If the other employees new salary is $2800.00, so his old salary was $3150.

$$\left(2400 \times \frac{1}{4}\right) + 2800 = \left(\frac{2800}{8}\right) + 2800 = 350 + 2800 = \$3150$$

***Unit Activity 3***

1. To find the product, take a fraction of a fraction. For example, take $\frac{1}{4}$ of $\frac{1}{2}$, which is $\frac{1}{8}$ of the total. Alternatively, multiply the numerator by the other numerator and the denominator by the other denominator.

$\frac{1}{2}$



$\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$

2. To solve $\frac{2}{5}x = \frac{3}{7}$:

$$\frac{2}{5} \times x = \frac{3}{7}$$

$$x = \frac{3}{7} \times \frac{5}{2}$$

$$x = \frac{15}{14}$$

## Answers for Self Assessment

1. **Error -** Misheck multiplies the first numerator and the second denominator and records the units digit of this product. If there is a tens digit, he remembers to add it later. He then multiplies the first denominator and the second numerator, adds the numbers of tens remembered, and records them as tens in the answer. Misheck's procedure seems to correspond with cross-multiplication.

   **Remedial Work** - Try to explain the meaning of $\frac{1}{4}$ of $\frac{2}{3}$ to Misheck, then model $\frac{1}{4}$ of $\frac{2}{3}$ using slicing the rectangular approach. Ask Misheck what part of the rectangle shows $\frac{1}{4}$ of $\frac{2}{3}$? Let Misheck model another problem. Ask him to generate the rule for multiplying proper fractions by observing a pattern among the examples you looked at.

2. (a) Teddy has three-quarters of a metre of cloth. He shares it among five people. How many metres will each get? ($\frac{3}{4} \div 5$)

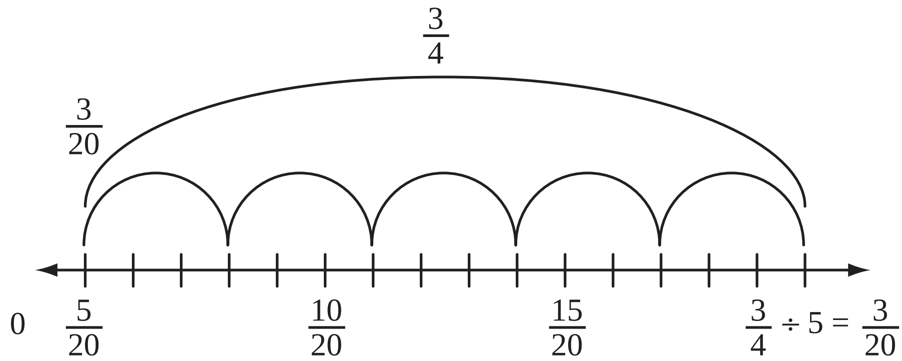



*Figure 3.9*

(b) Edith has $2\frac{1}{2}$ m of cloth to make children's dresses. If each dress requires $\frac{2}{3}$ m, how many will she able to make? ($2\frac{1}{2} \div \frac{2}{3}$)

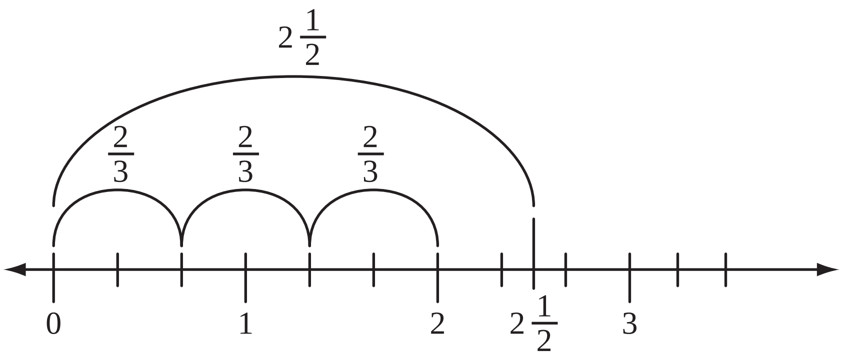



*Figure 3.10*

$\frac{2}{3} = \frac{4}{6}$. She will be able to make 3 dresses with $\frac{1}{2}$ m remaining.

3. It has no reciprocal because $\frac{1}{0}$ is undefined.

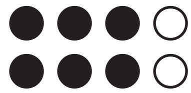
4. Neo divided the numerators, but not the denominators although they are common. Maybe Neo is trying to use the common denominator algorithm. Look at the problem as measurement division. Model the problem using length model, probably strips. Ask Neo how many $\frac{3}{5}$ make $\frac{4}{5}$?
Model more examples, then generate with her the common denominator algorithm.

## Answers to Unit 3 Test:

1. Nawa knows that in division there is an invert-and-multiply algorithm. But she cannot remember which one to invert. Instead of inverting the divisor, she inverts the dividend.

2. Using set model

$\frac{3}{4} \div \frac{1}{8}$ The whole set is 8

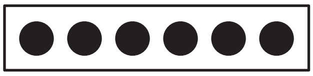


$\frac{3}{4}$ is 6 counters

$\frac{1}{8}$ is 1 counter

How many sets of $\frac{1}{8}$ in $\frac{3}{4}$?

How many **sets of 1** in **a set of 6**?

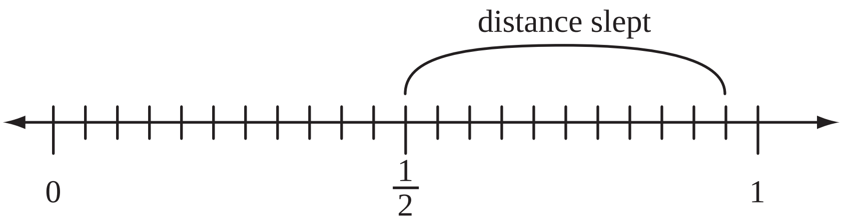


So $\frac{3}{4} \div \frac{1}{8} = 6$

3. (a) 17 is not a multiple of 2, 9, and 3. So we need a multiple such as 18.

   (b) $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{9}$ do not add up to 1, but to less than 1. This means the whole set will not be used, and that a part will remain.

4. (a)




Distance she slept is $\frac{10}{11}$ of half of the journey ($\frac{10}{11}$ of $\frac{1}{2}$).

$\therefore$ Distance she slept is $\frac{5}{11}$ of entire distance.

or

(b) Let $x$ be distance she slept.

$$x + \frac{x}{10} = \frac{1}{2}$$

$$\frac{10x + x}{10} = \frac{1}{2}$$

$$2(11x) = 10$$

$$22x = 10$$

$$x = \frac{10}{22} = \frac{5}{11}$$

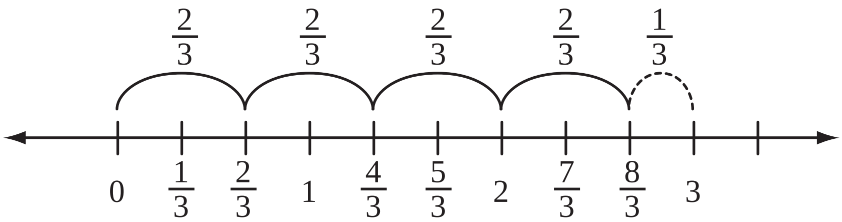
5. Joyce has difficulty determining the denominator. She looks at shaded and unshaded parts of the whole as unrelated, especially since she has already counted the shaded parts to represent the numerator. Joyce has not clearly understood the concept of denominator. She thinks denominator represents the unshaded part only, which has not been counted yet. To help Joyce, the teacher should use real objects.

6. (a) If a mother has $1\frac{1}{2}$ kg of corn, how many loaves of cornbread could she make if each loaf requires $\frac{2}{3}$ kg?

$1\frac{1}{2} \div \frac{2}{3} = \frac{3}{2} \div \frac{2}{3} = \frac{3}{2} \times \frac{3}{2} = \frac{9}{4} = 2\frac{1}{4}$

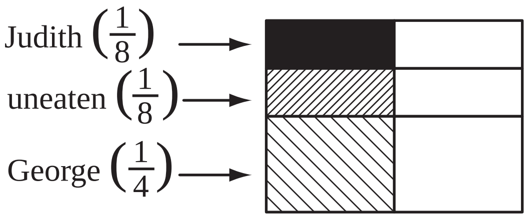
She could make $2\frac{1}{4}$ loaves of cornbread.

(b) An insect has to cover a distance of 3 km. If it covers $\frac{2}{3}$ km in a day, how many days will it take to cover the distance?




The remaining $\frac{1}{3}$ is $\frac{1}{2}$ of $\frac{2}{3}$

$\therefore 3 \div \frac{2}{3} = 4\frac{1}{2}$

7. 


$\frac{1}{2} > \frac{1}{8}$

$\therefore$George ate the biggest part of nshima. $\frac{1}{8}$ of nshima was left after Judith had eaten.

8. The cause could be that Lazarus over-generalised the rule of adding numerators when adding fractions. He could have thought that since numerators are added, so are denominators. Or maybe Lazarus has already learned how to multiply proper fractions where numerators are multiplied and denominators also multiplied. So he thinks that in addition, the numerators can be added as well as the denominators.

Lazarus needs a lot of modelling in addition. Maybe try to put the addends in one rectangular slab, if this will help.

# Unit 4: Decimals

## Introduction

Decimals are a way of representing fractions or parts of a whole. This unit has been prepared to extend the knowledge and skills you gained in Units 1, 2, and 3 on fractions. Decimals are yet another way of representing numbers. You will be exposed to ways of enabling your pupils understanding of base 10, place-value and fraction concepts and clarifying these ideas encountered in out-of-school activities. In many real life situations, people read scales, measure lengths, weigh objects, and deal in money, all areas where decimals are met. These will form focal points for learning decimals in this unit.

## Objectives

After working through this unit you should be able to:

- represent and use decimal fractions in a variety of equivalent forms in real life and mathematical situations
- develop number sense for decimal fractions in your pupils
- enable your pupils to carry out calculations with decimals using the four operations

## Introducing Decimals

When you were taught decimals, you were probably introduced to them by changing common fractions, and then by the direct division method. This approach works well, if fractions were grasped by pupils. It has been shown through research (Cockcroft 1982) that many pupils have problems in understanding the meaning of two or more decimals places, i.e., hundreds, thousands, etc.

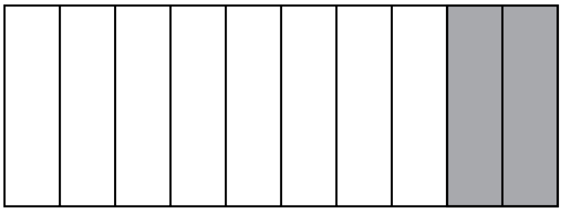
## Unit Activity 1

1. Construct five tasks to test pupils' understanding of decimals.
2. Interview at least four pupils on the task above.
3. Discuss with a colleague what you observed and learned from the outcome of these tasks and the interviews.
4. How do you think decimals should be introduced? Before you go on, read the next few pages and write down ideas of your own.

### *Model for Introducing Decimals*

***Proportional model – tenths***

Pupils should form concepts of decimals and learn the meaning of each place in a decimal. Introduce a decimal first as a fractional part of a unit. For this idea, you need a diagram, like *Figure 3.1* below, with 10 equal parts.



The shaded part is written as 0.2 and is read as “two-tenths”

*Figure 4.1*

Ask students to shade or colour fractions such as $\frac{1}{10}$, $\frac{4}{10}$, $\frac{5}{10}$, etc. State that there are two ways of representing the ’shaded’ or coloured areas. Pupils will be familiar at this stage with the common-fractions form. Introduce the decimal notation form. This should be modelled by how it is written.

e.g. .1, .4, .6

At this stage two peculiarities of decimal notation should be mentioned. First, for numbers less than 1, the decimal is written as 0.1 or 0.4 or 0.6 etc. (not just as .1) and is read as one-tenth, four-tenths, six-tenths, etc. Secondly, whole numbers are sometimes recorded using a decimal point, e.g., 4.0, 6.0, 82.0 indicating that they are whole numbers with no tenths.

## Practice Activity 1

Find objects or figures with ten equal parts or better still improvise by drawing on card-board boxes grids you can cut out.

1. Ask pupils to colour two-tenths, seven-tenths, three-tenths, nine-tenths, etc.
2. Ask pupils to write these parts as decimals.
3. Write different decimals (tenths), and ask pupils to read them out aloud, one at a time.

Ensure that pupils know that 10-tenths is the equivalent of a whole (1) and that 17-tenths is more than a whole. These ideas should have been developed when pupils dealt with fractions. Mixed numbers involving whole numbers and decimal parts are written and read as follows:

**2.4 is commonly read as ‘two point four’ but should be ‘two and four tenths’.**

Delay the use of ‘point’, as in ‘two point four’, because this brings out the meaning of the place 4 holds (four-tenths). Give pupils maximum opportunity to practice reading decimal numbers (e.g., read 5.6 as five and six-tenths).

Make simple translations between fractions and decimals notations. For example, ask pupils to calibrate a number line in tenths.

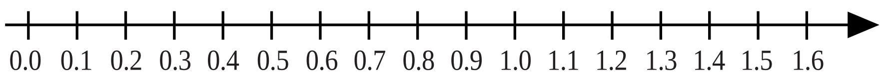



*Figure 4.2*

Develop concepts of which is bigger, e.g., between 0.9 or 1.2 by posing several questions. If this is related to fractions, pupils should be able to grasp this easily. At this stage you may want to relate this to place value ideas, or you may have already done so. For easy explanation, we have deliberately separated the two.

Before we explain decimals in terms of place value, and assuming your pupils have had enough practice, we would move on to hundredths.

**Hundredths**

When extending decimals to hundredths, the sequence is similar to that of tenths. In this case ensure that pupils have $10 \times 10$ squared-paper diagrams as shown in *Figure 4.3* below.

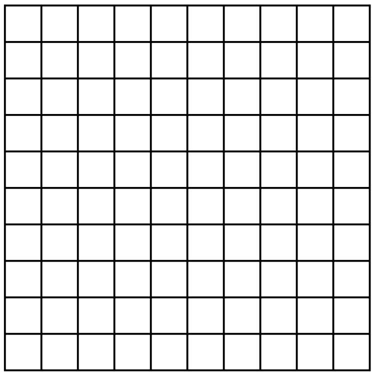


*Figure 4.3*

Discuss and ask pupils to distinguish between one-tenth and one one-hundredth of the square. Ask pupils to say how many hundredths of the square make one-tenth of it.

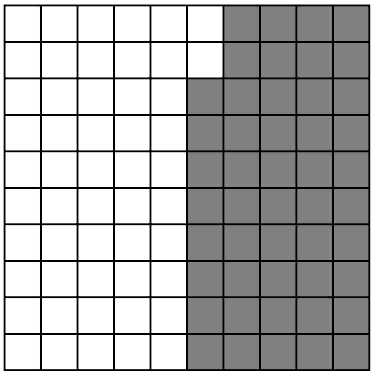


*Figure 4.4*

In the example above, ask pupils to say what part of the whole is shaded or coloured. Get three responses, $\frac{4}{10} + \frac{8}{100}$ and $\frac{48}{100}$. Then, as a way of introducing addition, write $\frac{48}{100} = \frac{4}{10} + \frac{8}{100}$. At this point, you may turn to place value interpretation and decimal notation of 0.48. For practice, ask pupils to similarly colour various fractions and record them in decimal form. Include cases where the fraction of a square is less than tenths, e.g., 0.03.

## Reflection

How would you introduce the interpretation of a fraction as a division calculation that leads to a decimal? e.g., $\frac{7}{10} = 10\overline{)7} = 0.7$

## Decimals Using Place Value

The place value of decimals is most useful in understanding algorithms with decimals. Also, after dealing with proportional parts in studying decimals, non-proportional material may now be considered.

## Practice Activity 2

Materials: For this activity you will need:

- place value charts
- abacus for pupils to practice and extend the place idea learned with whole numbers.

Review names of places on place chart, e.g., 467, 38, etc. Ask pupils what each numerical position stands for.

| Thousands | Hundreds | Tens | Ones . | Tenths | Hundredths | Thousandths |
|---|---|---|---|---|---|---|

Extend the chart while discussing the meaning of tenths, hundredths, etc. It is important to indicate the 'point' on the place value chart, as shown above. This will consolidate the role of the 'point' in decimal notation.

Use the grid to write examples and ask pupils to read and write some on the place value grid, e.g.,

1.

| T | O | $t^{th}$ | $h^{th}$ |
|---|---|---|---|
| 3 | 7 . | 2 | 4 |

Read as 37 and 24 hundredths or 37 and 2 tenths, 4 hundredths.

2. What is sixty-three hundredths?

| T | O | $t^{th}$ | $h^{th}$ |
|---|---|---|---|
| | . | 6 | 3 |

3. What is sixty-three tenths?

| T | O | $t^{th}$ | $h^{th}$ |
|---|---|---|---|
| | 6 . | 3 | |

This example will help pupils to be able to write $\frac{63}{10}$ as a decimal.

*Continues on next page*

4. Write 7 hundredths

| T | O | $t^{th}$ | $h^{th}$ |
|---|---|---|---|
| | . | | 7 |

Ensure that pupils realise there are 0 tenths, and therefore the decimal notation will be 0.07.

Ordering and rounding off decimals should be introduced at this stage. Ask pupils to compare decimal numbers, e.g., 23.94 and 23.89. Which is larger?

Further explore with pupils the idea of rounding off to the nearest unit. Here you need to ask the same types of questions as for whole numbers, e.g., round off 16.68 to the nearest tenth. Establish that 16.68 lies between 16.60 and 16.70. Ask pupils to state whether it is nearer to 16.60 or 16.70. Pupils should be able to get the result 16.7.

## Practice Activity 3

Find or make value charts or abacuses. Ensure each pupil has one. In the absence of these, let pupils improvise by drawing columns and using sticks or stones or bean seeds to ensure each pupil participates actively.

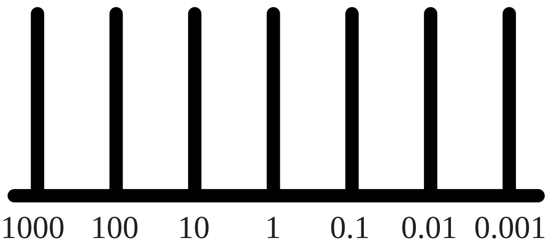



*Figure 4.5: An abacus made from a board and vertical wires to hold counting beads or washers. Each wire must hold up to 19 beads or washers.*

a) Let pupils put the appropriate number of stones or other counters in columns. Give them as many exercises as possible involving numerals from 0, 1, …, …, …, 9.

b) Ask pupils to place 10 or more ‘objects’ in the hundredths or tenths column. Observe their reaction! Remind pupils at this stage about ‘rules’ of place value. Extend these ideas to decimal places, i.e., $10 \times 0.1$ is 1.

c) Extend the place value to thousandths, ten thousandths, and so on.

## Addition and Subtraction of Decimals

Calculations involving decimals follow the same rules as for whole numbers. If pupils understand calculations with whole numbers, they can easily transfer that knowledge to decimals. Before you engage pupils in practical activities, let them relate whole number addition to decimal additions through estimating/approximating the expected sums.

## Self Assessment 1

Do the sums below mentally:

4 + 4 =

42 + 43 =

600 + 400 =

0.4 + 0.43 =

0.42 + 0.43 =

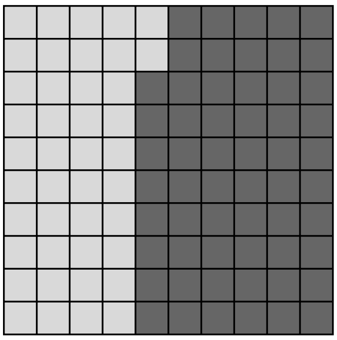
0.006 + 0.004 =

These mental calculations should be followed by practical demonstrations using the place value charts, abacus, or proportional grids. You may have used the rule of ‘carry over’ to next unit in the last problem above—10 tenths is same as 1, 10 hundredths is the same as 1 tenth, 10 thousandths is the same 1 hundredth, etc.

Addition and subtraction are done in a similar way as for whole numbers. Always ensure that the two numbers in an addition or subtraction have the same number of digits after the decimal point. If one has fewer digits, fill those with ‘place holders’, i.e., zeros.

## Practice Activity 4

a) Ask pupils to colour the hundredths grid, using two colours. Ask them what fraction of the grid is coloured by a certain colour.



b) Prepare simple addition exercises for pupils to use the sticks or objects on the place value chart or the abacuses to find the following sums:

0.12 + 0.53

0.8 + 0.1

0.024 + 0.235, etc.

c) Addition and subtraction of decimals should be related to situations such as calculations involving costs to shopping bills or measurements. Prepare as many real-life addition questions as possible for the pupils.

Example: Munthali went to town to shop. He bought a packet of sugar costing  $2.45 and 2 loaves of bread at $0.85. What was his total bill?

A carton contains items weighing 3.25 kg, 1.5 kg, and 0.85 kg. Find the total weight of the items.

Encourage your pupils to find real-life problems involving currencies and measurements. This will help them relate school activities to out-of-school experiences.

## Multiplication of Decimals

Introduce multiplication with decimals **after** decimal concepts and multiplication of whole and fractional numbers have been thoroughly dealt with.

**Multiplication as repeated addition model**

This model works if the multiplier is a whole positive number, e.g., $3 \times 0.4$. The context could be: Three sticks lined up end-to-end. Each stick is 0.4 m long. How long is the line of sticks? From this problem and previous work on multiplication with whole numbers, pupils should be able to deduce that this is a case of 'repeat addition'.

$$0.4 + 0.4 + 0.4 = 1.2 \text{ m}$$

The meaning for the above problem could also be modelled with the use of squared-paper diagrams:

$$3 \times 0.4 = 1.2$$

or in common fraction form:

$$3 \times 0.4 = \frac{4}{10} + \frac{4}{10} + \frac{4}{10} = \frac{12}{10} = 1.2$$

This model can be extended to multiplication of the form (decimal $\times$ whole number) using the distributive law. For example:

$$0.42 \times 4 = 0.42(1 + 1 + 1 + 1) = 0.42 + 0.42 + 0.42 + 0.42$$

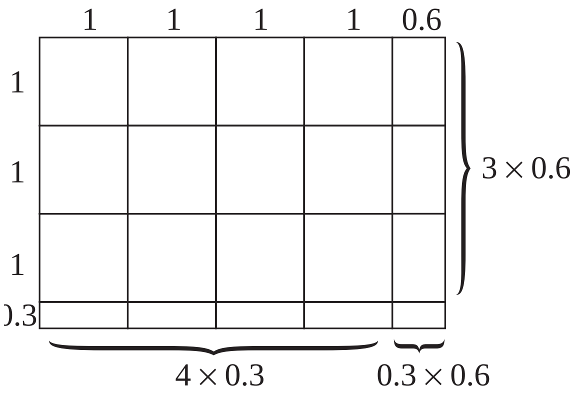
**Enlargement–Reduction model**

The case of $0.25 \times 2.4$ does not fit in the meaning of repeated addition. This could be written as 0.25 of 2.4. The multiplier in the product can be seen as a scalar factor, enlarging or reducing the multiplicand. The context could be: A wheel with a circumference of 0.25 m makes 2.4 revolutions when rolling from the door to the gate. What is the distance from the door to the gate? The required computation is $2.4 \times 0.25$, which can be seen as two complete revolutions and 0.4 of a revolution:

$$2.4 \times 0.25 = 2 \times 0.25 + 0.4 \times 0.25$$

**The area model for multiplication**

The area model gives another visual representation and interpretation of multiplication. If the multiplier and the multiplicand are the lengths of the sides of a rectangle, then the product represents the area enclosed by the rectangle. For example, the length of a hall is 4.6 m and the width is 3.3 m. What is the area of the hall? This gives the equation $4.6 \times 3.3$, which is also represented in the following diagram:




$$\begin{aligned} & (4 \times 3) + (4 \times 0.3) + (3 \times 0.6) + (0.3 \times 0.6) \\ = \ & 12 + 1.2 + 1.8 + 0.18 \\ = \ & 15.18 \end{aligned}$$

Adding the parts of the diagram ($4 \times 3 + 4 \times 0.3 + 3 \times 0.6 + 0.3 \times 0.6$) will give the numerical value of the product.

## Unit Activity 2

Explain the shortened algorithm of noting the decimal places in the question, carrying out the calculation with whole numbers, and putting back the decimal point.
(Hint: think of common fractions with denominators, 10, 100, 1000, etc.)

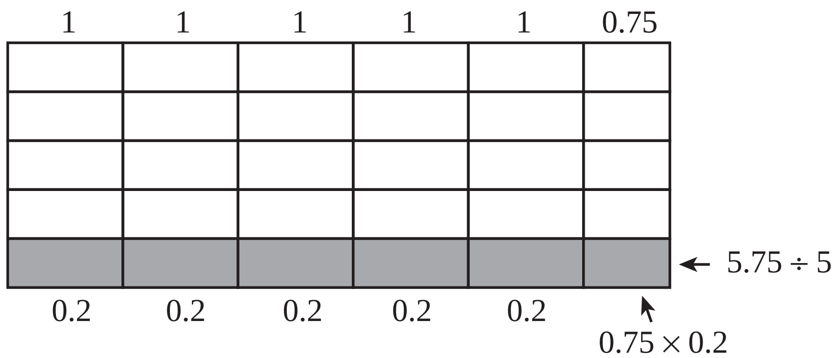
Think of life occurrences where multiplication with decimals would arise. For example, there are four and half containers of milk and each container holds 3.25 litres. How much milk is there in total? Prepare as many questions as possible for your pupils.

## Division with Decimals

The division algorithm with decimals is done in a similar way as for whole numbers. The context from which the calculation arises suggests a way of handling it. For example, if $5.75 is shared equally among five people, how much will each person get?

**Sharing / partitioning model**

The problem above suggests partitioning 5.75 into five equal parts. The sharing model works as long as the divisor is a whole number.



This model fails in cases where the divisor is negative, or is a fraction or decimal. Calculations such as $7.45 \div 3.2$ cannot be described with the sharing model.

**Grouping or "goes into" model**

This model is based on continuous subtraction. For example, how many bottles containing 0.35 litres can be filled from a storage bottle containing 8.75 litres? The useful phrase is 'how many times does 0.35 go into 8.75'? The total is known (8.75), each part is known (0.35), but the number of parts needs to be found. Many situations can be described using this model as long as the dividend is greater than the divisor.

**Fractional "goes into" model**

Example: A bag contains 4.5 kg of rice and 3.5 kg of it is used for cooking. What fraction of the total amount was used? The question 'what fraction of 4.5 gives 3.5' is the key element in the model. Thus, the desired answer comes from $? \times 4.5 = 3.5$

## Self Assessment 2

Use the models above to do the following exercises:

1. A bag contains 75 kg of sugar. Small packets of 1.5 kg each are filled from the bag. How many 1.5 kg packets can be filled?
2. 24 kg of flour is placed in bags of 0.8 kg each. How many will be filled?
3. 2.2 m of cloth costs $4.95. How much will a metre cost?
4. 1 kg of cheese costs P16. What will you pay for 0.2 kg?
5. A rectangular piece of land measures 40.4 m by 30.6 m. What is the area?
6. What is the total cost of four packets of milk at $0.98 each, 5 kg of potatoes at $1.07 per kilogram and three tins of tomatoes at $0.58 a tin?
7. Petrol is sold at K1548.45 per litre. How much fuel can be purchased for K40 000?

## Unit Activity 3

Write story problems for the following calculations:

$32.1 \times 3$
$0.36 \div 12$
$0.24 \times 4.5$
$0.48 \div 0.56$

### *Multiplication of a Decimal Fraction by 10, 100, 1000*

The study of what happens when a decimal number is multiplied repeatedly by 10 shows that the decimal place moves two places when multiplied by 100, three places to the right by 1000, and so on.

For example:

| Th | H | T | O | $T^{th}$ | $H^{th}$ | $Th^{th}$ | |
|---|---|---|---|---|---|---|---|
| | | | 2 . | 7 | 4 | 3 | $\times$ 10 |
| | | 2 | 7 . | 4 | 3 | | $\times$ 10 |
| | 2 | 7 | 4 . | 3 | | | $\times$ 10 |
| 2 | 7 | 4 | 3 . | 0 | | | |

In fact, this conforms with the rule pupils established earlier in the module. Any place holder will hold nine ‘elements’ or less. If there is a tenth, it moves one place to the left.

For example: ten hundredths is one tenth
ten tenths is one
ten ones is ten, etc.

Provide your pupils with as many examples as possible on the multiplication of decimals. Extend these to include problems like $0.32 \times 0.017$.

## Practice Activity 4

Organise pupils in groups. Give them problems involving multiplication by 10, 100, 1000, and so on. Let the groups compete. Speed and accuracy is the key.

## Unit Activity 3

1. Write down the sequence of steps and activities you would use to introduce and teach a lesson on division with decimal numbers.
2. Use this list to prepare the lesson. Remember to indicate the materials you will need, and get them ready in advance.
3. Try out the lesson with your class. Invite a colleague to observe the lesson and make notes. After the lesson, write down your own observations. Keep your observations and those of your colleague in your portfolio.
4. Prepare twenty questions on decimal fractions and give this test to your class. Keep the questions and the test results in your portfolio for future consideration.

## Unit Activity 4

If the school or a number of your pupils have calculators, then you can explore and establish a number of generalisations with decimals when multiplying and dividing by 10s, 100s, 1000s, etc.

For example, multiply 0.48912 by 10 five times. Write down the answers immediately under the multiplicands.

Multiply 0.489127 by 100, three times.
Divide 100 by 5
″ 0.5
″ 0.05
″ 0.005
″ 0.0005, etc.
Divide 2378.46 repeatedly by 10, 100, etc.

Let pupils explain what happens in each case. In all these activities or tasks, encourage pupils to first estimate the results. It is no use for them to blindly punch figures on the calculator and copy the resulting answers without appreciating the values they get. Identification of patterns in the above examples is key.

## Summary

This unit dealt with decimals. We learned that to construct concepts of decimals, we have to understand the meaning of decimals and form ideas about decimal place values. The use of materials and charts helped achieve this understanding. Also, the exploration of equivalences between fractions and decimals facilitates easy development of ideas of decimals, because fractional concepts were dealt with earlier.

Calculations with decimals were dealt with later in the unit. It was established that estimation of the result is key before the actual calculation, and mental computation was encouraged before using paper and pen. When pupils do calculations with decimals, they may have to first compute as for whole numbers and then correctly place the decimal point.

Ways of guiding pupils to develop these ‘rules’ were described in this section, including suggestions for extensions. It was advised to base the explanation of multiplication and division of decimals by 10 or its power on the principle of place value, i.e., digits move rather than the decimal point. It is helpful for pupils to apply these concepts to realistic situations involving money or measurements.

As your pupils move on to ratios and percentages, give them practice exercises from time to time about proper fractions. Proper fraction operations are used less than decimal operations and students tend to forget proper fraction algorithms unless they are reviewed over an extended period of time.

## Self Assessment 3

1. Describe how decimals and common fractions are alike and different.
2. How should computation rules for the operations with decimals be taught? Why?
3. Explain the meaning of how $2.7 \times 3.8$ is 10.26.
4. Model the problems below in terms of either measurements involving litres, grams, length, or money.
   a) $2.600 - 0.125$
   b) $4.120 + 2.3$
   c) $6.34 \times 7$
   d) $8.80 \div 4$
   e) $16.74 \times 9.4$
5. Given that $3 \times 78 = 234$ find:
   a) $3 \times 7.8$
   b) $0.3 \times 78$
   c) $0.03 \times 0.078$
6. Express the following decimals as common fractions:
   a) 0.07 b) 0.43 c) 0.16
7. Express the following fractions as decimals:
   a) $\frac{17}{100}$ b) $\frac{4}{100}$ c) $\frac{9}{10}$ d) $\frac{4}{12}$
8. Give everyday examples of decimals numbers with one, two, or three figures after the decimal point.
   a) Use these to plan a lesson to round off decimal numbers to the nearest whole number.
   b) Use these to plan a lesson to estimate the product of two decimal numbers.

## Unit 4 Test

**Section A**

1. Rewrite $10 + \frac{1}{10} + \frac{6}{100} + \frac{4}{10000}$ as a decimal.
2. Hanjobvu has a balance of \$713.64 in his account. He wrote two cheques for \$17.10 and \$103.75. What is the balance left in his account?
3. Find the area of a rectangular wall that is 6.5 m long by 2.7 m high.
4. Tilye, Mukuma, and Mubita went to a restaurant for a meal. The bill came to \$38.50, which they agreed to share equally. What was the minimum each person had to pay?
5. Chileshe walks at the rate of 80 steps per minute. If the average length of her step is 0.875 m, find the time she takes to walk 4.62 km.

**Section B**

1. Write a lesson for learning the ordering of decimals.
2. Describe the steps in introducing division with decimals when both divisor and dividend are decimals.

## Answers for Self Assessments

***Self Assessment 1***

8, 85, 1000, 0.83, 0.85, 0.010

***Self Assessment 2***

1. 50 packets
2. 30 bags
3. 42.25 per metre
4. P3.2
5. 1236.24 $m^2$
6. $11.01
7. 25.832284 L

***Self Assessment 3***

1. Both decimals and common fractions represent parts of a whole. Unlike common fractions, decimals always work on a base 10 system, whereas common fractions represent sets of whichever number base. (Other answers are possible.)
3. $2.7 \times 3.8$ can be seen in additive terms:
   $2.7 + 2.7 + 2.7 + (2.7 \times 0.8) = 10.26$ because $2.7 \times 3.8$ is the same as adding 2.7, 3.8 times, which results in 10.26.
4. a) 2.475
   b) 6.42
   c) 44.38
   d) 2.2
   e) 157.356
5. a) 23.4
   b) 23.4
   c) 0.00234
6. a) $\frac{7}{100}$
   b) $\frac{43}{100}$
   c) $\frac{16}{100} = \frac{4}{25}$
7. a) 0.17
   b) 0.04
   c) 0.9
   d) $0.3\overline{3}$

## Answers to Unit 4 Test

1. $10 + 0.1 + 0.06 + 0.0004 = 10.1604$
2. \$592.79
3. 17.55 $m^2$
4. \$12.83
5. 66 min or 1 h 6 min

# Unit 5: Ratios, Rates, and Proportion

## Introduction

In this unit, you will expand your knowledge of fractions to include ratios. You will also be introduced to the concept of proportional reasoning in different contexts.

Direct proportion is used in a number of everyday situations, for example rates of exchange or time taken for a job. Two quantities are said to be in direct proportion if the increase (or decrease) in one is matched by an increase (or decrease) in the other.

## Objectives

At the end of the unit you should be able to:

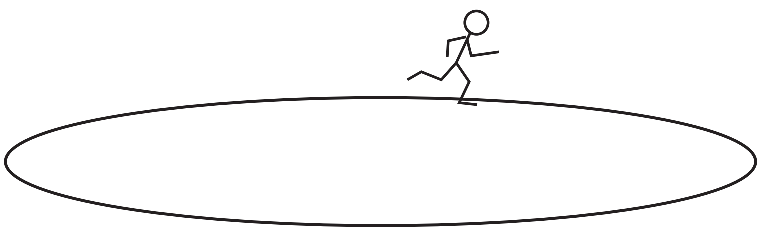
- solve ratio and proportion tasks in different contexts, including situations involving measurements, prices, geometric and other visual contexts, and rates of all sorts
- reflect, discuss, and experiment with predicting and comparing ratios
- help your pupils to relate proportion reasoning to existing processes, i.e., the concept of unit fractions and unit rates

## Unit Activity 1

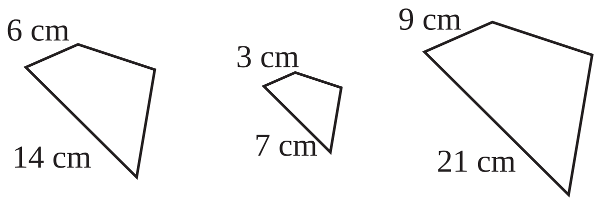
The following exercises cover the concepts of ratio and proportion. You may do them alone or discuss them with a colleague. We will use these same exercises to introduce different types of ratios, proportions, and rates.

1. Mr. Hara sells sugar cane according to the number of nodes on each cane. A cane with three nodes sells for 50 cents. Mr Hara has a longer cane that is worth $1.50. How many nodes does this cane have?

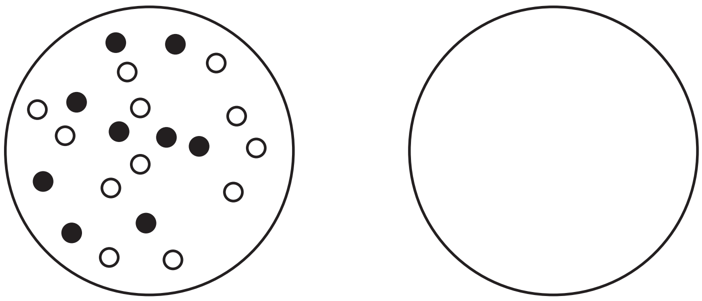

2. Yesterday, Zifile counted the number of laps she ran at track practice and recorded the amount of time it took. Today, she ran fewer laps in more time than yesterday. Did she run faster, slower, or about the same speed today, or can you tell? What if she had run more laps in more time?



3. Here is a set of three geometrical shapes. Investigate the relationship(s) between the corresponding sides.




4. Fill the second circle with a similar mix of black and white dots. Draw your dots about the same size and with the same spacing as the dots in the left circle. Do not count the dots in either drawing until you are done. Try to get the mixes of black and white to look the same. When finished, count and compare the number of black and white dots in the left with number of black and white in your drawing (circle).



How can these counts help you decide if your drawing has the same mixture? Can you adjust the number of either or both colours so that you could argue that they are in the same mix as those in the left circle?

The exercises above represent ratios in different settings of contents. Ratios can compare two parts of the same whole. In the dot mixture exercise, 9 black dots and 12 white dots are in one whole set of 21 dots. The ratio or fraction of black dots to the whole is 9 to 21. But you could also compare the black to the white dots instead of one colour to the total. A comparison of black to white is not a **fraction** but a **ratio** of one part to another. The black and white dots are in the ratio 9:12 or 3 to 4. Can you think of other examples of part-to-part ratios?

## Reflection

The fraction $\frac{2}{3}$ is a comparison of 2 parts to the 3 parts that make up the whole. Such a comparison of parts to a whole is a ratio. Is it correct to say that **ALL** fractions are ratios? What about vice-versa—are all ratio fractions? Or is a ratio a fraction? Illustrate your opinion with examples.

## Rates

In the sugar cane exercise, you compared nodes and money. In other words, you were comparing measures of two different things or quantities. You should have figured out that for every 50 cents you have 3 nodes, so the cane for $1.50 should have 9 nodes. This is an example of a rate—a comparison of the measure of two different things or quantities. This is an example of a ratio that is also a rate. All prices are rates, and they are also ratios of money to a measure of quantity.

Question 2, involving the comparison of time to distance, is also an example of a rate. All rates of speed, for example, driving 40 km per hour or running a kilometre in three minutes, are ratios of time to distance.

## Self Assessment 1

Against the statements below, indicate whether each is **True** or **False** – i.e., whether the statement is a ratio or not:

- Circumference of a circle to the diameter, a relationship designated by the Greek letter $\pi$ (pi)
- The probability of an event occurring
- The distance scales on a map
- Trigonometric functions in a right-angled triangle

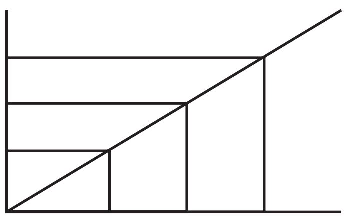
From the exercises above, you should have realised that a ratio is an ordered pair of numbers of measurements that are used to express a comparison between the numbers or measures. This is written as $a : b$ where $b \neq 0$. Ratios cover a wide range of situations, but the principle of all ratios is essentially the same.

In the activities that follow, pupils will be expected to select an equivalent ratio among others presented. The focus is on intuitive rationale for why the two ratios selected are the same or why they **“look”** the same. In other cases/activities, pupils will be expected develop numeric methods or provide equivalent ratios to validate and explain their reasoning.

## Practice Activity 1

Ask your pupils to classify rectangles. Make a collection of ten or more different rectangles for students to cut from paper. Prepare the rectangles so that each is similar to two or more other rectangles. Make three or four sets of similiar rectangles.

The following method will help you draw similar rectangles.

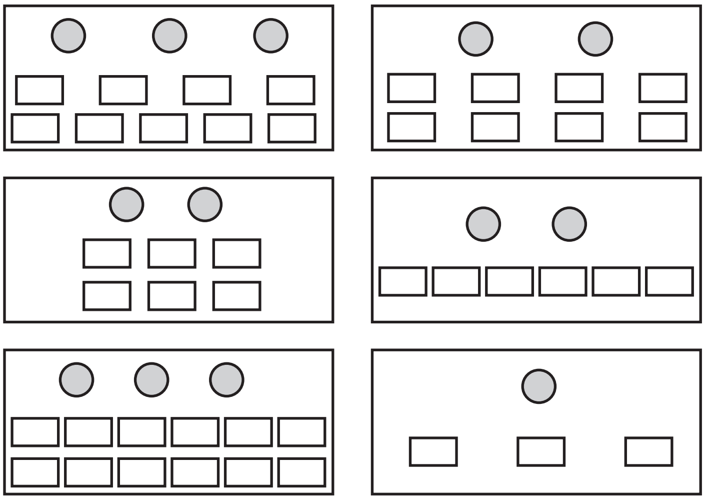


Draw a diagonal for each set of similar rectangles. All rectangles with the same diagonal will be similar. Trace each rectangle and cut it out.

Let pupils put those rectangles that are **“alike”** together. When they have grouped the rectangles, have them make additional observations to justify why they go together. Encourage as many ways as possible to describe how the rectangles are alike.

## Practice Activity 2

Prepare cards with two distinctly different objects as shown below. Give one card to each student or group. Pupils are then to select a card where the ratio of the two types of objects is the same. This activity offers an introduction to the notion of rates as ratios.



Also allow pupils to compare circles to circles and boxes to boxes.

## Practice Activity 3 (price as a ratio)

In this activity, let pupils pair coins or bills with objects. This will demonstrate price as a ratio.

## Practice Activity 4 (scaling)

In this activity, pupils should fill in charts where paired entries are related in some way. Scaling up is a matter of providing entries with larger numbers and scaling down is entering smaller numbers.

| | | |
|---|---|---|
| Examples: | 30 minutes | $\frac{1}{2}$ hour |
| | 60 minutes | 1 hour |
| | 90 minutes | ? hours |
| or | | |
| | 1 mango | K100.00 |
| | 3 mangoes | K300.00 |
| | 7 mangoes | ? |

## Practice Activity 5

Set the following questions for pupils:

1. In classroom A, there are 12 boys and 15 girls. In classroom B, there are 8 boys and 6 girls. In classroom C, there are 4 boys and 5 girls.
   (a) Which two classrooms have the same boys-to-girls ratio?
   (b) On one occasion classroom C joined classroom B. What was the resulting ratio of boys to girls?
   (c) On another occasion, classroom C joined classroom B. What was the resulting ratio of boys to girls?
   (d) Are your answers to parts (b) and (c) equivalent? What does this tell you about adding ratios?

2. (a) If 1 cm on a map represents 35 km, how many kilometres are represented by 3 cm, 10 cm, and 12 cm?
   (b) Lusaka is about 600 km from Chipata. About how many centimetres apart would Chipata and Lusaka be on this map?

## Proportional Reasoning

Proportional reasoning involves the ability to compare ratios and to predict or produce equivalent ratios. It requires the ability to mentally compare different pieces of information and to make comparisons, not just of the quantities involved but of the relationships between quantities as well.

Thus, pupils need opportunities to construct the type of comparative thinking that is fundamental to proportional reasoning. Pupils should experience and reflect on a variety of examples of ratios so that they begin to see different comparisons that are the same ratio.

In the exercises at the beginning of this unit, you were provided with opportunities to construct relationships between quantities or measures. You were able to think about these relationships and apply them to similar or different situations. In many cases, you had to construct each ratio mentally before it could be applied proportionally to the next measurement.
Pupils also require these opportunities if they are to develop their abilities to reason proportionally. From the experiences in the exercises, we can conclude that a proportion is a statement of equality between two ratios. Different notations for proportions are used:

2 : 6 = : 12 or $\frac{2}{6} = \frac{4}{12}$

These are read "2 is to 6 as 4 is to 12" or "2 to 6 is in the same ratio as 4 to 12". A ratio that is a rate usually includes the units of measures. For example,

$$\frac{\text{K1548.00}}{\text{1 litre}} = \frac{\text{K4644.00}}{\text{3 litre}}$$

Finding one number in a proportion when given the other three is called **'solving a proportion'**.

## Reflection

A very real distinction exists between a proportion and the idea of equivalent fractions. Distinguish between two equivalent fractions and a proportion.

(Hint: two equivalent fractions are different symbols for the same quantity or amount; they represent the same rational number in different forms. What about a proportion?)

## Unit Activity 2

**Intuitive Methods for Solving Proportions**

Solve the two problems below using the approach that seems easiest or most reasonable:

1. Ngoma bought 3 maize cobs for K2400.00. At the same price, how much would 10 maize cobs costs?
2. Ngoma bought 4 maize cobs for K3755.00. How much would 12 maize cobs cost?

In the first problem, it is easiest to determine the unit rate or unit price of one cob of maize. This can be found by dividing the price of 3 maize cobs by 3. Multiplying this unit rate of K800 by 10 will give the answer. This is referred to as the **Unit-Rate-Method** of solving proportions.

In the second problem, a unit-rate approach could be used, but the division does not appear easy without a calculator. But since 12 is a multiple of 4, it is easier to see that the cost of a dozen is three times the cost of four. This way is referred to as "factor-of-change" approach. This approach is frequently used when the numbers are compatible.

Graphs provide another way of thinking about proportions and connect proportional thought to algebraic interpretations. All graphs of equivalent ratios are straight lines through the origin. If the equation of one of these graphs is written in the form $y = mx$, the slope $m$ is always one of the equivalent ratios.

These intuitive approaches to ratio and proportion, as well as the procedural algorithm of using cross product, should all be explored.

The activities suggested below will provide your pupils with opportunities to develop proportional reasoning. The activities are not designed to teach specific methods or algorithms for solving proportions, although some can be modified to suit different situations or thought processes.

## Practice Activity 6

*This activity requires:*

- a balance or scale that weighs to the nearest gram. The activity will proceed more quickly if there are two to four scales.
- a set of four to six “matching” cups or containers for each group of students. “Matching” means that each group’s set of containers has containers of identical shape, but of differing sizes. A set of kitchen measuring cups (quarter-cup, third-cup, half-cup, etc) is a good set for one group. Small, medium, large, and larger soup tins make another set. Nesting cups from a child’s toy or different-sized beakers from a chemistry set can make the other sets.
- enough of an easily weighable material (sand, rice, mealie meal, sugar, etc.) to fill, at the minimum, each group’s largest container. The activity will progress more quickly if each group has enough material to fill all its containers at one time. Have different groups use different materials.

*Procedure:*

1. Each group weighs its empty containers to the gram.
2. Each group fills all its containers with the same substance, then weighs them all again. (Groups with insufficient material can fill and weigh one container at a time.)
3. Group members calculate the net weight of material in each container, and add the net weights for a total weight of material in all containers.
4. Group members plot a large pie graph in which the pie wedges represent the material weights in the individual containers.

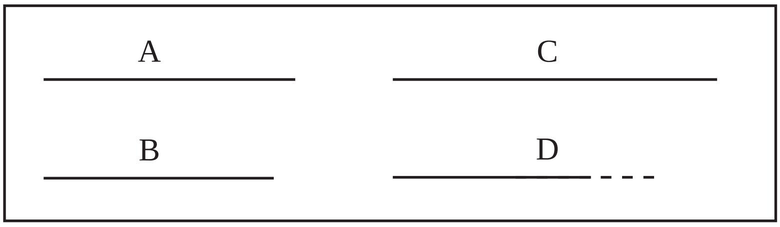
*Discussion:*

Engage pupils in a discussion by asking questions about the graphs:

- For groups that had identical or nearly identical containers, but different materials, ask if their pie graphs are similar or different? Why?
- How do the graphs compare between groups that had the same material but differing sets of containers?
- From the graphs, can you tell which group had the set of largest containers? The set of most containers? The set of containers that are closest to or farthest apart from one another in size?
- Pointing to various graphs, ask:
    - what fraction of a group’s material is in the smallest, the largest, the two smallest, or all but the largest containers?
    - what is the ratio between one group’s smallest and largest container?
- Ask students to find a graph that shows the same fraction of material in one container as in two containers, or in one container and in three containers.

## Practice Activity 7

Draw two lines labelled A and B on the board. Draw a third line, C, that is significantly different from A. Begin drawing a fourth line D, under the line C. Let pupils tell you when to stop drawing so that the ratio of C to D is the same as the ratio of A to B.




Where should line D end so that A : B = C : D. Measure all four lines and compare the ratios with a calculator.

## Practice Activity 8

Use a price chart like the one below:

| Mango | 3 | 6 | 9 | 12 | 15 |
|---|---|---|---|---|---|
| Price | K249 | K498 | K747 | K996 | K1245 |

Let pupils select any two ratios from the chart that they think should be equal and write them in an equation as fractions. For example,

$$\frac{3}{249} = \frac{12}{996}$$

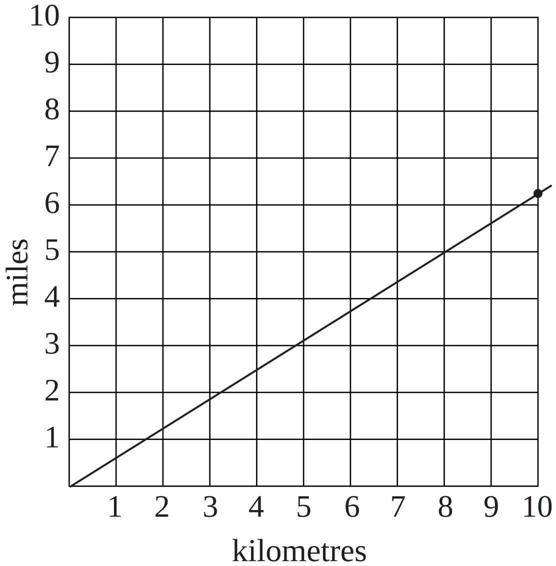
The cross product for this becomes $3 \times 996 = 12 \times 249$. Will the numbers always work out that way in a proportion? Let pupils investigate and establish the answer using other data and checking the cross-products with a calculator.

Erase K498 from the chart. What is the cost of six mangoes? Allow pupils a lot of oportunities to explore the unit-ratio approach and the factor of change approach.

**Note**: Two ratios are equal if one is a multiple of the other. Therefore, all equal ratios are of this form $\frac{a}{b} = \frac{ac}{bc}$ where $c$ is a non-zero rational number.

## Practice Activity 9 (using graphs)

In the exercise below, let pupils draw a graph and use it to answer the questions that follow:




Let pupils use the graph to estimate:

a) 3 miles in km
b) 6 km in miles
c) 8 miles in km
d) 10 km in miles

## Self Assessment 2

1. Mr. Hara works a 30 hour week and gets K160 000.00. If he worked a 40 hour week (at the same rate) how much would he get?
2. Miss Jere drives 400 km in 5 hours. How far would she go in 6 hours? How long would it take her to travel 300 km?
3. A post 4 m high casts a shadow 2.5 m long. A building is 18 m high. What length of shadow will the building cast at the same time?

## Reflection (Quantities not in Proportion)

Tengayumo the First had fourteen wives. How many wives had Tengayumo the Third?

Does this problem fit into a discussion of proportional reasoning introduced at the beginning of this unit? If no, why not? If your answer is yes, then how?

Some problems do not have quantities/measures that are in proportion. You need to make sure that quantities have a rule connecting their proportion before carrying out any calculations. For example, Royal Arts Studios is advertising video tapes for sale. Here is a chart they have made:

| Number of Tapes | 3 | 5 | 10 | 20 |
|---|---|---|---|---|
| Price | K5000 | K23 500 | K44 000 | K85 000 |

The number of tapes and the cost are not in proportion. If you pay K5000.00 for one tape, you would expect to pay K25 000.00 for five tapes.

## Self Assessment 3

In the questions that follow, some of the quantities are in proportion, others are not. Answer questions in which the quantities are in proportions. Indicate **'NOT in proportion'** for those that are not.

1. Three metres of curtain material costs K3600. Mrs Nyirenda wants to buy five metres. How much will four cost? How much material could she buy for K10 000.00?
2. A car hire company charges the following rates:

| Kilometres | 10 | 15 | 20 | 50 | 100 |
|---|---|---|---|---|---|
| Charges in $ | 4 | 5 | 6 | 15 | 35 |

   How much will it cost to drive a hired car for 150 kilometres?
3. Mary can type a letter containing 300 words in six minutes. How long will it take her to type a letter containing 700 words?
4. Mulenga goes to the market to buy 5 kg of potatoes at K8500.00. Find the cost of 6 kg of potatoes and also the weight of the potatoes one can get for K12 000.00.
5. Three friends win a jackpot. They put in what they could afford and agreed that if they won, the money would be split in proportion to the amount each contributed that week. They won K50 000 000.00 and cannot agree on how to split the money. Chadili put in K80 000.00, Mautho paid K100 000.00 and Thandekile paid K75 000.00. How should the jackpot be split?

## Inverse Proportion

Inverse proportion means that two quantities are connected in a different way. As one quantity increases, the other decreases. For example the chart below shows that the faster you travel the less time you take.

| Speed | 48 km/h | 64 km/h | 96 km/h |
|---|---|---|---|
| Time | 12 hours | 9 hours | 6 hours |

At what speed should Mulenga travel to complete the same journey in three hours?

Note that the ratios are in reverse order.

## Self Assessment 4

1. Ng’endwa’s Café ordered enough food to last twenty people six days. How long would the food last if the average number of customers in a day was 25?
2. Mr. Molefhi wrote a book of 310 pages with an average of 250 words per page. One edition is printed using large type, which can only fit an average of 200 words per page. How many pages will this edition contain?
3. Miss Muhone was going to meet Mbonyiwe at the airport. If she travelled at 100 km/h, she would arrive one hour early and if she travelled 50 km/h, she would arrive one hour late. How far was the airport?

## Summary

Pupils will need a considerable amount of class time for intuitive thought and reflection. However, this time will be well spent because it will give them an opportunity to:

- Discover relationships
- Express these relationships in their own words
- Connect concepts and procedural knowledge

The use of a symbolic or mechanical method, such as the cross-product algorithm for solving proportions, does not develop proportional reasoning and should not be introduced until pupils have had considerable experience with more intuitive and conceptual methods.

## Unit 5 Test

**Section A**

1. “Proportional thinking is more than just the ability to solve a proportion”. Discuss this statement.
2. Prepare a lesson that introduces proportional reasoning through activities for your class.
3. Identify two equal ratios from any of the experiences or activities that you have done. Pretend that one of the measures in the two ratios is unknown, then set up a proportion to find it. Explain how you would solve the proportion in terms of a unit rate. How would you solve it in terms of a factor of change?

**Section B**

4. A car uses petrol at the rate of 1 litre for every 6.5 km travelled. How many litres does it use on a journey of 117 km?
5. A boy cycles 16 km in an hour and a girl runs 4.4 m in a second. Who is faster?
6. A car journey takes $4\frac{1}{2}$ hours when the car travels at an average speed of 120 km/h. Calculate the time taken for the same journey when the average speed is reduced to 108 km/h.

## Answers for Unit Activities

***Unit Activity 1***

1. 9 nodes
2. Slower. If she had run more laps in more time, it would be difficult to know if she changed her speed unless her time per lap was measured on both days.
3. The first shape's dimensions are twice those of the second shape, while the third shape's dimensions are three times those of the second shape. Otherwise, the three are the same shape.

***Unit Activity 2***

1. K8000.00
2. K11 265.00

## Answers for Self Assessments

***Self Assessment 1***

- True
- True
- False, although can be expressed as a ratio. That is, a 1:50 000 scale **is** a ratio, but the typical scale of km is not.
- True

***Self Assessment 2***

1. K213 333.33
2. 480 km, 3.75 hours
3. 11.25 m

***Self Assessment 3***

1. 4 metres would cost K4800; she could buy 8.3 metres with K10 000
2. Not in proportion
3. 14 minutes
4. 6 kg of potatoes costs K10 200; with K12 000, one can buy just over 7 kg of potatoes
5. Chadili should receive K15 685 000, Mawtho should receive K19 610 000, and Thandekile should receive K14 705 000.

***Self Assessment 4***

1. The food would last almost 5 days (5 full days with 24 people).
2. The large print edition has 387.5 pages with 200 words per page.
3. 200 km

## Answers for Unit 5 Test

4. 18 litres
5. The boy cycles 0.04 seconds faster than the running girl, therefore, he travels only slightly faster.
6. 5 hours

# Unit 6: Percentages

## Introduction

This unit extends your knowledge of common fractions, decimals, and ratios to yet another representation of numbers—**percent**. Since percent is also a proportion, it is used in the context of ratios and proportions, and builds on the information and activities from the previous unit.

In this unit, you will explore the meaning of percent and carry out calculations involving percents. You will also have an opportunity to practice ways to teach percentages to pupils.

## Objectives

After completing this unit you should be able to:

- state and explain the meanings of the terms percent and percentage
- use percentages to express proportions of a quantity or set
- convert fractions and decimals to percent and vice-versa
- carry out calculations involving percents
- teach percents effectively to your pupils

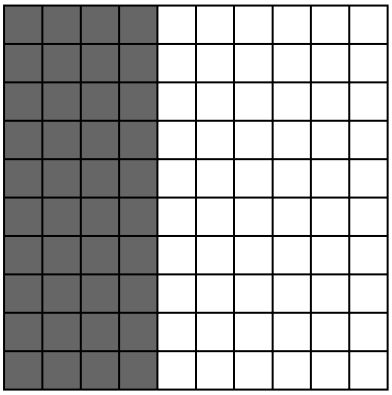
## Unit Activity 1

Using a square grid of $10 \times 10$, carry out the following activities:

1. Shade 25 squares in one corner of the grid, 15 in another, 10 in another, and a half square of the grid in the remaining section. Shade these squares using different colours.
2. (a) Write down what the shaded parts represent as fractions.
   (b) Write down what the shaded parts represent as decimals.
3. Add up:
   (i) all the shaded squares on the grid.
   (ii) the three largest shaded areas on the grid.
   (iii) the two smaller shaded areas on the grid.
   Write down what these parts represent as fractions and as decimals.
4. What can you say about fractions and decimals?

## Meaning of Percent

From the activity you have just done, you should have found the following answers to part 2, a and b : $\frac{25}{100}$, $\frac{15}{100}$, $\frac{10}{100}$, and $\frac{0.5}{100}$ in terms of fractions and 0.25, 0.15, 0.1 and 0.005 in terms of decimals. When the denominator for a fraction is 100, a special term is used to refer to the numerator. The term “percent” is used. The term percent means ‘out of hundred’ or ‘for each hundred’. The term ‘percent’ is symbolised as ‘%’. Thus, for a proportion of 40 out of 100, we may write this as ‘40%’ read as ‘40 percent’. This may be represented diagrammatically as shown below:



Percents are common in everyday life. They provide a much easier way to compare proportions. For example, which is bigger, $\frac{3}{8}$ or $\frac{5}{16}$? Contrast this with 37.5% or 31.25%. In this latter case, it is easy to tell that 37.5% is bigger while in the other case it is not as easy. The convention to a standard figure (100) makes it convenient to compare two or more proportions.

## Expressing a Proportion as a Percent

The commonest and easiest proportions are the ones involving money where, for example, 100 cents make 1 dollar or 100 ngwees make 1 Kwacha or 100 thebes make 1 Pula, etc. In measurements, we have 100 cm making 1 metre. Thus, problems like what percent is 60 cents to 1 dollar or what percent is 7 ngwee to 1 Kwatcha or what percent is 47 cm to 1 metre are easily determined as 60%, 7%, and 47% respectively. However, the ‘bases’ or proportions are often not expressed as ‘so many per hundred’, therefore, we have to find an equivalent proportion for a population of 100, as examples 1 and 2 below illustrate.

*Example 1*
**Using a Ratio Table**

a) What is the percent of girls in a class of 20, where there are 15 boys?

| Number of pupils | 20 | 100 |
|---|---|---|
| Number of girls | 5 | 25 |

$\times 5$

Thus, 5 girls out of 20 is the same proportion as 25 out 100.

Therefore, 25% of the class are girls.

b) A shop owner ordered 350 electric toys and of these 140 were defective. What percentage does this represent?

| | | | |
|---|---|---|---|
| Electric toys | 350 | 700 | 100 |
| Defective toys | 140 | 280 | 40 |

$\xrightarrow{\times 2}$ $\xrightarrow{\div 7}$

140 out 350 is the same proportion as 280 out of 700 (multiply each by 2)

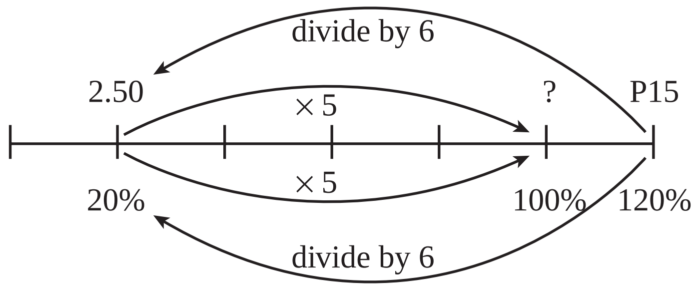
If we divide the resulting proportion by 7, we obtain 40 out of 100, thus 140 out 350 is 40%.

Therefore **40%** of the toys were defective.

*Example 2*

**Using a double number line**

Ten packets of razor blades cost P15. This cost includes 20% tax. What is the cost of razor blades without tax? Use a double number line to illustrate your answer.

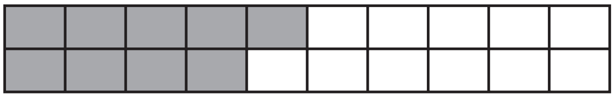



The unknown cost of the razor blades is set at 100%, and adding tax makes 120%. Dividing by 6 gives 20%, which corresponds to P2.50. Now multiply by 5 to get from 20% to 100%. Doing likewise above the number line gives 5 × P2.50 = P12.50 as the untaxed price.

The models allow a better understanding of the concepts and algorithm involved. Standard rules should not be the starting point. With time, “rules” will be discovered, but these rules must be firmly based on a context that pupils can relate to. If the “rule” is forgotten, pupils can go back to the context and “build it up again”.

## Self Assessment 1

1. Make use of equivalent proportions in the question below.

   In the diagram above, 11 out of 20 squares are shaded. What percent does this represent of the whole diagram?

2. Model the following problem using a grid.

   Mr. Banda received $60 for delivering the produce to the hotel on time. He has been promised a 10% raise if he delivers his produce even earlier. How much will the raise be?

Solve the following questions using a ratio table and/or a double number line:

3. A Housing Authority increased its house rents by 15% as of 1$^{st}$ April. If Ms. Ngoma was paying $540 house rent in March, what will be the house rent after April?
4. In a school, 25 pupils are on a special diet. If there are 400 pupils in the school, what percent are on a special diet?
5. In a village with 250 people, 16% of the population is over 60 years. How many people in the village are over 60 years of age?
6. In a factory, parts are made for a car. The diameter of the parts produced is 2.5 cm. The parts are rejected if the measured diameter deviates by more than 2%. Find the range in millimetres of the diameter of accepted parts.
7. Mrs. Nyirenda has a 400-acre farm and she has planted beans covering 30% of her land. How many acres are used for growing beans? (Hint—model the solution by picturing the whole field and covering 30% of it.)
8. Mr. Zondwayo got K500 off a neck-tie price that was originally K2000. What percent was taken off?

## Practice Activity 1

The tasks that follow require that you work with your pupils.

Using a metre-rule (enlarged), cover part of the metre stick. Ask pupils to write down the part covered as a fraction and as a decimal:

a) Tell pupils that another way to express the same part (proportion) is ‘percent’ – per hundred or for each 100. Write the percent, including the symbol, next to the common fraction and to the decimal.

b) Allow pupils to practice by covering several different ‘lengths’ and writing the parts as percents.

c) Extend this work to a $10 \times 10$ grid and let pupils practice with this model to convert percents to fractions and decimals and vice-versa.

## Practice Activity 2

In this activity, pupils will extend their understanding of percent by considering bases/groups that are not 100.

Begin by drawing a ten-grid rectangular shape on the board and shading three squares.

Ask pupils how much of the strip is shaded. Probable answers are 0.3 or $\frac{3}{10}$. Ask pupils what percent of the strip is shaded. What must they do to change $\frac{3}{10}$ to a fraction with a denominator of 100? (Multiply both the numerator and denominator by 10.)

Give further examples of the same type. Introduce the ratio method of calculation:

3 : 10 is equivalent to … : 100

or

$$\frac{3}{10} = \frac{?}{100}$$

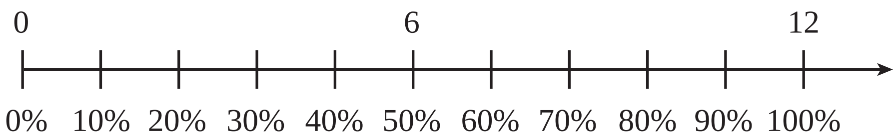
Introduce examples where the group or base is not a unit of 10, e.g.,

6 : 12 is equivalent to … : 100

or

$$\frac{6}{12} = \frac{?}{100}$$

This may be better shown to pupils by using the idea of a double number line. For example:




This demonstrates that if the unit of measure is 100, i.e., signifying the whole, then parts of it becomes ‘out of 100’. Thus, 6 is equivalent or is another way of expressing 50%. Alternatively, 6 could be seen as 50% of 12.

Make clear to the pupils that these may not always be ‘whole numbers’. For example,

Emusa played eight matches in a League game. Out of these they won five matches. What percent did they win?

$$\frac{5}{8} = \frac{?}{100}$$

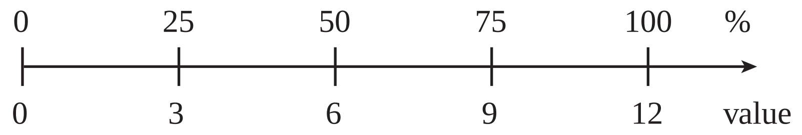
The idea of using a number line may not achieve the exact answer, though it would give an estimate or approximate one. The ratio method may be more appropriate.

$$\frac{5}{8} = \frac{x}{100}$$

$$x \times 8 = 5 \times 100$$

$$x = \frac{5 \times 100}{8} = \frac{5 \times 25}{2} = 62.5\%$$

The double number line model can be used to solve many percent questions. For example, using the following number line, the questions below can be dealt with easily.




What is 75% of 12?

Place 0 under 0% and 12 under 100%. The number that would be under 75% is 9.

The same model can be used to answer questions such as:

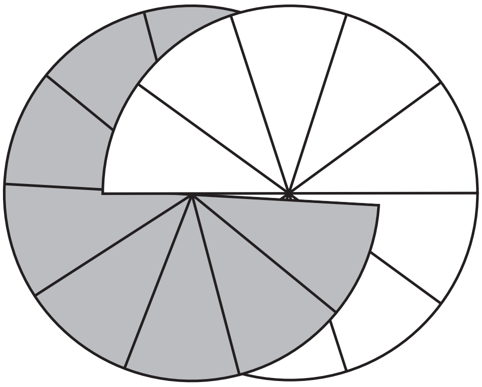
9 is 75% of what number?

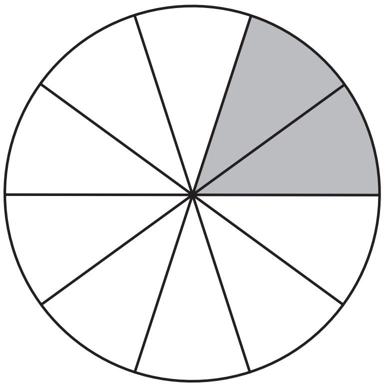
9 is what percent of 12?

Pupils can be asked to make a percent ruler to visualise this work.

**Circular Model**

Cut out two circles of manila of the same size but of different colour. Use a protractor to divide each into 10 equal sectors. Cut along one radius to the centre of the circle. Insert one circle inside the other so that rotating the white circle can expose anything from 0% to 100% of the coloured circle. Using the marked sides, pupils can count by 10% units to get a visual reinforcement of various percents. Using the unmarked sides, pupils can gain experience in visual estimation. The teacher should make the 'template' available to each pupil so that each pupil can make the 'percent circles'.



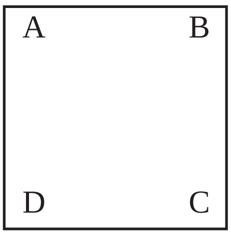


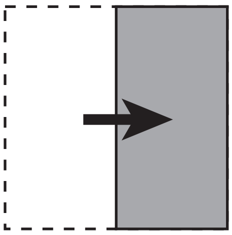
Rotate to show 20% shaded

## Practice Activity 3

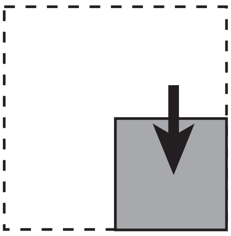
1. Make a percent model as illustrated above. Let pupils use it to gain experience in visual estimation.

2. Use paper folding to illustrate percent. Concrete activities provide valuable experience and allow you to teach percent using a geometric rather than an arithmetic model.

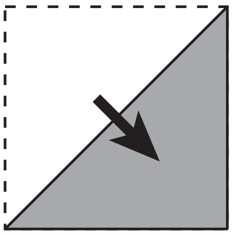


Fold A to B (50%)



Fold B to C (25%)



Fold A to C (50%)

What percent of the original square is remaining when:

| | |
|---|---|
| A is folded to the midpoint of AB | (75%) |
| A, B, C, and D are folded to the centre of the square | (50%) |
| A is folded to C, then B is folded to C | (37.5%) |

3. Without using calculators, pupils are to find the term that does not belong in each question, and explain their thinking.

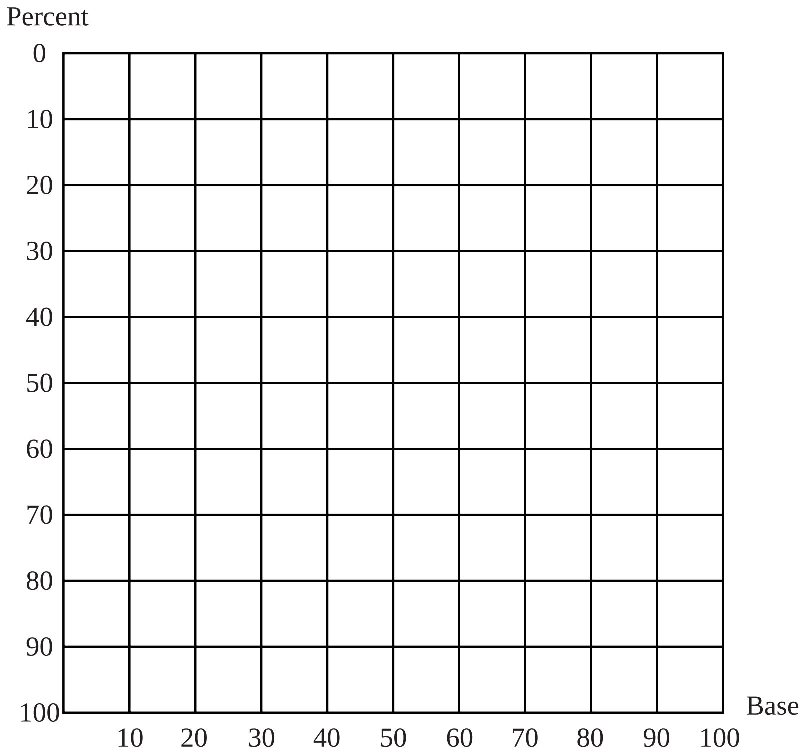
| | | | | |
|---|---|---|---|---|
| a) | $\frac{3}{4}$ | 0.75 | 0.34 | 75% |
| b) | 20% of 40 | 40% of 20 | 10% of 80 | 5% of 20 |
| c) | $3\frac{1}{4}$ | 0.325 | 3.25 | 325% |
| d) | 50 % of 18 | $18 \times 0.5$ | $\frac{1}{2}$% of 18 | 18% of 50 |

e) P100 at 6% for 2 years — P100 at 8% for 1 year
P100 at 2% for 4 years — P100 at 4% for 2 years

f) Percent change from 60 to 120 — Percent change from 66 to 99
Percent change from 40 to 80 — Percent change from 48 to 72

4. **Percent Chart**

A percent chart drawn on graph paper relates the topic of percent to proportions found in similar triangles.



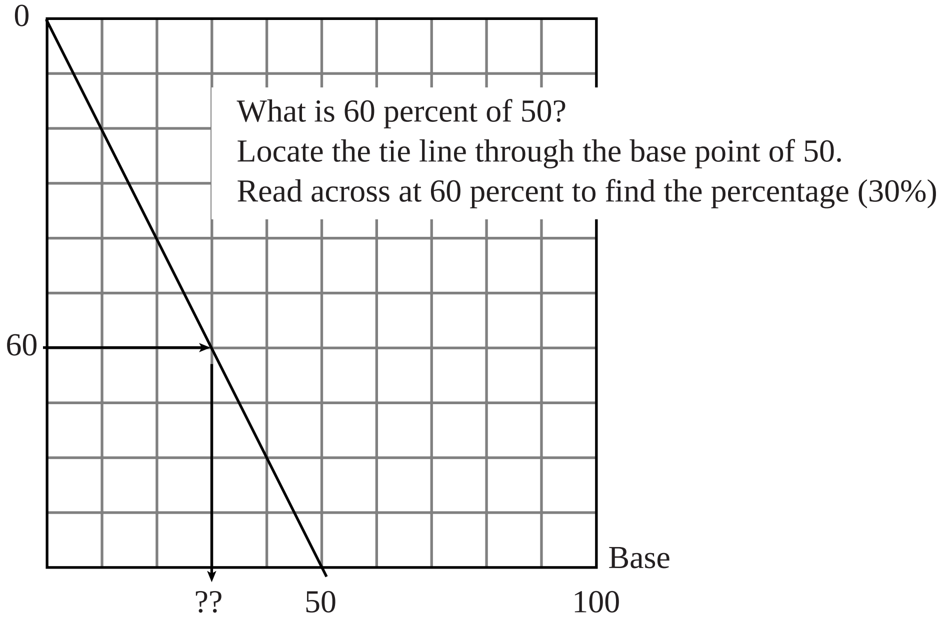
Pivot a ruler or straight edge at the 0% point and move it along the base to serve as the tie line:

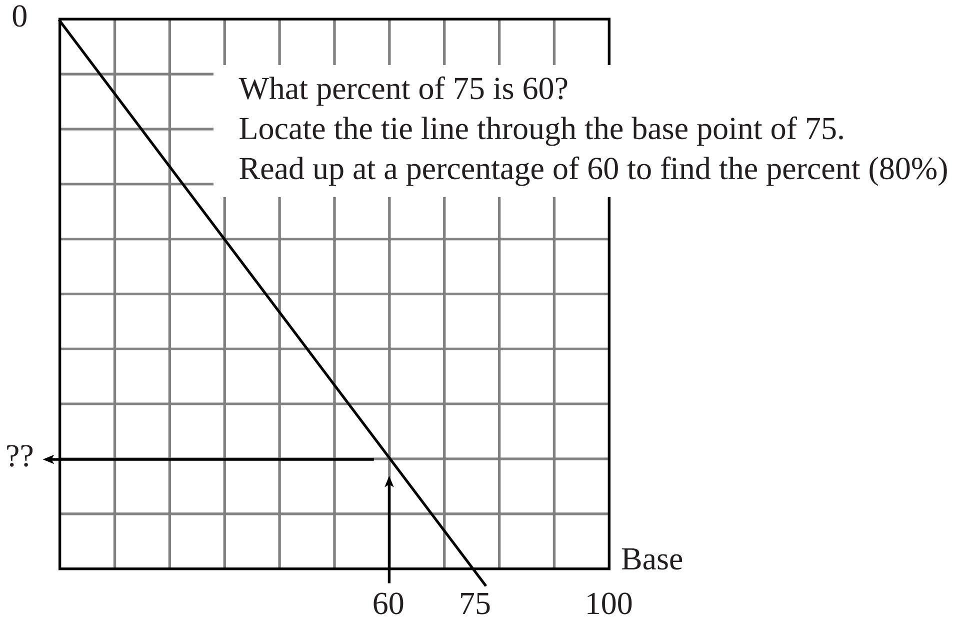
a) The percentage can be read given the rate and the base. For example, what is 60% of 50?

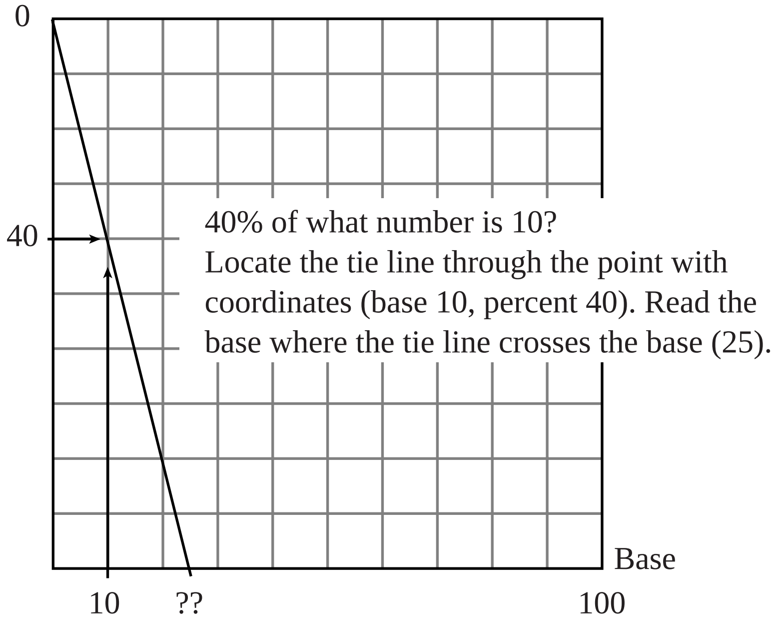
b) The rate can be read given the percentage and the base. For example, what percent of 75 is 60?

c) The base can be read given the rate and the percentage. For example, 40% of what number is 10?

The sketches below illustrate how to locate the answer in each case.






## Problems of Percent Increase and Decrease

In everyday life, you will come across situations of either increase in percents or decrease (or discount) in percent. Very common are increments to salaries or the cost of services or goods. It is important that pupils learn to make estimates of the reduced or increased price. To make estimates involving percents, it is useful to know the fractions' equivalent to the most common percents (and the other way round). The most common fractions with corresponding percents are listed in the following table. Note that 33.3% is an approximation, the other values are exact.

| fraction | $\frac{1}{10}$ | $\frac{1}{8}$ | $\frac{1}{5}$ | $\frac{1}{4}$ | $\frac{1}{3}$ | $\frac{3}{8}$ | $\frac{1}{2}$ | $\frac{5}{8}$ | $\frac{3}{4}$ | $\frac{7}{8}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| percent | 10% | 12.5% | 20% | 25% | 33.3% | 37.5% | 50% | 62.5% | 75% | 87.5% |

Example on making estimates:

a) What is the income tax at 18.5% on K1949?

Approximate 20% × K2000 = $\frac{1}{5}$ × K2000 = K 400

b) A watch that originally sold for \$198 is now priced at \$149. By what percent was the price reduced? The price was approximately reduced by 25%.

c) A radio sold for P149 a week ago. Now it is priced at P198. What is the increase percent? $\frac{50}{150} = \frac{1}{3}$ = 33.3% (1 dp).

d) A teacher got an increment of 15% on a salary of K150 000.00. What is her new salary? You could reason as follows:

100% represents K150 000, then an increment of 15% implies 115%, which we have to find;

i.e. $\frac{115}{100} \times 150\ 000.00$ = K172 500.00

The previous four examples illustrate one rule for percentage increase and decrease: the change is always expressed as a percent of the first number, not the second. So you would say “reduced from P198 by 20%” rather than “reduced by 20% to P149”.

## Reflection

1. Percent is out of 100, i.e., the whole quantity being considered is 100. However, it is not unusual to hear of 110% or more. How do you explain this?

2. Do you think it is helpful to have pupils memorise most common percents and their related fraction and decimal forms? For example:

| Decimal | Fraction | Percent |
|---|---|---|
| 0.1 | $\frac{1}{10}$ | 10% |
| 0.2 | $\frac{2}{10}$ | 20% |
| 0.25 | $\frac{1}{4}$ | 25% |
| 0.5 | $\frac{1}{2}$ | 50% |
| 0.70 | $\frac{70}{100}$ | 70% |
| 0.75 | $\frac{3}{4}$ | 75% |

## Practice Activity 4

Display a number chart ($10 \times 10$ grid) with the numbers 0 to 99. Ask pupils to estimate the percentage of numbers between 1 and 100 that:

- contain only even digits
- contain only odd digits
- contain both even and odd digits
- contain the digit 2
- contain digits that add to 10
- are even
- are multiplies of 3

## Practice Activity 5

Have pupils play the following game in pairs. Each pair is given a set of 10 cards. On the card a price and a discount percent are given. Write down the estimated price discounts, then use a calculator to find the exact discounted price. The pupil who has estimated the discounted price nearest to the calculated value scores a point. If both are near, each gets a point.

a) Some sample cards:

| | | | |
|---|---|---|---|
| \$80 | 12% | \$120 | 24% |
| \$20.75 | 20% | \$231 | 5% |
| \$245 | 35% | \$85.75 | 15% |

The same cards can be used again by taking the stated price as the price after the discount, or by pupils estimating the original price before the discount. A set of cards with cost price and selling price of articles can be used for estimating the percent profit.

b) 25% of the seats in a 60 000 seat stadium were not occupied. The easiest way to calculate the number of unoccupied seats is to convert the percent to a fraction. One quarter of the 60 000 seats is 15 000. Therefore, 15 000 seats were empty.

c) 7.8% interest is charged on a P1500 loan. The easiest way (with a calculator) to work out $0.078 \times 1500$ is through a conversion of the percent to a decimal.

d) The house rent of P560 was increased by 15%. What will be the price of the new rent? Most pupils will use "addition" properties: 10% will give P56. Halve this (for 5%) to give P28. Adding the two P56 + P28 = P84. The new house rent will be P560 + P84 = P644.

Sometimes the change in a given quantity is a reduction. For example, if an article costing \$60 000 is reduced by 5%, what will be its new price? Solution: we may proceed as in the case above. First obtain 5% of \$60 000 and then take this away from the original price.

$$\text{i.e., } \frac{5}{100} \times 60\,000 = \$3000$$
$$\$60\,000 - \$3\,000 = \$57\,000$$

Or we could proceed as follows: \$60 000 represents 100%, and a reduction of 5% implies we find 95% of \$60 000

$$\frac{95}{100} \times 60\,000 = \$57\,000$$

## Self Assessment 2

1. What percent of the whole numbers between 1 and 100 are:
   a) prime
   b) multiplies of four
   c) divisible by 5
2. Change
   a) $\frac{3}{7}$ into a percent
   b) 55% into a fraction
3. Lundazi reported that 65% of the total number of families in the district have enough food to last the month. If 450 families are in the district, how many have enough food?
4. The football team won 60% of the six games they played this season. How many games were lost?
5. During a sale a shop reduces its prices by 20%. What was the original price of a CD player now sold for P420?
6. What are the advantages of using a ratio table to solve questions involving percents and percentages?
7. 40% of the population of a village is male. If there are 800 males in the village, how many people live in the village?
8. A company increases its prices by 10% but due to the drop in the number of items sold, decides to drop the price again by 10%. Are they selling the items at the original price again? Justify your answer.
9. The price of petrol is increased by 5% and three months later, it is increased by 10%. Would it have made a difference if the first increase had been 10% and the second 5%? Justify your answer.
10. A farmer sold two cows for P1200 each. He made a profit of 10% on one, but lost 10% on the other. Did the farmer make a profit or loss on the transaction?
11. A shop prices its goods to make 25% profit. During a sale, the shop decides to give a 10% discount for cash sales. What is the percent profit the shop makes on cash sales?

## Summary

Percent is a ratio between some number and 100, and it is widely used in everyday life. Introduce pupils to percent by using concepts they have already learned, i.e., fractions and decimals. Emphasise the meanings of percent, base, and percentage, and to help your pupils see the relevance to everyday life, prepare problems using materials and situations from your local community.

## Unit Test

**Section A**

1. Chiyumba sells newspapers. She gets a commission of 2% on the total sales for the day. One Sunday, she sold papers worth K50 000. How much commission did she get?
2. A salesman bought a bicycle at \$60 and later sold it at \$65. What was his percent profit?
3. A lady's dress is normally priced at K150 000. What is the price during a sale when a 14.5% discount is allowed?
4. Tshupo is paid 4% commission on the value of all goods sold. What value of goods should he sell in order to earn a commission of \$540?
5. Mrs. Munga sold an article for \$32.50, making a profit of 8%.
   a) Calculate the original price of the article.
   b) For how much should she have sold it to make a profit of 20%?

**Section B**

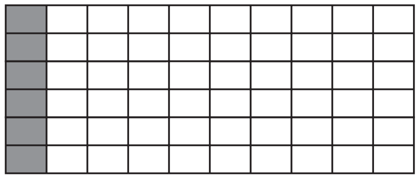
6. Plan and write down a 40-minute lesson that uses a metre stick graduated in millimetres to introduce percent to a Grade 5 class. Include such percents as 25%, 2.5%, and 0.25%.
7. Describe how you would teach percent of quantities whose bases are not 100.
   a) A school has an enrolment of 550 pupils. 44 of the pupils are wearing spectacles. What percent is this? Show how you expect a pupil to write down the solution to this question using
      (i) a ratio table
      (ii) double number line
   b) Using a ratio table, show how you expect a pupil to change $\frac{4}{5}$ to a percent.

## Answers for Self Assessments

***Self Assessment 1***

1. 45%
2. $6

10% →



3. $621
4. 6.25%
5. 40 people
6. Parts must be within 0.5 mm of 2.5 cm (or between 2.55 cm or 2.45 cm)
7. 120 acres have beans grown on it
8. 25% off

***Self Assessment 2***

1. a) 23%
   b) 25%
   c) 20%
2. a) 42.86%
   b) $\frac{11}{20}$
3. 292.5 families
4. 2 games (if won 67% games; otherwise they lost 2.4 games)
5. P525
7. 2000 people
8. The items are **not** selling at the original prices after the increase and decrease of prices by 10%. The drop of 10% is decreasing the prices by 10% of the inflated (or increased) prices and, thus, the prices are slightly less than the original prices.
9. The result would have been the same because multiplication is commutative. Thus it does not matter in which order the multiplication takes place to find the same result.
10. Because the farmer sold the two cows for the same price and made 10% profit of that price on one cow but lost 10% of that price on the other cow, the farmer broke even, neither losing nor gaining on the transaction.

## Answers for Unit 6 Test

1. K1000
2. 8.33%
3. K128 250
4. $13 500
5. a) $30.09 is the original price
   b) $36.11

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